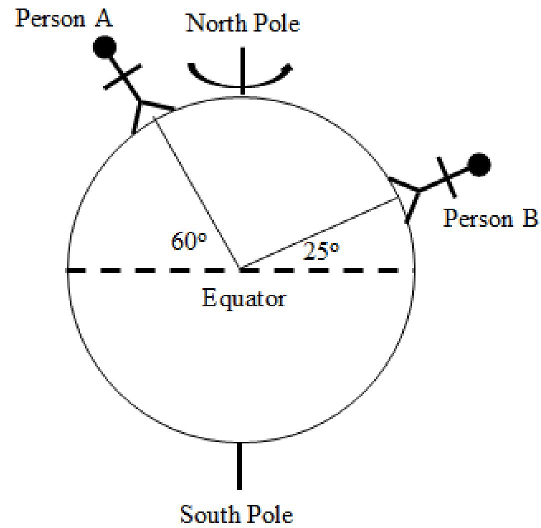
First Name:____________________ Last Name:________________________________

Two people are standing on the Earth. Person A is at a latitude of 60° N. Person B is at a latitude of 25° N.



1. Compare the magnitude of the centripetal acceleration of Person A with the magnitude of the centripetal acceleration of Person B. As a reminder, $a_{\text{centripetal}} = \frac{v_{\text{tangential}}^2}{r} = r\omega^2$. Do not actually calculate them, just think! Explain your reasoning.

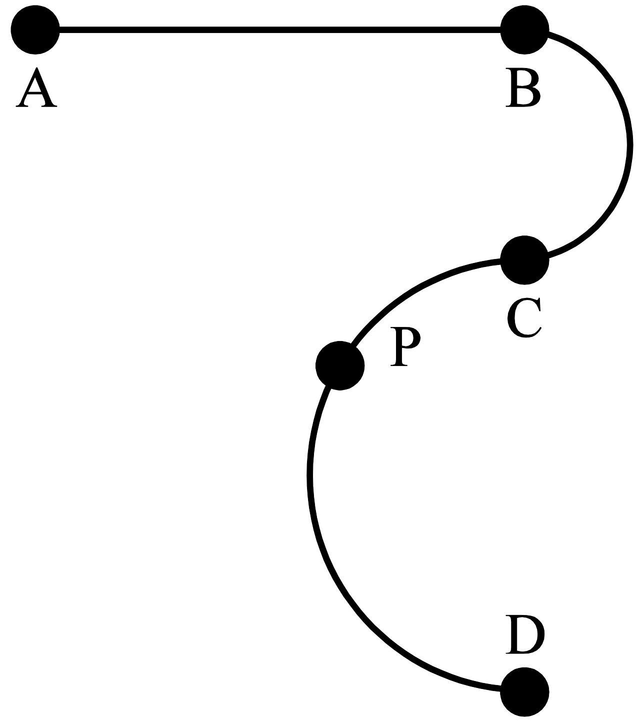
2. Two students are discussing their answers to Question #1.

Student 1: *Since v and r are the same for both Person A and Person B and the time to complete one rotation is the same, I think both will have centripetal accelerations with the same magnitude.*

Student 2: *I agree with your conclusion but not your logic. I think v and r will be smaller for Person A than for Person B since Person A moves in a smaller circle at that higher latitude. But since v and r are smaller by the same factor I would still expect the magnitudes of their centripetal accelerations to be the same.*

Do you agree or disagree with either or both of the students? Explain your reasoning

3. A car is traveling at constant speed along the road ABCD. Section AB is straight while sections BC and CD are semi-circular. Rank the magnitude of its acceleration in those three sections from smallest to largest. If any of the accelerations are zero, please indicate that. Explain your reasoning. (Note that point P will be referenced in the next question.)



4. On the diagram in Question #3, draw in the acceleration vector for the car when it is at point P. Make sure you label it. Explain your reasoning.

5. Assume that the friction force between the car's tires and the road suddenly vanishes when the car reaches point P. On the diagram in Question #3, draw in the path the car would follow and make sure you label it. Explain your reasoning.

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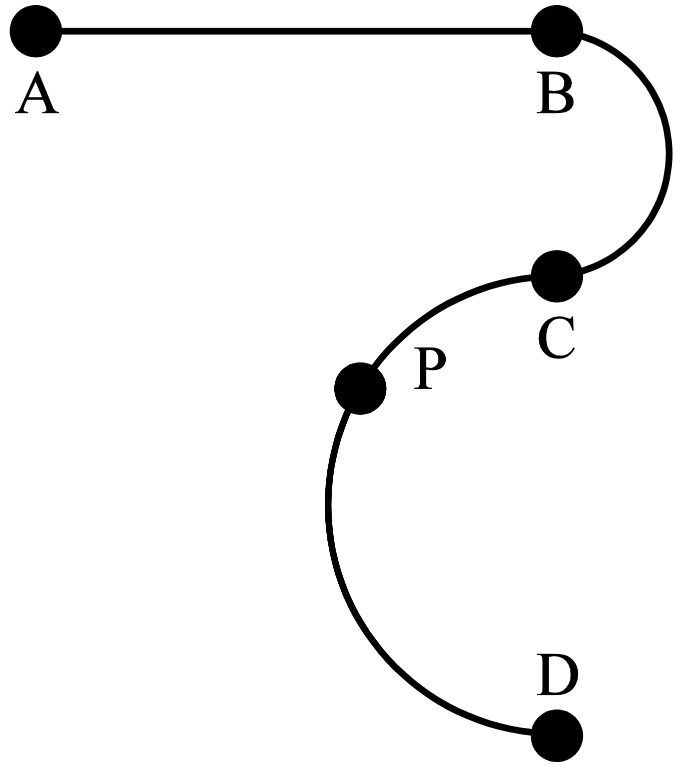
6. Now imagine that the car begins speeding up along path CD. Draw in the car's acceleration vector at point P on this diagram and make sure you label it. Your vector will probably not have the right magnitude but the **direction** of the acceleration should be qualitatively correct. Explain your reasoning. Hint: the direction is NOT the same as it was in Question #4.



7. Now that the car is speeding up, is the magnitude of its acceleration larger, smaller or the same size as it was in Question #3? Explain your reasoning. Note that there are two different factors to consider here. Before continuing on to Question #8, have a learning assistant check to see if you have correctly included both of those factors.

8. Now imagine that the car begins slowing down along path CD. Draw in the car's acceleration vector at point P on the diagram in Question #6 and make sure you label it. As in Question #6, the direction of the acceleration should be qualitatively correct. Explain your reasoning.

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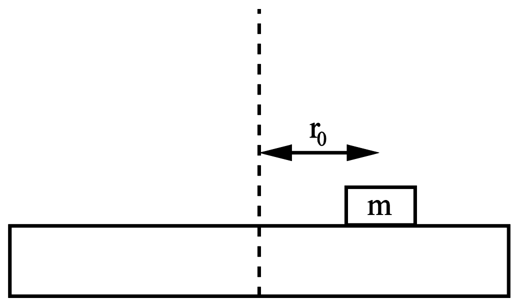
A small block of mass *m* rests on a horizontal turntable a distance r_0 from the center of the turntable as is shown in a **side view** here. There is friction between the block and the turntable described by μ_s and μ_k. The angular velocity of the turntable (about a vertical axis through the center of the turntable shown with the dashed line) is gradually increased until the block slides off the turntable. The maximum angular velocity for which the block does not slide will be labeled as the critical angular velocity ω_{critical}. It describes the transition between the block sticking to the turntable and sliding across it.



9. Draw two free body diagrams with qualitatively correct vector lengths for *m*. In the first case assume that $\omega = \frac{1}{2}\omega_{\text{critical}}$. In the second case, assume that $\omega = \omega_{\text{critical}}$. The diagrams should also be correct relative to each other. If you are uncertain about the length of any of the vectors, explain why.

10. Two students are discussing their free body diagrams in Question #9.

Student 1: *I think that kinetic friction acts on the mass since it is clearly sliding on the turntable. It also points radially inward so it is responsible for keeping the mass moving in a circle. Since the angular velocities differ by a factor of two, the magnitudes of the friction forces in the free body diagrams also differ by a factor of two.*

Student 2: *I agree about the magnitudes of the friction forces. However, while the mass is sliding on the turntable, it is not sliding in the direction of the friction force. There is no relative motion in the radial direction so I think that it is static friction that keeps the mass moving in a circle and not kinetic friction.*

Do you agree or disagree with either or both of the students? Explain your reasoning.

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11. You now repeat the original experiment but with a mass 2*m*. Would the critical angular velocity in this situation be larger, smaller or the same size as the critical angular velocity determined in the original experiment above? If ω_{critical} has changed, determine the factor by which it changed. Explain your reasoning.

12. Two students are discussing their answers to Question #11.

Student 1: *I think that ω_{critical} would be larger. Since the mass is larger, the normal force will also be larger. That means that the static friction force acting on the mass will have a larger maximum value so it should be able to rotate faster without sliding.*

Student 2: *While I agree that the static friction force will be larger, the larger mass is also more difficult to accelerate. Specifically, its inertia is twice as large. Since the static friction force* ***and*** *the force required to produce the centripetal acceleration both increase by the same factor of two, ω_{critical} should be the same.*

Do you agree or disagree with either or both of the students? Explain your reasoning.

13. You now repeat the experiment again but this time you start the block at a radius of $2r_0$. Would the critical angular velocity in this situation be larger, smaller or the same size as the critical angular velocity determined in the original experiment? If ω_{critical} has changed, determine the factor by which it changed. Explain your reasoning.

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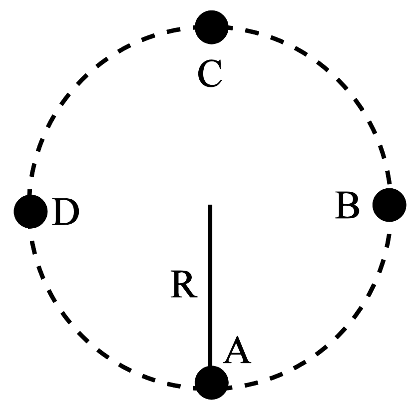
14. You now take your experiment to the moon where the gravitational acceleration is smaller. Specifically, $g_{\text{Moon}} \simeq \frac{1}{6}\, g_{\text{Earth}}$. You again place the block at radius r_0. Would the critical angular velocity in this situation be larger, smaller or the same size as the critical angular velocity determined in the original experiment above? If ω_{critical} has changed, determine the factor by which it changed. Explain your reasoning.

A ball of mass *m* moves in a **vertical** circle of radius *R* under the influence of gravity while connected to a massless cable. The ball makes it completely around the circle and the cable never goes slack.



15. Rank the speed of the mass at points A-D from largest to smallest.

Largest Smallest

Explain your reasoning.

16. Rank the magnitude of the centripetal acceleration required to keep the mass moving in a circle at points A-D from largest to smallest.

Largest Smallest

Explain your reasoning.

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17. Draw free body diagrams with qualitatively correct vector lengths for the mass when it is at each of the points A-D. The diagrams should also be correct relative to each other.

18. Explain, as accurately as you can, why the tension in the cable varies. There are two different reasons why this happens.

19. Determine the magnitude and direction of the ball's tangential acceleration when it is at each of the points A-D.

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20. Determine the minimum speed the ball can have at point C and still make it around the loop. Hint: if the ball is moving too slowly the cable will go slack.

21. The ball is traveling too slowly and the cable suddenly goes slack. What is the shape of the trajectory that the ball follows from that time forward (or at least until it drops far enough that the cable becomes taut again)? Explain your reasoning.

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Comments about this tutorial:

This has been used in both algebra-based and calculus-based physics courses.

This is certainly a long tutorial. It really consists of four different sets of Questions (#1-2, #3-8, #9-14, and #15-21) so you can certainly choose which sets of questions to use. If you have them start from Question #1 and just work through it most groups will be somewhere between Questions #9-11 at the end of class.

Questions #6-8 are definitely challenging for them. Many faculty focus on **uniform** circular motion and these questions are obviously meant to teach them about the acceleration vector when the speed is changing.

Question #10 focuses on the typical issue of "Is it static friction or kinetic friction when an object moves in a circle on a horizontal surface?" Our experience has been that their difficulty with this question is entirely determined by whether they have seen it in class or not.

Questions #1-2 and #9-14: Note that these questions use angular velocity. In calculus-based physics courses, the textbooks often don't introduce angular velocity until much later. In the algebra-based physics courses, angular velocity and centripetal acceleration are generally introduced at the same time. I chose to introduce angular velocity early in the calculus-based course so that I could ask them these questions. If you prefer not to do that, you can certainly eliminate these questions and have them solve Questions #6-8 and #15-21.

Question #10: Most of the students will have the relative magnitudes of the friction forces incorrect.

The proportional reasoning Questions (#11-14) definitely unintentionally trick many students.

Questions #15-21: While I have seen small numbers of groups get this far, I haven't seen enough students work on these to have any specific comments.

Tutorial Source(s):

Questions #1-2 were written by Colin S. Wallace at UNC – Chapel Hill and the remaining questions were written by Drew Milsom.

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