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Aharonov–Bohm/Casher effect in a Kondo ring

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Abstract

The influence of a magnetic impurity or ultrasmall quantum dot on the spin and charge persistent currents of a mesoscopic ring is investigated. The system consists of electrons in a one-dimensional ring threaded by spin-dependent Aharonov–Bohm/Casher fluxes, and coupled via an antiferromagnetic exchange interaction to a localized electron. The problem is mapped onto a Kondo model for the even-parity channel plus free electrons in the odd-parity channel. The twisted boundary conditions representing the fluxes couple states of opposite parity unless the twist angles ϕ_α satisfy $\phi_\alpha = f_\alpha \pi$, where f_α are integers, with spin index $\alpha = \uparrow, \downarrow$. For these special values of ϕ_α, the model is solvable by the Bethe ansatz.

Keywords: Bethe ansatz; Kondo effect; Persistent current; Quantum dots

Kondo physics of quantum dots is of considerable current interest, both experimentally [1,2] and theoretically [1]. An important problem is to understand how the Aharonov–Bohm/Casher (ABC) effect [3,4] in a ring coupled to a quantum dot [5], and hence e.g. persistent currents [6–9], would be modified by the many-body correlations present in the Kondo regime.

To address this question here, we study a simple model where electrons in a one-dimensional (1D) ring threaded by a magnetic flux and a charged string are coupled via antiferromagnetic spin exchange to a localized electron, representing a magnetic impurity or an ultrasmall quantum dot. A detailed analysis shows that this model can be mapped onto the integrable Kondo model for special values of the magnetic flux and the charge on the string, allowing for an exact solution of the problem.

The system is described by the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_\alpha \int_0^L \mathrm{d}x \psi_\alpha^\dagger(x)\partial_x^2\psi_\alpha(x) + \lambda \sum_{\alpha,\beta} \psi_\alpha^\dagger(0)\boldsymbol{\sigma}_{\alpha\beta}\psi_\beta(0)\cdot \boldsymbol{S}, \quad (1)$$

where $\lambda > 0$ is an antiferromagnetic Kondo coupling, L is the ring's circumference, and $\boldsymbol{S}$ is the impurity spin (located at $x = 0$), and ψ_α is an electron field with spin index $\alpha = \uparrow, \downarrow$. The effect of the magnetic flux and the line charge threading the ring has been gauged away [6] and encoded in twisted boundary conditions $\psi_\alpha(L) = e^{i\phi_\alpha}\psi_\alpha(0)$. Here $\phi_{\uparrow,\downarrow} = 2\pi(\Phi/\Phi_0 \pm 4\pi\tau/F_0)$, where Φ is the magnetic flux enclosed by the ring and τ the line charge density of a charged string passing through the center of the ring. $\Phi_0 = hc/e$ is the elementary flux quantum, with $F_0 = hc/\mu$ its dual.

We are interested in an exact solution of the problem, in particular, how the Kondo interaction may affect the spin and charge persistent currents induced by the phase shifts ϕ_α. Since the essential physics of the system is confined to a small region around the left and right Fermi points, we can linearize around $\pm k_\mathrm{F}$ and introduce left (l) and right (r) moving chiral fields. After introducing a basis of definite parity fields $\psi_{\mathrm{e/o},\alpha}(x) = (\psi_{\mathrm{r},\alpha}(\pm x) \pm \psi_{\mathrm{l},\alpha}(\mp x))/\sqrt{2}$, the Hamiltonian becomes $H = H_0^\mathrm{o} + H_0^\mathrm{e} + H_\mathrm{imp}^\mathrm{e}$, where

$$H_0^{\mathrm{e/o}} = \mp \frac{v_\mathrm{F}}{2\pi}\sum_\alpha \int_0^L \mathrm{d}x \psi_{\mathrm{e/o},\alpha}^\dagger(x) \mathrm{i}\partial_x \psi_{\mathrm{e/o},\alpha}(x),$$

describe independent relativistic electrons, and the impurity contribution is now diagonal:

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$$H^{e}_{imp} = \lambda \sum_{\alpha,\beta} \psi^{\dagger}_{e,\alpha}(0)\boldsymbol{\sigma}_{\alpha\beta}\psi_{e,\beta}(0)\cdot\boldsymbol{S}. \quad (2)$$

We recognize $H^{e}_{K} \equiv H^{e}_{0} + H^{e}_{imp}$ as the chiral Hamiltonian of the spin-S Kondo model.

While the even and odd parity channels are decoupled in the Hamiltonian, the twisted boundary conditions couple states of opposite parity. However, for the special values $\phi_\alpha = f_\alpha \pi$, where f_α are integers, the even and odd parity states decouple from each other entirely. One can then solve H^{e}_{K} by the Bethe ansatz (For a recent review see [10]). Thus, our original problem has collapsed to an exactly solvable problem for $f_\alpha \in Z$, consisting of a left-moving odd-parity branch of independent relativistic electrons, together with a (decoupled) right-moving even-parity branch defined by the 1D Kondo model. For generic values of ϕ_α it is not possible to choose a basis which renders H and the boundary conditions simultaneously diagonal, strongly suggesting that the model is not integrable in general.

The leading mesoscopic behavior of the charge (spin) persistent current is then $I(\phi) = -D\phi/L$ where $D = D_c(D_s)$ is the corresponding charge (spin) stiffness. This equation holds on general grounds independent of whether the model is integrable or not. The stiffness constants can be determined by the Bethe ansatz for finite systems, developed previously for the 1D Hubbard model [11,12].

The charge stiffness, for example, can be evaluated as a finite difference using the integrable points $f_\uparrow = f_\downarrow = 0, 1$. When $\phi_\uparrow = \phi_\downarrow = \phi$, we find $D_c = ev_F/\pi$, unaffected by the Kondo scattering. This indicates that, for spin-independent fluxes, spin-charge separation holds even at the mesoscopic scale in this model.

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