CAPACITORS and RC CIRCUITS

Part II AC Response

GOAL: To continue investigating how capacitors behave in electrical circuits, especially those circuits with sinusoidally time varying applied voltages. Methods will be developed to measure the capacitance and the impedance (a generalization of resistance) of capacitors.

INTRODUCTION:

A general introduction to capacitors was developed for the previous experiment.

For a capacitor the generalization of resistance is called the *capacitive reactance* or more generally the *impedance*, and is denoted as $\mathbf{X_C}$. The unit for reactance or impedance is the ohm, the same unit in which resistance is measured. The capacitive reactance is defined to be the ratio of the peak voltage across the capacitor to the peak current flowing in or out of the capacitor.

$$X_C = \frac{V_{PEAK}}{I_{PEAK}}. \qquad \textbf{(1)}$$

This peak voltage can easily be measured with an oscilloscope. The peak current can be found from the voltage drop across a resistor in series with the capacitor, because the instantaneous voltage across the resistor is proportional to the instantaneous current through it. (V = IR)

Please read the theory section for further details.

Q i. What is the relationship between the angular frequency, **ω**, that is measured in radians per second, and the frequency, **f**, that is measured in cycles per second?

Q ii. What is the relationship between angular frequency, **ω** and the period, **T**, of a sinusoidal oscillation, i.e., time it takes to complete one cycle?

Q iii. How will the current through the capacitor be measured in this experiment?

Q iv. How will the resistance of the oscilloscope change the actual circuit and how will you take these changes into account?

Q v. What is the capacitance of a capacitor consisting of two 40 cm X 40 cm parallel plates separated by 1.5 mm of paper? (use κ = 3.5) ? (See last week's writeup.)

PROCEDURE:

Before class, review the operation of the oscilloscope, as given in the appendix.

Overview: A sinusoidally varying external voltage will be used to test capacitors and their reactance or impedance will be measured. A parallel plate capacitor will be constructed and its capacitance measured.

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First just observe the sine wave. Connect the output (not the **TTL** output) of the function generator to the scope's **Channel # 1** input (red banana) and the function generators ground to the black grounded input for Channel # 1.

Set the function generator to produce a sine wave. Set the amplitude and DC offset to about half the full setting. Select the 1000 Hz button and about 2 on the frequency knob.

On the scope set the **SECOND/DIV** to about 1 millisecond, the **VOLTS/DIV** to 0.5 volts and the **TRIGGER MODE** to **AUTO**. This should get something on the scope so that the best view of the output of the function generator can be found by changing scales and twiddling other knobs on the scope. Try changing the frequency, amplitude, and DC offset of the function generator.

At one frequency, sketch the sine wave, include the relevant setting of function generator and oscilloscope. Measure the frequency of the function generator using the oscilloscope and compare it to the setting of the function generator. Typically the two readings should be close but the reading from the oscilloscope should be more accurate.

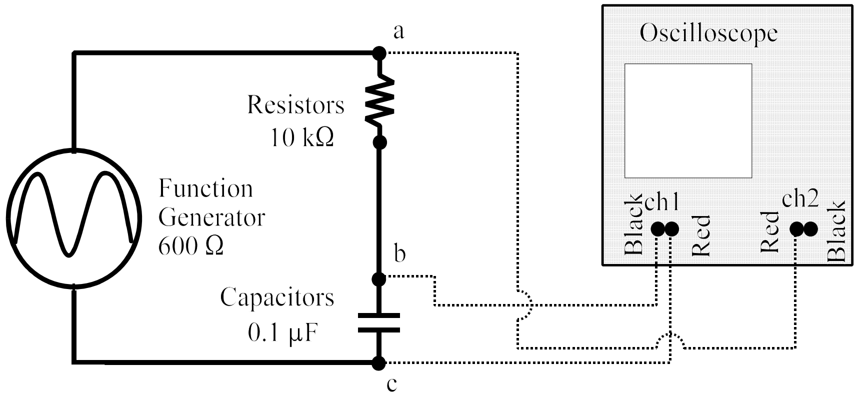

Figure 1 The circuit for measuring the impedance of a capacitor .Because the input resistance of the scope is much larger than the other parts of the circuit, it has basically no effect on the functioning of the circuit.

I. Measure reactance and capacitance with a sine wave and an oscilloscope

The circuit used in this part of the experiment is similar to that used last week and is shown in fig. 1 except that the function generator is set to produce a sine wave. To measure the reactance one must measure both the peak voltage across the capacitor and the peak current flow into the capacitor. In the circuit shown, the voltage across the capacitor measured in channel #1 that is between points **c** to **b**.

The current flowing into the capacitor is the same as the current flowing through the resistor. If the voltage across the resistor could be measured and the resistance of the resistor known, the current through it could be determined (**V** = **IR** !)

The oscilloscope is wired to read the voltage between points **a** to **b** and points **c** to **b** in the circuit shown in fig.1. Note that one of these voltages is measured in the reverse direction compared to the other because the ground of the scope is connected to the common point **b**. (The ground of the function generator floats relative to earth ground because of the 2-to-3 prong adapter)

For a given resistor-capacitor pair measure the peak voltage and the peak current for at least five frequencies preferably in a ratio of approximately 1:3:10:30:100.

1. From these values find the capacitive reactance, $\mathbf{X}_C$ at each frequency.
2. Plot this $\mathbf{X}_C$ vs. $1/\omega$. Determine the capacitance, **C** from the slope of the line. (The slope = 1/**C**.)
3. Plot $\mathbf{X}_C$ vs. ω.
4. Try to determine if the peak voltage occurs before or after the peak current, and by what fraction of a complete cycle. Convert this fraction of a cycle to radians or degrees.
5. Repeat for other pairs of resistors and capacitors.

II. Build a capacitor and measure its electrical characteristics

Using the 2 hardboard squares, and aluminum foil as plates, and 1 posterboard square as a separator make a capacitor. Measure its capacitance. Offset the plates to reduce the effect plate area to half the original area and again measure the capacitance. Attempt to measure the dielectric constant for the posterboard spacer using eq.3 from the writeup of last week's experiment.

EQUIPMENT:

Oscilloscope
& 2 BNC-to-banana adapters
Function generator
& 2-to-3 AC plug adapters

Resistor board (~ 10 Ω to 22 MΩ)
Capacitor (~ 0.005 to 0.47 μF)
& Inductor (~ 2.5-100 mH) board

In room
Lead wires
Stopwatches
Semi-log paper
Large hardboard squares
large posterboard squares
Aluminum foil
Large weights
Meter sticks & Calipers

Generator/motor + capacitor demo

The oscilloscope has finite resistance. This resistance may effect the circuit and change your results. Scope 1 MΩ

Theory: Sine Wave Response

For a resistor the peak applied voltage $\mathbf{V}_{R,0}$ and the peak observed current flow $\mathbf{I}_{R,0}$ occur at the same time and the ratio $\mathbf{V}_{R,0}$ / $\mathbf{I}_{R,0}$ is just the resistance, **R**.

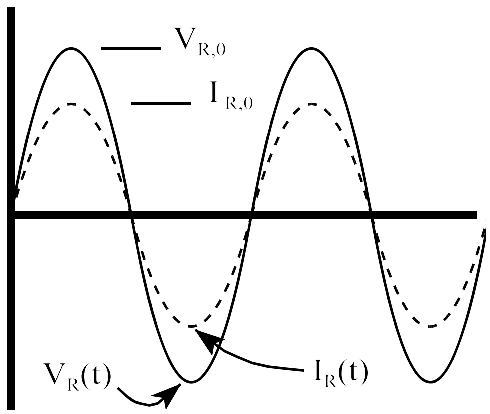

Figure 2 The current, $\mathbf{I}_R$(t)through a resistor of resistance, **R**, when the applied voltage is $\mathbf{V}_R(t) = \mathbf{V}_{R,0} \sin(\omega t)$. In this case, $\mathbf{I}_R(t) = \mathbf{I}_{R,0} \sin(\omega t)$, and $\mathbf{I}_{R,0} = \mathbf{V}_{R,0} / \mathbf{R}$.

For a capacitor the peak observed current flow $\mathbf{I}_{C,0}$ occurs before the peak voltage $\mathbf{V}_{C,0}$ across the capacitor and the ratio $\mathbf{V}_{C,0}$ / $\mathbf{I}_{C,0}$ is the *capacitive reactance* (or impedance) $\mathbf{X}_C$. If the capacitor's capacitance is **C** and the driving voltage is a sine wave with an angular frequency, **ω**, then

$$X_C \equiv \frac{V_{C,0}}{I_{C,0}} = \frac{1}{\omega C} . \qquad \textbf{(2)}$$

This suggest an alternative method of measuring the capacitance, **C**. First measure $\mathbf{V}_{C,0}$, $\mathbf{I}_{C,0}$ and **ω**, and then plot $\mathbf{V}_{C,0}$/ $\mathbf{I}_{C,0}$ vs 1/ **ω**. The slope of this line should equal 1 / **C**. Or plot $\mathbf{I}_{C,0}$ / $\mathbf{V}_{C,0}$ vs **ω**, and the slope should equal **C**.

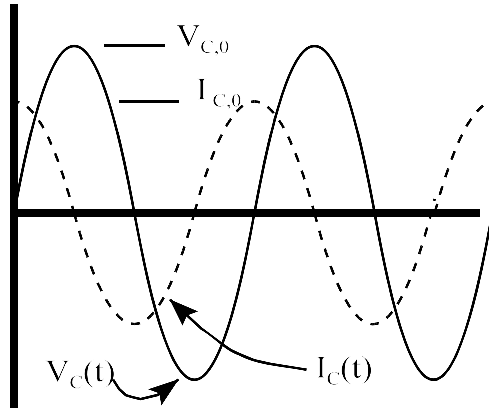

Figure 3 The current, $\mathbf{I}_C(\mathbf{t})$ through a capacitor of capacitance, **C**, when the voltage across the capacitor is $\mathbf{V}_C(\mathbf{t}) = \mathbf{V}_{C,0} \sin(\omega t)$. In this case, $\mathbf{I}_C(\mathbf{t}) = \mathbf{I}_{C,0} \sin(\omega t + \pi/2)$, and $\mathbf{I}_{C,0} = \mathbf{V}_{C,0} / \mathbf{X}_C$.

One fundamental difference in response of a sinusoidal signal between resistors and capacitors is the timing of the peak current flow relative to the peak voltage across the device. Another difference is that for capacitors the ratio $\mathbf{V}_{C,0}$ / $\mathbf{I}_{C,0}$ decrease with increasing frequency, **ω** of the signal.

ANALYSIS:

A i. Two methods of measuring capacitance have been developed. Last week, one measured capacitance by measuring the related RC constant. This week capacitance was measured by measuring capacitive reactance. Compare these two methods for measuring the capacitance.

A ii. For the capacitor which you made, was the capacitance proportional to the area of overlap for two plates?

A iii. In the *Field Mapping Experiment*, it was observed that the electric field connecting parallel plates extended beyond the length of the plates. The part of the field beyond the plates is called the fringing field. How does the presence of a fringing field effect the magnitude of the capacitance of the capacitor that you made in class?