PHYSICAL REVIEW B VOLUME 56, NUMBER 8 15 AUGUST 1997-II

Interaction-induced enhancement and oscillations of the persistent current

C. A. Stafford*
Institut de Physique Théorique, Université de Fribourg, CH-1700 Fribourg, Switzerland

D. F. Wang
Institut de Physique Théorique, EPF Lausanne, CH-1015 Lausanne, Switzerland

(Received 21 May 1997; revised manuscript received 19 June 1997)

The persistent current I in integrable models of multichannel rings with both short- and long-ranged interactions is investigated. I is found to oscillate in sign and increase in magnitude with increasing interaction strength due to interaction-induced correlations in the currents contributed by different channels. For sufficiently strong interactions, the contributions of all channels are found to add constructively, leading to a giant enhancement of I. Numerical results confirm that this parity-locking effect is robust with respect to intersubband scattering. [S0163-1829(97)50236-0]

The absence of macroscopic persistent currents in normal metal rings even at $T=0$ is a consequence of a parity effect stemming from Fermi statistics, first discussed by Byers and Yang:[1] For N spinless electrons in a purely one-dimensional (1D) ring, the persistent current I is diamagnetic if N is even and paramagnetic if N is odd, independent of disorder and interactions,[2,3] and takes a maximum value $I_0=ev_F/L$ for a clean ring, where v_F is the Fermi velocity and L the circumference of the ring. In a ring with many independent channels, I is the sum of many such diamagnetic and paramagnetic contributions, and is thus very small.[1] Büttiker, Imry, and Landauer[4] argued that persistent currents should, nonetheless, exist at the mesoscopic level, and calculations with noninteracting electrons[5] predicted typical currents of order $I_0\ell/L$ for a diffusive metallic ring, where $\ell \ll L$ is the elastic mean free path. Subsequently, persistent currents were observed experimentally in both normal metal[6,7] and semiconductor[8,9] rings. Surprisingly, the persistent currents observed in metallic rings were roughly two orders of magnitude larger than those predicted by theories neglecting electron-electron interactions,[5,10] being of order I_0 in individual rings.[7] Mean-field calculations[11–13] for multichannel rings, renormalization group results for 1D rings,[14] and exact diagonalization studies[15] found that the persistent current in disordered systems can be enhanced by repulsive interactions due to the suppression of charge fluctuations, but the very large experimentally observed magnitude of I is still generally considered to be a mystery.

In this paper, we propose a nonperturbative interaction effect which can lead to a large enhancement of I in multichannel rings, *even in the ballistic regime*, due to the fact that the parities of different channels are no longer independent in an interacting system. We investigate two integrable models of interacting M-channel rings, $SU(M)$ fermions with inverse-square $V(x)=g/d(x)^2$ and delta-function $V(x)=U\delta(x)$ interactions. In the prior model, we show that for $g>0$ the persistent currents of all channels add constructively, provided $k_FL>2\pi M$, leading to a large persistent current whose parity depends only on the total number of electrons N. In the $SU(M)$ delta-function gas, I is found to oscillate in sign and increase in magnitude with increasing U due to a progressive condensation of electrons into the lowest subband, leading to a parity-locking effect for $U>16M^4\hbar v_F/3\pi$, where v_F is the Fermi velocity. A disordered two-channel ring with interchannel interactions is also investigated numerically, and shows, importantly, that the parity-locking effect persists even when the subbands are mixed strongly by disorder. Qualitatively, the parity-locking effect arises in a thin ring because for sufficiently strong repulsive interactions, electrons can no longer pass each other. The elimination of transverse nodes in the many-body wave function (which are necessary for two electrons to pass each other) leads to a state whose parity depends only on the total number of electrons, as discussed by Leggett,[2] and whose persistent current is consequently large.

Spinless electrons in a nondisordered ring of arbitrary cross section with M transverse channels, threaded by an Aharanov-Bohm flux $(\hbar c/e)\phi$, may be represented by 1D $SU(M)$ fermions. The transverse degrees of freedom may be represented by an $SU(M)$ spin variable $\sigma=1,\ldots,M$. Disorder and interactions will in general lead to intersubband scattering, which breaks this $SU(M)$ symmetry. In order to preserve the integrability of the model, we consider a special class of interactions without intersubband scattering, i.e., interactions which depend only on the electrons' coordinates along the ring. Intersubband scattering will be shown not to alter the results obtained for these integrable models in a fundamental way. The Hamiltonian of the system is

$$H=-\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+\sum_{i<j}V(x_i-x_j)+\sum_{\sigma=1}^{M}K_\sigma\varepsilon_\sigma, \qquad (1)$$

where K_σ is the number of electrons in subband σ, $N=\Sigma_\sigma K_\sigma$, and ε_σ is the energy minimum of subband σ. Units with $\hbar=m=1$ are used. The Aharanov-Bohm flux leads to the twisted boundary condition[1]

$$\Psi(x_1\sigma_1,\ldots,(x_i+L)\sigma_i,\ldots,x_N\sigma_N) = e^{i\phi}\Psi(x_1\sigma_1,\ldots,x_i\sigma_i,\ldots,x_N\sigma_N). \qquad (2)$$

For simplicity, let us consider equally spaced subbands $\varepsilon_{\sigma+1}-\varepsilon_\sigma=\Delta\equiv E_F/M$. The subband splitting Δ plays the

0163-1829/97/56(8)/4383(4)/$10.00

role of an $SU(M)$ magnetic field. As we shall see, the effect of repulsive interactions is to renormalize this effective field, causing a condensation of electrons into the lowest subband.

At $T=0$, the equilibrium persistent current is given by $I(\phi)=-(e/\hbar)\partial E_0/\partial\phi$, where $E_0(\phi)$ is the ground-state energy of Eq. (1), subject to the boundary condition (2). I is a periodic function of ϕ with period 2π, and may thus be expressed as a Fourier series,

$$I(\phi)=\sum_{n=1}^{\infty} I_n \sin(n\phi). \tag{3}$$

The value of I at $\phi=\pi/2$ (1/4 flux quantum) is determined by the odd harmonics, $I(\phi_0/4)=I_1-I_3+I_5-\cdots$, and may be taken as a measure of the first harmonic, assuming the higher odd harmonics are small. For $V(x)=0$, one finds

$$I(\phi_0/4)=\frac{e\hbar\pi}{2mL^2}\sum_{\sigma=1}^{M}(-1)^{K_\sigma}K_\sigma. \tag{4}$$

For $M\gg 1$, this leads to the well-known[5] result $|I|\sim M^{1/2}I_0$ due to the random parities of the different channels. The system may be either diamagnetic or paramagnetic, depending on the channel occupancies K_σ.

Let us next consider a model with long-range interactions: $V(x)=g/d(x)^2$, where $d(x)=(L/\pi)|\sin(\pi x/L)|$ is the chord length along the ring. This model was introduced and solved by Sutherland[16] for the case $M=1$ and $\phi=0$, and can be shown to be integrable[17] for arbitrary M, ϕ, and $g\geqslant 0$. For $g>0$, the ground state is highly degenerate in the limit $L\to\infty$ in the absence of $SU(M)$ symmetry breaking ($\Delta=0$) due to the strong repulsion of the potential at the origin, which prohibits particle exchange. In a finite ring, these states differ in energy by at most $\pi\hbar v_F/L$ due to boundary effects; all electrons will thus be condensed into the lowest subband for $\Delta>\pi\hbar v_F/L$, i.e., for $k_FL>2\pi M$, which is satisfied provided the ring is sufficiently thin. The ground state of the system in this ''ferromagnetic'' state has the Jastrow product form

$$\Psi(\{x\})=\exp\left(i\frac{\phi-a}{L}\sum_{k=1}^{N}x_k\right)\times\prod_{1\leqslant i<j\leqslant N}\left|\sin\left(\frac{x_i-x_j}{L}\pi\right)\right|^{\lambda}\sin\left(\frac{x_i-x_j}{L}\pi\right) \tag{5}$$

for $0\leqslant\phi\leqslant\pi$, where $a=0$ if N is odd and $a=\pi$ if N is even. Here $\lambda=\sqrt{g+1/4}-1/2$. One readily verifies that Ψ is an eigenstate of Eq. (1), has the correct symmetry, and obeys the twisted boundary condition (2). This eigenstate is a positive vector when the particles are ordered, and is therefore the ground state of the system. The ground-state energy is found to be

$$E_0(\phi)=\frac{\pi^2(\lambda+1)^2N(N^2-1)}{6L^2}+\frac{N}{2}\left(\frac{\phi-a}{L}\right)^2. \tag{6}$$

The corresponding persistent current is

$$I(\phi_0/4)=(-1)^N\frac{e\hbar\pi N}{2mL^2}\sim(-1)^N MI_0. \tag{7}$$

The condensation of all electrons into the lowest subband caused by the strong repulsive interactions thus leads to an enhancement of the typical persistent current by a factor of $M^{1/2}$ in the ballistic regime, due to the suppression of the parity effect of Byers and Yang.

Let us next consider a model with short-ranged interparticle interactions $V(x)=U\delta(x)$. This system is integrable for arbitrary U, and the eigenenergies with twisted boundary conditions may be determined from a straightforward generalization of the nested Bethe ansatz of Sutherland[18] to the case $\phi\neq 0$. The energy of the system may be expressed as

$$E=\sum_{j=1}^{N}k_j^2/2+\sum_{\sigma=1}^{M}K_\sigma\varepsilon_\sigma, \tag{8}$$

where the pseudomomenta k_j are a set of N distinct numbers which satisfy the coupled equations

$$\exp[i(Lk_j-\phi)]=\prod_{\alpha=1}^{N-K_1}\frac{k_j-\Lambda_\alpha^{(1)}+iU/2}{k_j-\Lambda_\alpha^{(1)}-iU/2}, \tag{9}$$

$$\prod_{\beta=1}^{N_{n-1}}\frac{\Lambda_\alpha^{(n)}-\Lambda_\beta^{(n-1)}-iU/2}{\Lambda_\alpha^{(n)}-\Lambda_\beta^{(n-1)}+iU/2}\prod_{\gamma=1}^{N_{n+1}}\frac{\Lambda_\alpha^{(n)}-\Lambda_\gamma^{(n+1)}-iU/2}{\Lambda_\alpha^{(n)}-\Lambda_\gamma^{(n+1)}+iU/2}=-\prod_{\beta=1}^{N_n}\frac{\Lambda_\alpha^{(n)}-\Lambda_\beta^{(n)}-iU}{\Lambda_\alpha^{(n)}-\Lambda_\beta^{(n)}+iU},\quad n=1,\ldots,M-1, \tag{10}$$

where $\Lambda_\alpha^{(n)}, \alpha=1,\ldots,N_n=N-\Sigma_{\sigma=1}^{n}K_\sigma$ are distinct numbers, with $\Lambda_j^{(0)}=k_j$. For $\Delta=0$, the ground state is an $SU(M)$ singlet when N is an odd multiple of M. As Δ increases, electrons in the higher subbands are transferred to lower subbands, until all electrons are condensed into the lowest subband for $\Delta>\Delta_c$. This phenomenon is analogous to the spin-polarization transition of the 1D Hubbard model in a magnetic field, studied by Carmelo *et al.* and by Frahm and Korepin.[19] Using the techniques of Ref. 19, one finds

$$\Delta_c=(1/4\pi)[U^2+(2\pi n)^2]\tan^{-1}(2\pi n/U)-Un/2, \tag{11}$$

where $n=N/L$. For $\Delta>\Delta_c$, the system is in a parity-locked state, with persistent current given by Eq. (7). It is useful to consider some limiting cases of Eq. (11): Using $k_F\simeq\pi n/M$, one finds $\Delta_c\simeq M^2E_F$ for $U=0$ and $\Delta_c\simeq 16M^3E_Fv_F/3\pi U$ for $U/v_F\gg M$. For fixed Δ, the critical interaction strength required to enforce parity locking is thus

$$U_c\simeq 16M^4v_F/3\pi. \tag{12}$$

For $M\gg 1$, very strong interactions are thus required to cause complete parity locking, which would lead to a *macroscopic* persistent current. This result is in contrast to that for the preceding model, which exhibited parity locking for any value of the interaction $g>0$, provided the ring was sufficiently thin.

In order to see how the parity-locking effect develops as a function of interaction strength, let us consider the simple case of a two-channel ring with N odd; then the parities of the two channels are necessarily opposite. For mesoscopic rings, $SU(2)$ excitations of the type considered by Kusmartsev[20] and by Yu and Fowler,[21] which lead to a ϕ_0/N periodicity of the persistent current, can be neglected.[21] The persistent current for $M=2$ is then given by

$$I(\phi_0/4)=(-1)^{K_1}(K_1-K_2)I_0/N. \tag{13}$$

For $\Delta \ll \Delta_c$, the polarization $(K_1-K_2)/L \simeq 2\chi\Delta$, where the susceptibility χ may be evaluated from Eqs. (8)–(10) using the method of Shiba;[22] one obtains $\chi=2/\pi v_F$ for $U=0$ and $\chi \simeq 3U/(2\pi v_F)^2$ for $U \gg v_F$. The magnitude of the persistent current is thus given by $|I| \simeq 4e\Delta/\pi\hbar N$ for $U=0$, and is increased by the factor $3U/8\pi v_F$ for $v_F \ll U \ll U_c$. As U is increased, the persistent current thus oscillates in sign and grows roughly linearly in magnitude due to the progressive transfer of electrons from the upper to the lower subband. For $M \gg 1$, the evolution of the system toward the parity-locked state as U is increased will of course be more complicated, but one nonetheless expects I to fluctuate in sign and increase in magnitude as electrons condense into the lowest subband.

A peculiarity of the integrable models considered up to this point is that the number of electrons in each channel is a constant of the motion. Both disorder and more realistic interactions which depend on the transverse coordinate will break this symmetry, and it is therefore important to verify that the parity-locking effect is not destroyed. To this end, we have considered a disordered two-channel ring, modeled in the tight-binding approximation, with a nearest-neighbor interchain interaction V included to induce interchannel correlations.[23] The Hamiltonian is

$$H=\sum_{i=1}^{L}\left[\sum_{\alpha=1}^{2}(e^{i\phi/L}c_{i\alpha}^{\dagger}c_{i+1\alpha}+\text{H.c.}+\varepsilon_{i\alpha}\rho_{i\alpha}) + \frac{\Delta}{2}(c_{i1}^{\dagger}c_{i2}+\text{H.c.})+V\rho_{i1}\rho_{i2}\right], \tag{14}$$

where $c_{i\alpha}^{\dagger}$ creates a spinless electron at site i of chain α, $\rho_{i\alpha}\equiv c_{i\alpha}^{\dagger}c_{i\alpha}$, and $\varepsilon_{i\alpha}$ is a random number in the interval $[-\varepsilon/2,\varepsilon/2]$. The interchain hopping determines the subband splitting Δ between the symmetric and antisymmetric states. The rms amplitude of the intersubband scattering is $\varepsilon/2\sqrt{3}$, and the expectation value of the difference in subband populations is given by

$$\langle K_1-K_2\rangle=-2\partial E_0/\partial\Delta. \tag{15}$$

Figure 1 shows the average subband occupancies and persistent current for an ensemble of 100 rings with 5 spinless electrons on 18 sites as a function of V, calculated using the Lanczos technique. The subband splitting $\Delta=0.8$ is chosen so that in the absence of disorder and interactions, $K_1=3$ and $K_2=2$, leading to a large cancellation of the persistent current due to the different parities of the two channels. The on-site disorder $\varepsilon=2>\Delta$, E_F mixes the two channels, but does not lead to strong backscattering (localization). Figure 1(a) indicates that the average difference in subband occupancies increases from 1.05 at $V=0$ to 4.55 at $V=20$, corresponding to a 96% condensation into the lowest subband. As the intersubband polarization increases, the first harmonic of the persistent current oscillates in sign and increases in magnitude, similar to the behavior of the integrable system given in Eq. (13). In contrast, the second harmonic remains paramagnetic, and does not exhibit a noticeable increase. The statistical width of the current distribution is $\delta I=(\int_0^{2\pi}[\langle I(\phi)^2\rangle-\langle I(\phi)\rangle^2]d\phi/2\pi)^{1/2}=(0.125\pm.01)I_0$ for the whole range of V. Note that $|\langle I_1\rangle| \ll \delta I$ for $V=0$, while $\langle I_1\rangle$ is diamagnetic with $|\langle I_1\rangle| \gg \delta I$ for $V>5$, as expected for a system with N odd due to parity locking.

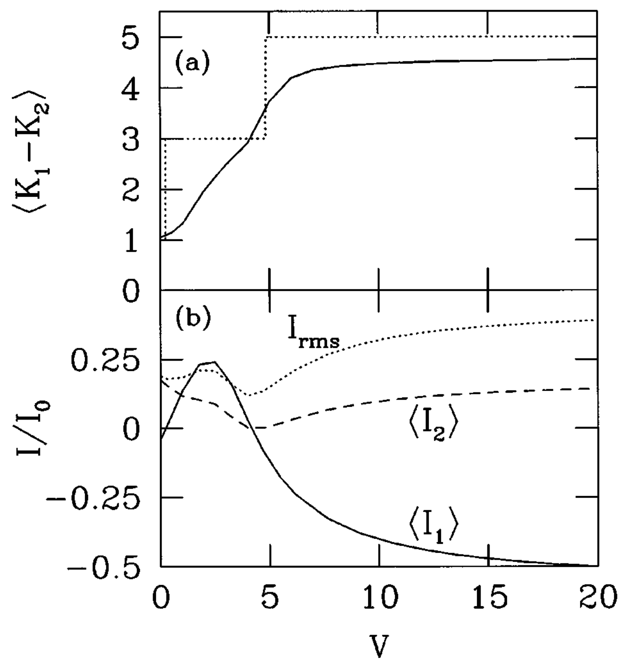

FIG. 1. Results for an ensemble of 100 disordered two-channel rings with 5 spinless electrons on 18 sites as a function of the interchain interaction V. The subband splitting is $\Delta=0.8$. (a) Difference of average subband occupancies $\langle K_1-K_2\rangle=-2\partial E_0/\partial\Delta$ at $\phi=\phi_0/4$ for disorder amplitude $\varepsilon=2$ (solid curve) and $\varepsilon=0$ (dotted curve). (b) Average first and second harmonics $\langle I_1\rangle$ and $\langle I_2\rangle$ of the persistent current and $I_{\rm rms}=(\int_0^{2\pi}\langle I(\phi)^2\rangle d\phi/2\pi)^{1/2}$ for $\varepsilon=2$. The relation of $\langle I_1\rangle$ to $\langle K_1-K_2\rangle$ is qualitatively similar to that in the model with unbroken $SU(2)$ symmetry, Eq. (13).

Figure 1 shows that the parity-locking effect is not significantly modified even in systems where the intersubband scattering is comparable to the subband splitting. While the subband occupancies are no longer constants of the motion in a disordered system, there is a corresponding topological invariant, namely, the number of transverse nodes in the many-body wave function[2] (i.e., nodes which encircle the AB flux ϕ). The lowest subband has no such nodes, while each electron in the second subband contributes one transverse node. In order for two electrons to pass each other as they circle the ring, such a transverse node must be present. As V increases, it becomes energetically unfavorable for electrons to approach each other, so transverse nodes in the many-body wave function will tend to be suppressed. In the strongly correlated limit, all such nodes will be eliminated, leading to a state whose parity depends only on the *total* number of electrons. In such a state, the persistent currents of all channels add constructively, leading to a large enhancement of I_1.

In conclusion, we have proposed an interaction effect which leads to a large enhancement of the persistent current (and in particular, of its first harmonic) in multichannel rings due to correlations in the contributions of different channels. Sufficiently strong interactions were shown to lead to an enhancement of the typical persistent current by a factor of $M^{1/2}$ in a ballistic ring with M channels, compared to the value for noninteracting electrons. It was shown that even when interactions are weak compared to those necessary to enforce complete parity locking, as is likely to be the case in metallic rings such as those studied in Refs. 6 and 7, the persistent current may still be substantially enhanced by interchannel correlations. It should be emphasized that the parity-locking effect is a pure interaction effect in multichannel rings; disorder plays no essential role. It is therefore complementary to mechanisms previously proposed for the enhancement of the persistent current,[11–15] which rely on the competition between disorder and interactions. It is likely that both such mechanisms are important to explain the anomalously large observed value[6,7] of the persistent current in disordered metallic rings.

C.A.S. thanks M. Büttiker and F. Hekking for valuable discussions. This work was supported by the Swiss National Science Foundation.

*Present address: Fakultät für Physik, Albert-Ludwigs-Universität, Hermann-Herder-Str. 3, D-79104 Freiburg, Germany.

[1] N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 46 (1961).

[2] A. J. Leggett, in *Granular Nanoelectronics*, edited by D. K. Ferry *et al.* (Plenum, New York, 1991), p. 297.

[3] D. Loss, Phys. Rev. Lett. **69**, 343 (1992).

[4] M. Buttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).

[5] H. F. Cheung, E. K. Riedel, and Y. Gefen, Phys. Rev. Lett. **62**, 587 (1989).

[6] L. P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).

[7] V. Chandrasekhar *et al.*, Phys. Rev. Lett. **67**, 3578 (1991).

[8] D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993).

[9] B. Reulet, M. Ramin, H. Bouchiat, and D. Mailly, Phys. Rev. Lett. **75**, 124 (1995).

[10] H. Bouchiat and G. Montambaux, J. Phys. (France) **50**, 2695 (1989); F. von Oppen and E. K. Riedel, Phys. Rev. Lett. **66**, 84 (1991); B. L. Altshuler, Y. Gefen, and Y. Imry, *ibid.* **66**, 88 (1991).

[11] U. Eckern, Z. Phys. B **82**, 393 (1991).

[12] A. Schmid, Phys. Rev. Lett. **66**, 80 (1991).

[13] D. Yoshioka and H. Kato, Physica B **212**, 251 (1995); G. Bouzerar and D. Poilblanc, J. Phys. (France) (to be published).

[14] T. Giamarchi and B. S. Shastry, Phys. Rev. B **51**, 10 915 (1995); H. Mori and M. Hamada, *ibid.* **53**, 4850 (1996).

[15] R. Berkovits and Y. Avishai, Europhys. Lett. **29**, 475 (1995); G. Bouzerar and D. Poilblanc, Phys. Rev. B **52**, 10 772 (1995); G. Chiappe, E. Louis, and J. A. Vergés, Solid State Commun. **99**, 717 (1996).

[16] B. Sutherland, Phys. Rev. A **4**, 2019 (1971); **5**, 1372 (1972).

[17] C. A. Stafford and D. F. Wang (unpublished).

[18] B. Sutherland, Phys. Rev. Lett. **20**, 98 (1968).

[19] H. Frahm and V. E. Korepin, Phys. Rev. B **43**, 5653 (1991); J. Carmelo, P. Horsch, P. A. Bares, and A. A. Ovchinnikov, *ibid.* **44**, 9967 (1991).

[20] F. V. Kusmartsev, J. Phys.: Condens. Matter **3**, 3199 (1991).

[21] N. Yu and M. Fowler, Phys. Rev. B **45**, 11 795 (1992).

[22] H. Shiba, Phys. Rev. B **6**, 930 (1972).

[23] Additional *intrachain* interactions were not found to modify the parity-locking effect in an essential way, but do lead to an enhancement of localization in models with spinless electrons.