23.5

(a) $E_i = K_i + U_i = \frac{1}{2}(0.0015\ \text{kg})(22.0\ \text{m/s})^2 + \frac{k(2.80\times10^{-6}\ \text{C})(7.50\times10^{-6}\ \text{C})}{0.800\ \text{m}}$

$= 0.608\ \text{J}$

$E_i = E_f = \frac{1}{2}mv_f^2 + \frac{kq_1q_2}{r_f}$

$\therefore v_f = \sqrt{\frac{2(0.608\ \text{J} - 0.491\ \text{J})}{0.0015\ \text{kg}}} = 12.5\ \text{m/s}$

(b) At the closest point, the velocity is zero:

$0.608\ \text{J} = \frac{kq_1q_2}{r} \Rightarrow r = \frac{k(2.80\times10^{-6}\text{C})(7.8\times10^{-6}\text{C})}{0.608\ \text{J}}$

$= 0.323\ \text{m}$

23.14

(a) $W = -\Delta U = qEd = \Delta K = 1.50\times10^{-6}\ \text{J}$

(b) Initial point was at higher potential since a positive charge moves toward lesser potential.

$\Delta V = \Delta U/q = \frac{(1.50\times10^{-6}\ \text{J})}{4.20\ \text{nC}} = 357\ \text{V}$

(c) $qEd = 1.50\times10^{-6}\ \text{J} \Rightarrow E = \frac{1.50\times10^{-6}\ \text{J}}{(4.20\ \text{nC})(0.06\ \text{m})}$

$= 5.45\times10^{3}\ \text{N/C}$

23.57

(a) Don't double count!

$$U = kq^2\left(-\frac{1}{d} + \frac{1}{\sqrt{2}d} - \frac{1}{d} - \frac{1}{d} + \frac{1}{\sqrt{2}d} - \frac{1}{\sqrt{3}d} + \frac{1}{\sqrt{2}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} + \frac{1}{\sqrt{2}d} - \frac{1}{d} + \frac{1}{\sqrt{2}d} + \frac{1}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} - \frac{1}{d} + \frac{1}{\sqrt{2}d} - \frac{1}{\sqrt{3}d} + \frac{1}{\sqrt{2}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} + \frac{1}{\sqrt{2}d} - \frac{1}{\sqrt{3}d} + \frac{1}{\sqrt{2}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} - \frac{1}{d} + \frac{1}{\sqrt{2}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} + \frac{1}{\sqrt{2}}d\right)$$

$$+ kq^2\left(-\frac{1}{d}\right)$$

$$U = kq^2\left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d}\right)$$

$$= -\frac{5.824\,q^2}{\pi \varepsilon_0 d}$$

(b) A negative energy means the ions are bound to each other. It would take energy to break them up.

23.73

(a) $$dV_p = \frac{k\,dq}{z+x} = \frac{kQ}{a}\frac{dz}{z+x}$$

$$V = \frac{kQ}{a}\int_0^a \frac{dz}{z+x}$$

$$= \frac{kQ}{a}\ln\left(\frac{x+a}{x}\right) = \frac{kQ}{a}\ln\left(1+\frac{a}{x}\right)$$

(b) $$dV_R = \frac{kQ}{a}\frac{dz}{r} = \frac{kQ}{a}\frac{dz}{\sqrt{z^2+y^2}}$$

$$V_R = \frac{kQ}{a}\int_0^a \frac{dz}{\sqrt{z^2+y^2}} = \frac{kQ}{a}\ln\left(\frac{\sqrt{a^2+y^2}+a}{y}\right)$$

(c) $\underline{x \gg a}$: $V_p \simeq \frac{kQ}{a}\frac{a}{x} = \frac{kQ}{x}$, since

$\ln(1+\alpha) \simeq \alpha$.

$\underline{y \gg a}$: $V_R \approx \frac{kQ}{a}\frac{a}{y} = \frac{kQ}{y}$

since $\ln\left(\frac{\sqrt{a^2+y^2}+a}{y}\right)$

$\approx \ln\left(\frac{y+a}{y}\right) = \ln\left(1+\frac{a}{y}\right) \approx \frac{a}{y}$

(a) $\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

$$\boxed{\vec{E} = -2xA\hat{i} + 3yA\hat{j} - 2zA\hat{k}}$$

(b) $W = \int_{\vec{a}}^{\vec{b}} q\vec{E}\cdot d\vec{s}$

$\vec{a} = (0, 0, 0.250\text{m})$

$\vec{b} = (0, 0, 0\text{m})$

$$\therefore W = \int_{\vec{a}}^{\vec{b}} (1.50\times10^{-6}\text{C})(-2xA\hat{i} + 3yA\hat{j} - 2zA\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= (1.50\times10^{-6}\text{C})A\int_{\vec{a}}^{\vec{b}}(-2x\,dx + 3y\,dy - 2z\,dz)$$

$$= (1.50\times10^{-6}\text{C})A\left[-x^2\Big|_0^0 + \frac{3}{2}y^2\Big|_0^0 - z^2\Big|_{0.250\text{m}}^0\right]$$

$$= (1.50\times10^{-6}\text{C})A(0.250\text{m})^2$$

$$= 9.375\times10^{-8}\ \text{C A m}^2$$

$$= 6.00\times10^{-5}\ \text{J}$$

$\therefore$ $\boxed{A = 6.4\times10^{2}\ \text{J/C m}^2}$ (also volt/m^2)

(c) $\vec{E}(0,0,0.250\text{m}) = -3.2\times10^{2}\,\hat{k}$ volt/m

(d) For y = constant, $V = \text{const} + A(x^2+z^2)$

So if V = constant, then $x^2+z^2 = \text{const}$ = circle.

(e) $[1280\text{ V} + 6.4\times10^2 \text{ volt/m}^2 \times 3\times(2.00\text{m})^2] \div 6.4\times10^2 \text{ volt/m}^2$

$= 1.4\times10\ \text{m}^2$, So radius $= 3.742$ m