Decoherence from spontaneous emission

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Starting from a generalization of the Weisskopf-Wigner model for the case of a two-level atom with a largely
spread center-of-mass wave function, we show that spatial correlations are destroyed to some extent by a
spontaneously emitted photon. We derive a particularly simple form of the corresponding decoherence function
and determine the dependence of the decoherence on orientation and size of a detector registering the outgoing
photon.

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I. INTRODUCTION

When describing an excited atom that is spontaneously
emitting a photon one usually considers the atom strongly
localized, i.e., pointlike compared to the wavelength of the
emitted light. Though this assumption is usually adequate in
regard to the extension of the electron cloud around the
center of mass of the atom, it need not be so in regard to the
atom’s center-of-mass wave function \( \phi(x,t) \). Through modern experimental technology it has become possible to
prepare atoms whose uncertainty in position exceeds consider-
ably the wavelength of the emitted light \([1–3]\). It was shown
in a recent pioneering experiment by Pfau et al. \([4]\) that in
this case the emitted light decoheres the spatial correlations
of the atomic density matrix. These authors also gave a short
theoretical treatment of the decoherence effect. Focusing on
one spatial dimension \((x)\), they started from the observation
that the averaged density matrix for the center-of-mass co-
ordinate, \( g_{i}^{(1)}(z) = \int \rho(z',z') \, dz' \), the transverse one-point
coherence function, is just the Fourier transform of the distri-
bution for the \( z \) component of the momentum. The final
momentum distribution is readily calculated \([5]\), and Fourier
transforming the result yields the final coherence function
\( g_{f}^{(1)}(z) \) in the product form

\[
g_{f}^{(1)}(z) = g_{i}^{(1)}(z) D(z),
\]

where \( g_{i}^{(1)} \) is the initial coherence function, and the deco-
herence function \( D(z) \) proves to be the Fourier transform of the
distribution for the momentum (in the \( z \) direction) of the
emitted photon.

In the present paper we will generalize this result in three
ways: (i) we will consider a wavepacket in the full three-
dimensional space; (ii) we will study the change of the
atomic density matrix \( \rho(x,x') \) itself, i.e., the true two-point
correlation function; and (iii) we will evaluate the decoher-
ce function on condition that the emitted photon has hit a
detector with variable size. We will show that a relation simi-
lar to Eq. (1) holds, namely \([6]\)

\[
\rho_{\text{emitted}}(x,x+r,t) = \rho(x,x+r) \, D(r),
\]

where the density matrix \( \rho_{\text{emitted}} \) describes atoms that have
undergone spontaneous emission, whereas \( \rho \) refers to atoms
that have not (provided they were treated identically except
for excitation and hence subsequent spontaneous emission).
We will derive an explicit expression for the decoherence
function \( D(r) \) and discuss its properties. Moreover, we will
clarify the physical assumptions and approximations on
which Eq. (2) rests.

First of all, let us illustrate the physical meaning of the
decoherence function in an atomic interference experiment
of Young’s type. We assume the impinging wavepacket to be
“quasimonochromatic”; i.e., to have a well-defined de Bro-
glie wavelength. Then, in perfect analogy to classical optics,
the “intensity”; that is, the probability to detect an atom, at a
given position in the observation plane, can be written as

\[
I = \rho(x) + \rho(x+r) + 2 \Re \{ \rho(x)\rho(x+r) e^{i\tau} \},
\]

where the argument \( \tau \) in the phase factor denotes the dif-
ference of the propagation times from the locations of the holes
\( x \) and \( x+r \) in the interference screen to the observation
point.

One learns from Eq. (3) that the visibility of the inter-
ference pattern is given by

\[
\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2 |\rho(x,x+r)|}{\rho(x) + \rho(x+r) + \rho(x,x+r)}. \quad (4)
\]

The influence of decoherence according to Eq. (2) is now
readily explained: It follows from Eq. (3) that the phase of
the decoherence factor \( D(r) \) gives rise to a shift of the in-
terference pattern, whereas its modulus, according to Eq. (4),
describes a reduction of the visibility.

II. GENERAL THEORY

We utilize a recently found solution \([7]\) to the problem of
spontaneous emission, in the dipole approximation, from an
extended wavepacket. This is an extension of the treatment
by Weisskopf and Wigner \([8]\), taking the center-of-mass mo-
tion into account. We are interested in the atomic state only
after the emission has happened; thus we consider the ini-
tially excited atoms only at times \( t \) larger than the mean
lifetime of the excited level \( \gamma_0^{-1} \). In this case the solution \([7]\)
for the wave function \( |\Psi(t)\rangle \) of the total system reduces to

\[
|\Psi(t)\rangle = \int d^3 p \sum_{j} \beta_j(p,k_j,t) |p,k_j\rangle,
\]

where
Here \( p \) denotes the atomic momentum, and \( k_j \) and \( \omega_j \) are the wave vector and the frequency of the radiation modes. The label \( j \) stands for both the wave vector and the polarization, \( \omega_0 \) denotes the atomic eigenfrequency, \( M \) is the atomic mass, and \( \alpha_0(p) \) denotes the initial center-of-mass wave function, for the atom, in momentum representation. The \( \lambda \)'s are the coupling constants (in units of \( \hbar \)) describing dipole coupling. Due to our assumption \( t \gg \gamma_0^{-1} \), the atom is in the ground state with certainty. Hence the internal atomic state does not affect the density operator for the center-of-mass dynamics and has therefore been discarded in Eq. (5).

In what follows, we will assume that no polarization-sensitive measurement is performed on the emitted photon. Choosing the two independent polarization directions associated with the wave vector \( k = k(\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta) \) as \( u_1 = (\cos \theta, \sin \varphi \sin \theta, -\sin \theta) \) and \( u_2 = (-\sin \varphi, \cos \varphi, 0) \), we can perform the summation over the two polarizations, for given \( k \), with the result

\[
\exp \left[ -i \frac{p^2}{2\hbar M} + \omega_j - \frac{\omega_0}{2} t \right]
\]

\[
\beta_j(p, k; t) = \lambda_j \alpha_0(p + \hbar k) \frac{i^j}{\hbar} \frac{\alpha_0(p + \hbar k) \alpha_0^*(p' + \hbar k) e^{-i(p^2 - p'^2)/2\hbar M}}{(\omega - \omega_0)^2 + \gamma_0^2},
\]

where Eq. (7) has been used. Upon substituting \( p + \hbar k = P \), \( p' + \hbar k = P' \), we can rewrite Eq. (9) as

\[
\rho_{\text{emitt}}(x, x'; t) = \text{const} \int \frac{d^3p}{(\Delta \Omega)} \frac{d^3k}{\hbar} \frac{1}{\Delta \Omega} \frac{1}{\hbar} \frac{1}{\hbar} \frac{\alpha_0(p + \hbar k) \alpha_0^*(p' + \hbar k) e^{-i(p^2 - p'^2)/2\hbar M}}{(\omega - \omega_0)^2 + \gamma_0^2},
\]

where \( \omega = \epsilon k \).

It is obvious from Eq. (10) that we arrive at an expression of the factorized form (2) if we neglect the kick the atom experiences due to emission, expressed by the creation \( \hbar \kappa t/M \) in the argument of the atomic center-of-mass wave function. When will this be justified? Inserting for \( t \) the value \( 10 \gamma_0^{-1} \) consistent with the derivation of Eq. (6), and assuming the atomic velocity in the laboratory frame to be of the order of 1000 m/s, we determine the ratio of the shift induced by the recoil of an optical transition (\( \omega_0 = 10^{15} / \gamma_0 = 10^9 / \gamma_0 \)) to the particle's de Broglie wavelength \( \lambda_{\text{dB}} \) and find: \( k \omega_0/(M \lambda_{\text{dB}}) = 2 \pi c/(\nu_0 \omega_0) \approx 2 \times 10^{-2} \). Clearly such a small relative shift can be neglected.

Dropping this correction in Eq. (10) we arrive at the desired result (2), where the decoherence factor is given by

\[
D(r) = \text{const} \int \frac{d^3k}{(\Delta \Omega)} \frac{\lambda_k^2}{k^2} e^{i k \cdot r} (\omega - \omega_0)^2 + \gamma_0^2,
\]

which is, in fact, the Fourier transform of the momentum distribution for the registered photons. Thus Eq. (11) is the extension of the previous result [4] in three respects: (i) it is the generalization to three dimensions; (ii) it allows us to study the influence of actual measurements on the emitted photons; and, what is most important, (iii) it describes the decoherence of the full density matrix. Of course, since the decoherence factor depends only on \( r \), it also describes the deterioration of the averaged density matrix \( \int d^3x \ \rho(x, x'; r, t) \), i.e., the one-point coherence function that was studied in [4] for the one-dimensional case. Finally, one learns from Eq. (2) that the normalization condition for the decoherence function simply reads \( D(0) = 1 \), thus determining the constant prefactor in Eq. (11).

III. DISCUSSION

Let us now discuss different experimental conditions, i.e., detectors that, seen from the atom's position, cover various solid angles \( \Delta \Omega \) in \( k \) space.
A. $\Delta \Omega = 4\pi$

This is the situation one encounters when a fictitious detector is employed that counts any photon, or, more physically, when no measurement is made at all. The integral, Eq. (11), can be split into two angular integrations over $\phi$, $\theta$, and one radial integration over $k$ running from 0 to $\infty$. The $k$ integral can be extended over the whole real axis introducing a negligible error [10]; hence this yields a residual integral and the intermediate result

$$D_{\Delta \Omega}(r) = \int_{0}^{\pi} d\phi \int_{0}^{\pi} d\theta \sin \theta \left| \lambda k_0 \right|^2 \frac{\pi}{\gamma_0} e^{-\gamma_0 |\cos \theta|} e^{i \omega_0 r \cos \theta}.$$

(12)

Neglecting the very smooth envelope $e^{-\gamma_0 |\cos \theta|}$ and inserting the coupling according to Eq. (7), we finally obtain

$$D_{\Delta \Omega}(r) = \frac{3}{2} \frac{1}{2d_{\parallel}^2 - d_{\perp}^2} \left\{ \frac{\sin(k_0 r)}{d_{\parallel}^2} + \left[ \frac{\sin(k_0 r)}{(k_0 r)^3} - \frac{\cos(k_0 r)}{(k_0 r)^2} \right] \right\} \times \left( 2d_{\parallel}^2 - d_{\perp}^2 \right),$$

(13)

where $d_{\parallel}$ and $d_{\perp}$ denote the parallel and the orthogonal components, respectively, of the dipole moment $d$ with respect to $r$ and $\sin(x) = (\sin x)/x$. It follows from the behavior of the sinc function that the coherence is destroyed for $k_0 r \geq 1$; i.e., when $r$ is of the order of the wavelength $\lambda$. This is indeed what one expects from a simple physical argument: The radiation could be collected by a microscope to image the emitter. It is well known from classical theory that in this way one can determine the position of the emitter with an accuracy that is roughly given just by $\lambda$. Hence, quantum correlations must be restricted to a spatial region of extension $\lambda$.

Moreover, one observes from the result (13) that coherence is damped off faster in the direction of $r$ parallel to the dipole moment than perpendicular to $d$. This feature becomes evident from Fig. 1.

In what follows we will specialize to the case of a randomly oriented dipole; then the second term in curly brackets of Eq. (13) vanishes.

B. $\Delta \Omega < 4\pi$

For a randomly orientated dipole the squared coupling constant (7), $|\lambda d|^2$, reduces to $g_{\perp}^2 d^2$. Even with this simplification, the integration over the polar angles in Eq. (11) becomes rather involved for the general case of a fixed detector position with collecting angle $\Delta \Omega$ as well as an arbi-
trary displacement \( r \). It is only when we adapt the shape of \( \Delta \Omega \), for given \( r \), to a polar coordinate system with its polar axis in the \( r \) direction, i.e., assume \( \Delta \Omega \) to be bound by \( \theta_1 \leq \theta \leq \theta_2 \), \( \varphi_1 \leq \varphi \leq \varphi_2 \), that the integration can be done analytically. Neglecting the effect of the finite atomic lifetime \( \gamma_0^{-1} \), the integrand, as a function of \( k \), can be approximated, up to a constant factor, by a delta function \( \delta(k-k_0) \), where \( k_0 = \omega_0/c \). Then the integral (11) is readily evaluated yielding

\[
D_{\Delta \Omega}(r) = \frac{\exp(ik_0r \cos \theta_1) - \exp(ik_0r \cos \theta_2)}{ik_0r(\cos \theta_1 - \cos \theta_2)}
\]

\[
= \sin c(k_0r[\cos \theta_1 - \cos \theta_2]/2)
\]

\[
\times \exp[ik_0r(\cos \theta_1 + \cos \theta_2)/2].
\]  (14)

Note that \( D_{\Delta \Omega}(r) \) does not depend on \( \varphi_2 - \varphi_1 \) when normalized such that \( D(0) = 1 \). We see from Eq. (14) that, in contrast to the case \( \Delta \Omega = 4\pi \), the decoherence factor is now a complex function. For the extreme case of a detector covering an infinitesimal solid angle \( d\Omega \), Eq. (14) simplifies to

\[
D_{d\Omega}(r) = \exp[ik_0r].
\]  (15)

This simple result shows that a photon emitted into a precisely defined direction does not decohere the atomic density matrix, but only shifts its phase. Hence it is not the photon's recoil that is primarily responsible for the decoherence effects, but the optical resolving power of the employed detector. The destruction of coherence is described by the modulus of \( D(r) \) that follows from Eq. (14) as

\[
|D(r)| = \left| \sin c(k_0r[\cos \theta_1 - \cos \theta_2]/2) \right|
\]

\[
= \left| \sin c\left( k_0r\sin \frac{\theta_1 + \theta_2}{2} \sin \frac{\theta_2 - \theta_1}{2} \right) \right|.
\]  (16)

One learns from Eq. (16) that \( |D(r)| \), for fixed \( \theta_2 - \theta_1 \), falls off faster the closer the detector's central axis along \( (\theta_1 + \theta_2)/2 \) comes to the equator of the polar coordinate system at \( \theta = \pi/2 \). Since the polar axis is given by the \( r \) direction, this means that the decoherence is strongest in the plane perpendicular to the direction of observation. Moreover, it becomes obvious from Eq. (16) that \( D(r) \) falls off slower, the smaller \( \theta_2 - \theta_1 \); i.e., the smaller \( \Delta \Omega \).

In the more practical case of a detector with a collecting angle, the form of which does not depend on the direction of \( r \), we will choose the central axis of \( \Delta \Omega \) as the polar axis and the detector to be a polar cap, \( 0 \leq \varphi \leq 2\pi, 0 \leq \theta \leq \theta_1 \). Then, characterizing the direction of \( r \) by the polar angles \( \phi \) and \( \Theta \), we have to calculate the integral

\[
D(r) = \text{const} \int_0^{\theta_1} \sin \theta \, d\theta \int_0^{2\pi} d\varphi \exp[ik_0r(\cos(\varphi - \phi)
\]

\[
\times \sin \theta \sin \varphi \cos \Theta].
\]  (17)

The integral over \( \varphi \) yields the Bessel function \( I_0 \); i.e., we arrive at the result

\[
D(r) = \text{const} \int_0^{\theta_1} \sin \theta \, d\theta \, I_0(k_0r \sin \theta \sin \phi)
\]

\[
\times \exp[ik_0r \cos \theta \cos \Theta].
\]  (18)

This integral has been evaluated numerically. The results are depicted in Fig. 2. One recognizes from Fig. 2(a) that the decoherence is much stronger in the direction of \( r \) orthogonal to the detector axis, compared to the parallel orientation. Moreover, it is obvious that with growing collecting angle \( \Delta \Omega \) the decoherence effect becomes stronger, as predicted also by the analytic solution (16).

In summary we have used a recent extension of the Weisskopf-Wigner theory of spontaneous emission to derive an explicit expression for the decoherence function. In position representation this decoherence function simply is a factor changing the density matrix of the atomic center-of-mass wave function. We have specified the underlying approximations and studied various observational conditions.

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[9] The denominator of Eq. (6) can be factorized into \( c^2(k-k_-)(k-k_+) \). Eventually an integral over all positive \( k \) is to be performed; see Eq. (11). The two poles contribute, but \( k_\pm \sim -2Me/h \) is very far off resonance and has to be discarded on physical grounds, whereas \( k_- = (k_0 - i \gamma_0/c)(1 - \cos \Theta/Mc) \) remains, with \( \Theta \) as the angle between \( p \) and \( k \). With our assumption \( v < c \) we therefore end up with the denominator \( 2Mc^2(\omega - \omega_0 + i \gamma_0)/h \) in Eq. (6) and hence with Eq. (9) as given above.