Decoherence by spontaneous emission in atomic-momentum transfer experiments

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The influence of spontaneous emission on the dynamical localization in atomic momentum transfer observed in recent experiments is investigated. Our numerical and analytical results indicate that contrary to previous expectation quantum diffusion due to decoherence by spontaneous emission is non-negligible in the experimental regime and may actually be visible in the measured data. Modifications of the experiments to strengthen this point are proposed. [S1050-2947(96)03412-9]

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Recently the dynamics of atomic de Broglie waves in a periodically modulated standing-wave laser field [1] has emerged as a very interesting example [2–6] for the experimental study [2–4] of the influence of the quantum theory on classical chaos. Indeed, in the latest experiment of Moore et al. [4] one of the basic models of “quantum chaos” showing the phenomenon of dynamical localization [7], the periodically kicked quantum rotor [8], has been realized. Here we shall be concerned with the quantum dynamics of atoms in sinusoidally phase-modulated standing-wave fields which, after a theoretical proposal in [1], has been measured in [2,3]. The theoretical analysis in [1] and in [2,3] and further theoretical work adjusted to the experimentally realized conditions [5,6] was based on numerical solutions of the Schrödinger equation of the center-of-mass motion of the atom in its ground state interacting with the off-resonant laser field only via virtual transitions to an internal excited state. In particular, effects of spontaneous emission were neglected due to the small occupation probability of the excited state.

The most interesting modifications which quantum theory imposes on classical chaos are quantum coherence effects, the most prominent of which are tunneling and dynamical localization. It is well known that under semiclassical conditions, when large quantum numbers or actions large compared to Planck’s constant are involved, these quantum coherence effects may become extremely vulnerable to decoherence effects like spontaneous emission. In fact, it is now widely believed that this is the fundamental reason why coherent superpositions of macroscopically distinct states are not commonly observed (see, e.g., [9]).

In view of this it is necessary, and in fact very interesting, to ask whether in the subtle quantum coherence effects observed in the experiments [2,3] decoherence due to spontaneous emission, however small its rate, may still show its influence. To address this question in some detail is the purpose of the present paper. Surprisingly, we find that contrary to previous expectation quantum diffusion induced by spontaneous emission should have a non-negligible influence on those data which have been interpreted as showing dynamical localization [2,3,5]. Moreover, that interpretation [10] is reinforced by the reasonable agreement between our theoretical prediction that the quantum diffusion is enhanced by the square of the localization length and the experimental value implied by the data.

The starting point of our analysis is a stochastic Schrödinger equation of a two-level atom in a periodically phase-modulated standing-wave laser field [13,14],

$$d|\psi(t)\rangle = \left\{ -i\frac{\hbar}{2} \mathcal{H} dt - \frac{\gamma}{2} |e\rangle \langle e| dt + \sqrt{\gamma e^{-i\kappa x u}} |g\rangle \langle e| \right\} |\psi(t)\rangle \times d\xi(t) / |\psi(t)\rangle.$$

Spontaneous emission with rate $\gamma$ is taken into account in (1) by a decay term for the excited state $|e\rangle$ and a corresponding stochastic repopulation term for the ground state $|g\rangle$; the latter term contains a Gaussian Ito force $d\xi(t)$ with $d\xi(t)=0$, $(d\xi(t))^2=dt$, and a phase factor accounting for the random-recoil momentum $\hbar k_L u$ in the direction of the plane wave along the $x$ axis with wave number $k_L$. The parameter $u = \sin \vartheta \sin \varphi$ is chosen at random from the distribution $P(\vartheta, \varphi) = 3/8 \pi (1-\cos^2 \vartheta) \sin \vartheta$ at each time step. The linear stochastic Schrödinger equation (1) is one out of an infinite family which are all equivalent to the same master equation for the statistical operator $\rho(t)=|\psi(t)\rangle \langle \psi(t)\rangle$. Another closely related member of the same family was used in [14] where the explicit form of the corresponding master equation can also be found. There a Poisson process $dP(t)$ appeared in (1) in place of the Wiener process $d\xi(t)$. Our particular choice of (1) is based on numerical convenience only. No microscopic interpretation of (1) beyond its stochastic equivalence with the master equation will be required here. In particular, in the experiments the spontaneously emitted photons are not registered, i.e., all expectation values are to be averaged over $\xi$.

The Hamiltonian [1]

$$H = \frac{P^2}{2M} + \hbar \delta_i |e\rangle \langle e| - \{ dE_0 \cos[k_L(x-\Delta L \sin \omega t)] |e\rangle \langle g| + \text{H.c.} \}$$

contains the relevant center-of-mass variables $x$, $P_x$, the atomic mass $M$ and dipole matrix element $d$, and the experimentally controllable parameters $\delta_i$ (detuning), $E_0$ (field amplitude), $k_L \Delta L = \lambda/2$ (modulation depth), and $\omega$ (modulation frequency). The experiments of [2,3] were performed for $\Omega = 2dE_0 / \hbar = 2 \pi \times 34.5 \times 10^7$ Hz and $\delta_i = 2 \pi \times 5.4 \times 10^9$ Hz, i.e., in a regime where only virtual transitions into the
excited state may occur. Therefore, Eq. (1) can be simplified by adiabatic elimination of the excited state [1] and we obtain instead for the stochastic wave function $|\psi_s(t)\rangle$ of the atom in its ground state [13]

$$d|\psi_s(t)\rangle = \left[ -\frac{i}{\hbar} H_{\text{eff}} dt - \frac{\gamma}{2} \cos^2[k_L(x - \Delta L \sin\omega t)] dt + \sqrt{\gamma} \cos[k_L(x - \Delta L \sin\omega t)] e^{-ik_L x} d\xi(t) \right] |\psi_s(t)\rangle \\
$$

with $\gamma = \gamma(\Omega/2\delta)^2$ and the effective Hamiltonian

$$H_{\text{eff}} = \frac{p_x^2}{2M} - \frac{\hbar \Omega_{\text{eff}}}{4} \cos^2[k_L(x - \Delta L \sin\omega t)].$$

In the following we shall use the experimental values [2] of the parameters $\Omega_{\text{eff}}/2\pi = \Omega/4\pi \delta L = 2.2 \times 10^7$ Hz, $\omega/2\pi = 1.3 \times 10^9$ Hz, $k = \hbar\delta_L^2/\omega^2 \delta_L = 0.34$, $k = 4k_L^2 h/M \omega = 0.16$, and $\gamma/2\pi = 0.98 \times 10^7$ Hz. Equations (3) and (4) differ by the stochastic terms from the Schrödinger equation solved in [1–6]. It is the influence of these terms which is our main concern here. For simplicity we shall focus on initial states with momentum $p_x = 0$ and two particular values for the parameter $\lambda = 2k_L \Delta L$, namely $\lambda = 3$ and $\lambda = 3.8$, which are both in the classically chaotic domain [1] $\sqrt{\pi \lambda} \leq 40k$, but correspond to drastically different dynamical behavior [3,5]: For $\lambda = 3.8$ the initial state has strong overlap, in phase space, with a classical resonance island, and the momentum transfer is localized by the KAM tori already for this classical reason [3,5,6]; on the other hand, for $\lambda = 3$ an initial state near $p_x = 0$ has considerably smaller overlap with the resonance island and the momentum transfer is localized only quantum mechanically and remains significantly smaller than the classically predicted value [2,3,5].

We begin by considering the effect of spontaneous emission, first for the case $\lambda = 3.8$. In Fig. 1 we plot the variance of the momentum in units of $2\hbar k_L$ as a function of $\omega t$ [15].

![FIG. 1. Variance of the distribution of momentum in units of $2\hbar k_L$ as a function of $\omega t$ for modulation depth $\lambda = 3.8$, classically (dashed line), quantum mechanically including spontaneous emission (solid line), and without it (dotted line).](image1)

We see that the variance of the momentum transfer saturates both classically and quantum mechanically at about the same average value ($\approx 6$), but quantum mechanically erratic partial coherent recurrences of states with relatively smaller variance occur. It can also be seen that the effect of spontaneous emission on the time-averaged variance is small but that it is readily noticeable in the revivals, which are indeed based entirely on quantum coherence. The behavior shown in Fig. 1 is consistent with the interpretation [3,5] that the momentum transfer in this case is restricted by a classical mechanism. This conclusion is also consistent with classical and quantum evaluations of the entropic localization length $l_S = \text{exp} (S)$ with $S = -\sum_i |c(i)|^2 \ln |c(i)|^2$, where the momentum variable is discretized in cells $p_x = 2\hbar k_L l$ both classically and quantum mechanically, and $|c(i)|^2$ is the occupation probability of the cell $l$. For $\omega t = 165$ we obtain the nearly equal classical and quantum results $l_S^{(c)} = 9.05$ and $l_S^{(q)} = 8.28$, respectively. Experimentally, the variance was measured at $\omega t = 81.7$ and $\omega t = 163.4$ and found to be time independent, within the accuracy of the measurement [2] in qualitative agreement with the present conclusions. The revivals so far are not accessible experimentally because they are characteristic of the individual pure initial state with vanishing variance, while experimentally only finite-temperature ensembles of initial states with variance $\approx 5.3$ have been used [16].

Let us now turn to the results for $\lambda = 3$. It has already been shown before [2,3,5] that the variance of the momentum transfer classically and quantum mechanically differ drastically in this case; this can be seen also in Fig. 2 by a comparison of the dashed classical and the dotted quantum-mechanical curve without spontaneous emission. The classical-quantum comparison of $l_S$, for $\omega t = 165$, now gives the different values $l_S^{(c)} = 38.3$ and $l_S^{(q)} = 19.36$, respectively. In other words, in this case already the time-averaged value of the variance of the transferred momentum is limited by a quantum-mechanical coherence effect – dynamical localization. If this interpretation is taken seriously one is now forced to predict, in view of the above results for $\lambda = 3.8$, a readily noticeable influence of spontaneous emission on the

![FIG. 2. The same plot as in Fig. 1 for modulation depth $\lambda = 3$. Measured data taken from [2] are indicated by solid circles and error bars corresponding to a 10% uncertainty in the variance [2].](image2)
time evolution of the variance. This comes as a surprise, because spontaneous emission was thought to be negligible in the experiments. However, the prediction is indeed borne out. The numerical data including spontaneous emission are plotted as the solid line in Fig. 2 [15]. They show that in the regime where the classical and quantum variances differ significantly the quantum-mechanical localization of the variance without spontaneous emission is replaced by a weak but readily recognizable quantum diffusion process induced by spontaneous emission, which is clear evidence of the coherent nature of the localization mechanism. This strongly reinforces, in a completely independent way, the claim in [3,5] that momentum transfer at $\lambda = 3$ for initial states near $p_x = 0$ is restricted by a quantum-mechanical coherence effect, vulnerable to spontaneous emission (see Fig. 2), and not by the much more robust classical phase-space structure, which would stabilize also against spontaneous emission (see Fig. 1).

For dynamical localization in the quantum kicked rotor coupled to a decohering environment the quantum diffusion effect is well known [13,14,17–20]. A simple rough formula for the quantum diffusion constant can be derived [13] by locally approximating the present system by a quantized standard map and adapting the considerations of [17] and [18] to the present case. One obtains [13] in the time regime where localization has established itself

$$\langle p_x^2 \rangle \approx \frac{1}{2} (2\hbar k_L)^2 \xi^2 \left( 1 + \beta \tilde{\gamma}^2 t \right)$$

with the localization length $\xi = (2\pi \alpha/k_L \Delta L)(\Omega_{eff}/8\omega)^{1/2}$. Note that the quantum diffusion constant induced by spontaneous emission is proportional to the square of the localization length, i.e., it is strongly enhanced by quantum coherence. The parameters $\alpha, \beta$ are of order 1 and are introduced here to account for the fact that Eq. (5) is obtained by qualitative arguments only. In particular, for $\beta = 1$ the estimate of the diffusion constant ignores the presence of boundaries of the classically chaotic domain and the existence of other classical phase-space structures like regular islands or cantori. Furthermore, the estimate assumes that the average time between spontaneous emission events in a single atom is large compared to the time $\Delta t = \xi/2 \pi/\omega$ in which a wave packet spreads over its localization length. If any one of these assumptions is violated the diffusion constant, and hence $\beta$, should be smaller. The comparison of Eq. (5) with the numerical results $\alpha = 0.54$, $\beta = 0.6$, i.e., Eq. (5) is a reasonable description. We have also checked numerically that the quantum diffusion constant indeed increases proportional to $\tilde{\gamma}$ [21].

Let us now turn to the experimental values which have been measured for $\omega t = 81.7$ and $\omega t = 163.4$; see Fig. 2. A first important observation is that both measurements were taken well inside the time regime $\omega t > 2 \pi \xi$ where the localization has had ample time to establish itself fully. As again a mixture of initial states has been used in the experiment, the coherent erratic oscillations present in the numerical trace for a single pure initial state cannot be present in the experimental result, i.e., the theory without spontaneous emission in the case of an initial mixture simply predicts that the variance will be time independent in the localized regime, just as for the case $\lambda = 3.8$. Instead, a slight but systematic increase of the variance with time is observed. Indeed, the measured variance for any value of $\lambda$ is always larger for $\omega t = 163.4$ than for $\omega t = 81.7$ (except for $\lambda$ values where resonances are present to prevent the increase like for $\lambda = 3.8$), ruling out the possibility that the experimentally observed effect is just due to the measurement uncertainty. If we decide to rule out the possibility that the increase is due to an unknown systematic error in the measurement we are obliged to take this increase seriously and encouraged to extract a diffusion constant from the slope of the variance [22]; see Fig. 2. Surprisingly enough the comparison with our theory shows that it has the correct size to be caused entirely by the effect of spontaneous emission. No other theoretical explanations for the observed increase have so far been proposed (the average collision rate between the atoms of $\tau^{-1} = 2.7 \times 10^{-3}$ s$^{-1}$ is much too small to explain this effect). We conclude, therefore, that the environment-induced quantum diffusion in a dynamically localized state may actually have been observed in these experiments.

There are three comparatively easy modifications of the experiment we can propose (besides, of course, many more involved ones), which would serve to verify or falsify this conclusion: (i) By increasing the interaction time, the further linear increase of the variance with the same slope could be checked. (ii) Decreasing the detuning $\delta_x$ and the intensity $\sim \Omega^2$ of the laser field, e.g., by a common factor of 1/2 would increase the quantum diffusion constant by 2 while leaving the localization length and all other relevant properties of the dynamical system unchanged. The effect of varying the spontaneous emission rate $\tilde{\gamma}$ in the stochastic Schrödinger equation is shown in Fig. 3. (iii) The proportionality of the quantum diffusion rate to the square of the localization length $\xi^2 = (\Omega^2/\delta_x \omega)^{1/2}$ could be tested by changing the modulation frequency $\omega$, the laser intensity $\sim \Omega^2$, and the detuning $\delta_x$, keeping $\lambda, k \sim \Omega^2/\delta_x \omega^2$ and $\tilde{\gamma} \sim (\Omega/\delta_x)^2$ fixed.

In conclusion, we note that dynamical localization is a subtle coherence effect where quasienergy states contain only a fraction of all available momentum basis states with

**FIG. 3.** The same plot as in Fig. 2. Dotted line, without spontaneous emission; full line, with spontaneous emission, as in Fig. 2; dashed and dashed-dotted line, with spontaneous emission and $\Omega^2, \delta_x$ changed by a common factor of 2 and 1/2, respectively.
support in the chaotic domain of phase space for no other reason than dynamically generated pseudorandomness. The vulnerability of this effect to decoherence effects like spontaneous emission manifests itself in the appearance of a quantum diffusion constant which is predicted to be proportional to $\xi^2$, times the spontaneous emission rate. This effect makes it hard to prepare such states but, at the same time, offers a characteristic hallmark of the dynamical localization effect which distinguishes it sharply from localization by classical phase-space structures. Presenting a theoretical analysis adjusted to the conditions recently realized in the laboratory we have shown that the measured data ought to be sensitive to the effect; and, unappreciated so far, indeed seem to show the effect in the predicted magnitude. We have proposed several simple or more ambitious modifications of the experiments which could further test our conclusions.

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[10] Recently the interpretation of the data in [2,3] as showing dynamical localization has been contested [6] on the basis of numerical work using an initial state with comparatively large positive initial average momentum $p_x/2\hbar k_L = 18$. In this case an increase in the variance of roughly the same size classically and quantum mechanically is found which, on first sight, seems to contradict dynamical localization. However, a closer scrutiny of this case reveals that the variance in the quantum case is due to quantum tunneling which occurs in the presence of dynamical localization [12] due to the symmetry $x \rightarrow -x$, $p_x \rightarrow -p_x$, $t \rightarrow t + \pi/\omega$ of $H_{eff}$, leading to a large variance but a quantum-mechanically limited localization length as measured, e.g., by the entropy of the momentum distribution, while the classical spreading is due to chaotic diffusion. The quantum tunneling can be suppressed by choosing an initial state with noninteger $p_x/2\hbar k_L = \hbar + n_\alpha$, $0.5 < n_\alpha \leq 0.5$ [11], which evolves under the Hamiltonian (4) with $p_x \rightarrow p_x - n_\alpha 2\hbar k_L$ and lacks the above symmetry.
[15] In the numerical simulations the wave function $|\psi_n \rangle$ is expanded in a finite basis of discretized-momentum states $|n \rangle$, $p_x |n \rangle = n(\hbar k_L/3)|n \rangle$, where the integer eigenvalue $n$ is ranging in the interval $[-240,240]$ corresponding to $p_x = -40, \ldots, 40$ in units of $2\hbar k_L$. The continuous random number $u$ is approximated by the nearest integer multiple of $\pm 1/3$ in consistency with the momentum discretization. Ensemble averages are taken over $N_u = 50$ and $N_\xi = 500$ independent trajectories of the random numbers $u$ and $\xi$, respectively.
[16] Initial states based on Bose-Einstein condensates here offer exciting new possibilities. The initial momentum spread of $\text{var}(l) = 5.3$ in the experiment [2] probably is also responsible for the fact that the measured variance of $l = 19$ is considerably higher than in Fig. 1. In [3] practically time-independent variances were reported not for $\lambda = 3.8$ but at the slightly larger value $\lambda = 4$, and again at $\lambda = 5.6$, where a resonance also dominates the initial state.
[21] In [19] it was argued that for the case of a "quantum kicked particle" roughly corresponding to the present situation the diffusion constant should vary $\sim \gamma^{1/3}$ (in our present notation) for $\gamma \rightarrow 0$. In the parameter domain set by the experiment we could not see this behavior numerically. The proposal (ii) we list near the end of this paper would permit an experimental check.
[22] The average slope of the variance is more significant than its absolute size for the comparison between theory and experiment because it is less influenced by details of the initial condition.