# NEWTON'S SECOND LAW OF MOTION

**GOAL:** To study acceleration from a constant force and the dynamics of a two-body system. To test Newton's second law of motion.

**INTRODUCTION:** Consider a body ( a cart) initially at rest, on a frictionless horizontal surface, with a constant horizontal force acting on it. What will happen? Intuitively, you know that the cart starts to move. Newton's first law of motion does not really provide much information because there IS a force acting on the body. By Newton's second law:

$$\vec{F} = m\vec{a}, \qquad (1)$$

$\vec{\mathbf{F}}$ is the force,
**m** is the mass and
$\vec{\mathbf{a}}$ is the acceleration,

the body will accelerate and in the direction of the force. Identifying the direction alone is important and is a testable hypothesis. **But how does one test the rest of Newton's second law?**

Experimentally the acceleration is often difficult to vary as an independent experimental variable, but the mass of the object and the applied force can more easily be varied. For testing purposes it might be better to rewrite eq.1 as

$$\vec{a} = \frac{\vec{F}}{m}. \qquad (2)$$

If one had a source of a constant force, $\vec{\mathbf{F}}$, one could vary the mass, **m**, and measure the acceleration, $\vec{\mathbf{a}}$. Then one could test if the acceleration was proportional to **1/m**.

What else can be tested? If one could produce several different constant forces, one could use a constant mass and again measure the acceleration. Then one could test if the acceleration really was proportional to the applied force.

To summarize the results so far, three tests of Newton's second law have been developed.

**Test #1** Does acceleration occur when a force is applied and in which direction is the acceleration?
**Test #2** Is the acceleration proportional to the applied force?
**Test #3** Is the acceleration inversely proportional to the mass?

Together, testing these three hypotheses, test all of the major aspects of Newton's second law of motion.

Before the experiment begins, one must consider how one will measure the mass, the force and the acceleration. Measuring mass is relatively easy, one compares the mass to be measured with known masses. This is done using a balance.

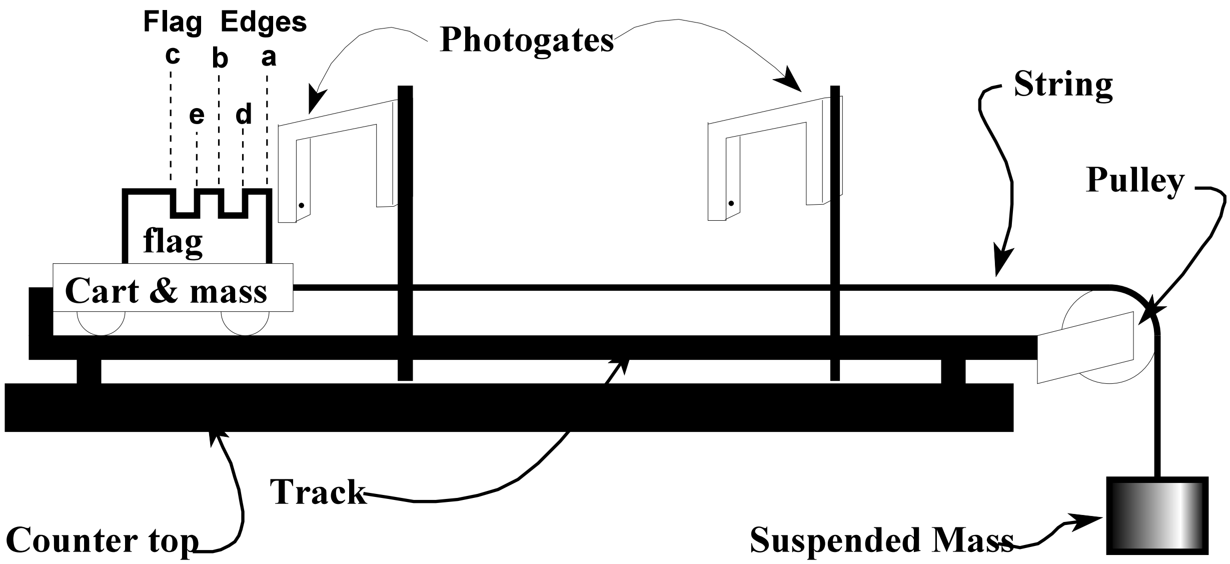
There are many ways to produce a force. Everything from someone pulling on a string, to springs, to an electric motor, to a rocket engine are possible. The problem is that one wants a force that is constant even as the object moves. The gravitational force on an object is constant (if the movement is not too far). Thus, a suspended mass can be used to provide a constant force, and this force can be redirected running the string over a pulley. But what if the system is accelerating? See the THEORY section for more details.

Finally, how does one measure acceleration? Measuring acceleration is difficult. Because measuring distances and times are relatively easy, one wants to use a trick to measure acceleration by measuring distances and times. **Method #1** involves measuring the time required to travel a given distance. In **Method #2**, the final velocity is measured after the cart has traveled a fixed distance. Velocity can be estimated by measuring a small $\Delta t$ for a small $\Delta x$. Details of these two methods are in the Theory section.

**PROCEDURE:** Simply, one applies a constant force to an object and measures its acceleration. The force can be applied by a suspended mass, and the acceleration found from a graph of measured distances and times. The mass of the cart and the applied force can be varied. A dynamic cart and track will be used for this experiment. Photogates and timer will be used to measure time.

Set up the equipment as shown in the figure below but without the photogates. How close to the ideal system described there is this system? Make **Test #1** that was developed in the introduction.

To make **Test #2** and **#3**, one must decide which of the two methods will be used to measure the acceleration. ☞ Quickly try both methods and select one.




**METHOD #1 Measuring travel time**

Method #1 involves eq. 8 and will use the photogate timer in the **PULSE** mode. In this mode the timer measures the time from when one photogate is blocked till the time of the next blocking. ( With **Memory** on, the time between a second pair of blockings is also measured.) Use a solid flag, i.e., no cutouts, and start the cart with the first photogate positioned just at edge **a.** Mark the position of the cart. To position the second photogate, it may be easier to move the cart some convenient distance along the track and position the photogate at the blocking point of edge **a**.

**METHOD #2 Measuring final velocity**

Method # 2 involves eq. 12 and can use either **PULSE** or **GATE** modes depending on how the flags are setup. In this method the photogate is removed from the starting position or the timer is reset after the cart has passed the first gate. In **GATE** mode, the time measured will be from the blocking at edge **a** till the unblocking at edge **d**. In the **PULSE** mode the time is from the blocking at edge **a** till the blocking at edge **b**.

In one of these two modes of operation, the actual travel distance between the two triggering events is different than the physical distance between the associated edges.

**A i.** Which mode and why?

Once a method of measuring the acceleration has been selected, one measures the acceleration for different applied forces to make **Test # 2** and with different cart masses for **Test #3.**

**Q i.** For each determination of acceleration, one should make several measurements while varying the total travel distance over as wide of a range as possible. Why?

**A ii.** Plot the data, extract the required slope and convert it into an acceleration.

Then repeat for several different combinations of applied force and cart mass, ( three cart masses, four suspended masses). A little thought and planning may speed data collection. One is varying the cart mass, the applied force and the travel distance.

☞ Which of these changes takes the longest to setup? Can you rearrange the order that you collect data so that this change is made least often? You may be asked by your instructor to only make Test # 2 or Test #3.

Experimentally determined accelerations should be compared against the predictions of eq.6.

**WARNINGS:**

- ♦ Do NOT drop things on your feet.
- ♦ Be careful with the equipment.
- ♦ Do NOT use more than 50 grams of suspended mass.
- ♦ Do not let the cart run into the pulley.
- ♦ Keep computer disks away from the equipment that may contains magnets.

**EQUIPMENT:**

Collision cart & masses
Track & pulley
Small masses & mass hanger
Timer & 2 photogates
Meter sticks & rulers
Small board and rubber bands as stops

In room:

Balances & levels
Card stock, tape string, & scissors
Track cleaning supplies

**Equipment notes:**

The track must be level and clean. The cart wheels should be clean.

See the appendix for general guidelines to using the photogates.

Use the indicator LED's on the top of the photogates to help set the photogate positions.

Note that in the **GATE** mode, the distance the cart travels between blocking the photogate and unblocking the gate is SHORTER than the width of the flag and must be measured experimentally.

Check the tracks and cart wheels for dirt.

There is a trick that allows one to keep the accelerating mass constant. Store on the cart all of the unused masses that can be placed on the mass hanger.

**THEORY:**

**FORCES**:

The motion and acceleration of the combined system of the cart and the suspended mass can be understood most readily by considering each body separately.

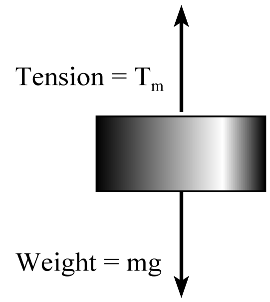
**Q ii.** For the suspended mass, under what circumstance is its weight and the tension, $T_m$ equal?

**Q iii.** If the track is level and frictionless what is the free-body diagram for the cart?

**Q iv.** What is the relationship between the weight of the cart and the normal force applied by the track?

**Q v.** What is the relationship between the tension, $T_M$, acting on the cart and $T_m$ in the free body diagram for the suspended mass?

Below is the free body diagram for the suspended mass:




**Q vi.** If the track is level and frictionless what is the free-body diagram for the cart?

**Q vii.** What is the relationship between the weight of the cart and the normal force applied by the track?

**Q viii.** What is the relationship between the tension, $T_M$, acting on the cart and $T_m$ in the free body diagram for the suspended mass?

Note the string constrains the cart and the suspended mass to move together, i.e., if the masses moves down 4.5 cm then the cart must move the same distance. Try it! If the changes in positions have the same magnitude for both objects, then the magnitude of their velocities and acceleration are also the same. Call the common magnitude of the acceleration **a**. For the suspended mass one has:

$$mg - T_m = ma, \quad \textbf{(3)}$$

and for the cart:

$$T_M = Ma. \quad \textbf{(4)}$$

**Q ix.** Because the tensions are equal, one can show that the applied force on the cart is:

$$T_M = m(g - a), \quad \textbf{(5)}$$

and

$$a = \frac{mg}{m + M}. \quad \textbf{(6)}$$

**Method #1**:

One can start with the general one-dimensional kinematics equation,

$$x = x_0 + v_0 t + 1/2\, a t^2, \quad \textbf{(7)}$$

where: is the position, $x_0$ is the initial position, $v_0$ is the initial velocity, and $t$ is the time. If one can impose the condition that $x_0$ and $v_0$ are both zero, then eq. 7 can be solved for acceleration in terms of only distances and times. An alternative to solving this simplified eq. 7 for each pair of **x** and **t,** is to consider this equation as one that describes a straight line with $x$ being the independent variable, and $t^2$ being the dependant variable. That is:

$$t^2 = Cx \qquad a = a(C) = ????? \tag{8}$$

**Q x.** Solve for the acceleration, $a$, in terms of the slope, $C$. What are the advantages and disadvantages of doing this?

A modification of this method can account for non-zero initial velocity, but the distance between the two gates is known. Label the position of the first gate as $x_0 = 0$. Eq. 7 becomes

$$x = v_0 t + 1/2\, a t^2\,, \tag{9}$$

$$\frac{x}{t} = v_0 + 1/2\, a t\,. \tag{10}$$

and dividing both sides by $t$ gives:
This is an equation of a straight line with $t$ as the independent variable and $x/t$ as the dependent variable. The "y-intercept" is $v_0$. The advantage of this method is that the initial velocity is measured and if it is not zero, it is correctly handled.

**METHOD #2**

One could also start by setting $v_0$ and $x_0$ to zero, in the following kinematic equation:

$$v^2 = v_0^2 + 2a(x - x_0), \tag{11}$$

Velocity can be estimated by measuring a small Δt for a small Δx.

**Q xi.** Derive the following including the acceleration in terms of a slope, **C'**.

$$1/(\Delta t)^2 = 2ax/(\Delta x)^2, \qquad 1/(\Delta t)^2 = C'x, \qquad a = a(C') = ????? \tag{12}$$

**ANALYSIS:**

**A iii.** The fundamental question is: at what level is your data consistent with or at odds with Newton's second law? This, of course, depends on the quality of your data. If the raw data had large uncertainties, then your confirmation or refutation will be weak. If the uncertainties are small, then your statement is stronger.

**A iv.** How is an uncertainty in the slope estimated?

**A v.** How is the error in a measurement propagated through the square of the measurement?

**A vi.** Consider what the role the friction of the cart on the track plays in this measurement.

**A vii.** How does the mass of the string affect the measurement?

**A viii.** How were the required initial conditions imposed? How well were they imposed?

**GOING FURTHER**: It is possible to measure the travel time and the final velocity with this system. Use the **PULSE** mode with **Memory** on. Start the cart with the first photogate at edge **c**, with the photogate set so that the timer starts as soon as the cart moves. The timer will stop when edge **a** passes the second photogate, but then the timer in memory restarts as edge **b** passes and stops with the passing of edge **c**. The displayed time will be the time from the {gate #1- edge **c**} to {gate # 2 - edge **a**.} Record this time. Push **Memory** switch to **Read** and the sum of this first time interval and the second time interval will be displayed. The first time can be used to calculate the acceleration using eq. 8 and the second time gives the acceleration using eq. 12.