

Dispersion Compensation for Atom Interferometry

Tony D. Roberts, Alexander D. Cronin¹, Martin V. Tiberg, David E. Pritchard

Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

¹ current address: University of Arizona, Tucson, Arizona 85721

(Dated: May 30, 2003)

A new technique for maintaining high contrast in an atom interferometer is used to measure large de Broglie wave phase shifts. Dependence of an interaction induced phase on the atoms' velocity is compensated by applying an engineered *counter phase*. The counter phase is equivalent to a rotation and precisely determined by a frequency, and can be used to measure phase shifts due to interactions of unknown strength. Phase shifts of 150 radians (5 times larger than previously possible) have now been measured in an atom beam interferometer and we suggest that this technique can enable comparisons of atomic polarizability with precision of one part in 10,000.

PACS numbers: 03.75.Dg, 39.20.+q

Keywords:

Atom interferometers are now precision tools for measuring interactions that cause a differential phase shift between atom waves in two separated paths. For example, measurements of platform rotation [1, 2] and acceleration [3, 4], gravity gradients [5], and atomic polarizability [6] have each been made using atom interferometers to directly measure a corresponding atomic de Broglie wave phase shift. The precision of these measurements made with cw atom beams can be improved by compensating for dispersion. *Dispersion*, i.e. a correlation between atom wavelength and phase shift, has limited the interferometer in [1, 6] to a maximum of 35 radians of interaction induced phase shift before contrast is reduced by $1/e$. In ref [2] a rotation induced phase of 10 radians could reduce the contrast by $1/e$. Here we present a new technique to maintain high contrast while studying large de Broglie wave phase shifts without reducing atom flux. It consists of two separated regions that induce time-dependant phases in a way that the net applied phase depends on the atom's velocity. We shall describe this technique and show that it is equivalent to rotation at an accurately known angular frequency.

The source of contrast loss addressed here comes from the experimental spread in atomic velocity combined with dispersion. Most interactions are dispersive because the interaction induced phase shift, or *interaction phase*, depends on velocity to some power: $\phi_{int}(v) \propto v^n$. The factor n equals -1 for interactions with phase shifts that depend on transit time, such as platform rotation or uniform fields applied to one arm of an interferometer. The factor n equals -2 for gravitationally induced phase shifts and also for electric or magnetic field gradients across an interferometer made with gratings. For $n \neq 0$, a spread in velocity leads to an inhomogeneous phase and hence a loss of contrast. Under such circumstances the statistical power in a measurement of interaction strength is optimized at a rather small interaction phase. For a Gaussian atomic velocity distribution, this phase is: $\phi_{int}(v_0) = |\frac{1}{n}| \frac{v_0}{\sigma_v}$, where v_0 is the average and

σ_v is the r.m.s. width of the velocity distribution. The supersonic atom beam in ref [1, 4] has $\frac{v_0}{\sigma_v} = 25$, which limits the most sensitive measurements to an interaction phase of 25 radians.

Dispersion compensation enables measurements of much larger interaction induced phase shifts. We demonstrate this by using an engineered *counter phase* to cancel dispersion, and regain high contrast. The technique has many advantages for precision measurements of an interaction strength—a very large interaction phase can now be measured, all the atom flux is used, and the need to precisely measure the velocity of the atoms is eliminated. It is a quantum extension of the classical velocity multiplexing technique that used mechanical choppers to modify the velocity distribution of the beam [7].

In an earlier proposal, Clauser [8] noted that a magnetic field gradient can compensate for gravitationally induced phase shifts. This idea in essence measures one interaction in terms of another, the overall error being a combination of the two errors separately. In contrast, the counter phase used here is determined by a frequency that can be set to a precisely known and stable value. Our technique is more closely related to methods developed in [9] and used in [2, 3, 4, 5] where dispersion compensation is achieved by moving the gratings to simulate platform rotation or acceleration.

To create the counter phase, two phase shift regions spaced a distance $L_{shifters}$ apart are used to produce differential phase shifts between the arms of the interferometer (Fig. 1). They apply a saw-tooth ramp — increasing the applied phase linearly from zero to $\pm 2\pi$ and then abruptly returning to zero—with frequency f . The phase shifts must be opposite in sign, as shown in Figure 2 and can be represented, modulo 2π , as

$$\begin{aligned}\phi_1(t) &= +2\pi ft \\ \phi_2(t) &= -2\pi ft.\end{aligned}\tag{1}$$

The sum of these phase shifts makes the counter phase

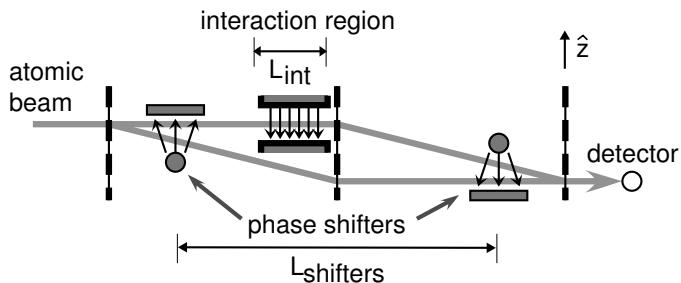


FIG. 1: A schematic of the three-grating atom interferometer. An interaction region is in the center, and the two phase shifters used for dispersion compensation are located on either side.

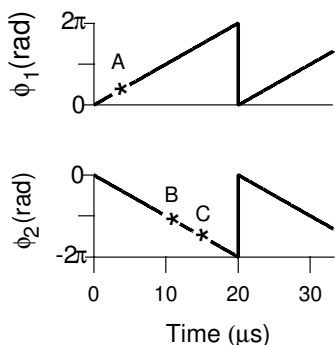


FIG. 2: The time-dependent phases, $\phi_1(t)$ and $\phi_2(t)$, introduced by each phase shifter. The sum, $\phi_1(t) + \phi_2(t)$ depends on velocity. For example, faster atoms may get phase shifts A and B, while slower atoms get the phase shifts A and C.

(modulo 2π):

$$\begin{aligned} \phi_{counter}(t, v) &= \phi_1(t) + \phi_2(t + L/v) \\ &= -2\pi f L_{shifters}/v. \end{aligned} \quad (2)$$

Importantly the total counter phase depends on the atom's time of flight between the two phase shifters, i.e. it has v^{-1} dispersion, but is independent of time. The dispersion scales with ramp frequency, hence it can compensate for a dispersive interaction of any strength. Fidelity of the two phase shifts $\phi_{1,2}$ to Eq. 1 becomes less critical when the ramp frequency is large as is discussed later.

The cancellation of dispersive effects in principle works perfectly if the interaction phase is proportional to v^{-1} . One type of interaction is a region of uniformly different potential of length L_{int} on one interferometer arm. The change in energy, $\hbar\omega_{int}$, for an atom inside the field is the same for atoms of all velocities, but the phase shift caused by the interaction depends on the transit time L_{int}/v of the atom passing through the region:

$$\phi_{int}(v) = \omega_{int} L_{int}/v \quad (3)$$

With this interaction alone, a spread in velocity from the atom source creates a spread in ϕ_{int} that destroys the contrast of the interference fringe and limits the size of ω_{int} that can be measured. By adding the counter phase, the total phase shift is:

$$\phi_{int}(v) + \phi_{counter}(v) = (\omega_{int} L_{int} - 2\pi f L_{shifters})/v \quad (4)$$

At the rephasing frequency, $f_{reph} \equiv \omega_{int} L_{int}/2\pi L_{shifters}$, the total phase shift is zero for all velocities. There is no net dispersion, and the fringe contrast should be ideal.

This method of rephasing the interference pattern can dramatically improve a precision measurement of atomic polarizability such as was made with the MIT interferometer [6]. We can now apply an interaction phase shift exceeding 150 radians for best signal to noise, compared with 25 radians previously. Furthermore, instead of measuring the atom beam velocity (and velocity distribution) and then modelling the phase shift, we need to measure only the ramp frequency and the distance $L_{shifters}$. In this proof-of-principle experiment, these advantages make it possible to determine the polarizability of sodium atoms with the same statistical precision as [6] in 1/15 the time.

We have implemented the phase shifters by using small regions of electric field gradient that create a phase shift by exposing each path of the interferometer to a different field. The gradient field is produced by a charged cylinder with radius $r = 0.5$ mm at a voltage of $V_0 \approx 2$ kV. The paths of the interferometer pass between the cylinder and a ground plane, which is a distance $a = 1.5$ mm from the cylinder axis (Fig. 3). The phase difference between the paths (separated by distance $w \approx 50$ μ m) is

$$\phi_{1,2}(t) \approx \pm \frac{\pi}{2} \ln^{-2}\left(\frac{2a}{r}\right) \frac{\alpha w}{v x^2} V_0(t)^2 \quad (5)$$

where x is the average distance of the two paths from the cylinder axis and α is the polarizability of the atoms. The two cylinders are oriented on opposite sides of the interferometer so that they can apply opposite relative phases as required by Eq. 1. To create a linear ramp in phase, a voltage must be applied to the cylinders with time dependence $V_0(t) \sim \sqrt{t}$. We approximate this ideal square-root shape in time by filtering a rectangle-wave high voltage (with duty cycle $p \approx 90\%$) using an RC circuit. During the off-cycle a diode drains the capacitor, quickly returning the voltage to zero. Using this voltage waveform, the phase produced by the gradient fields ramps approximately linearly from 0 to 2π during the on cycle:

$$\phi_{1,2}(t) = \pm \gamma (1 - e^{-t/RC})^2 \quad \text{for } 0 < t < p/f. \quad (6)$$

The parameter γ , proportional to the strength of the gradient field, can be changed by adjusting the amplitude of

the square wave or the position x of each cylinder. Setting to $\gamma = .83\pi$ rad and $RC = 2.4/f$ best approximates the perfect ramp defined by Eq. 1. High fidelity to Eq. 1 is not required if the ramp frequency is much larger than the inverse time of flight between the two phase shifters, since the deviation in phase is small compared to the additional phase of $2\pi m$ corresponding to m ramp cycles.

Applied to the measurement of polarizability [6], the phase shifters are used in conjunction with an interaction phase produced by a parallel plate capacitor (with voltage difference V and plate separation d) that makes a constant electric field \mathcal{E} surrounding one path of the interferometer for a length L_{int} . The change in energy of an atom in the electric field (with magnitude $\mathcal{E} = V/d$) is given by the polarizability α :

$$\hbar\omega_{int} = \frac{1}{2}\alpha V^2/d^2 \quad (7)$$

To determine α , the phase-shifted interference pattern is measured as oscillating atom beam intensity versus the transverse position z of one grating. For atoms of velocity v , the interference pattern without a counter phase is

$$I_v(z) = N + A \cos[k_g z + \phi_{int}(v)], \quad (8)$$

where N is the average intensity, A is the amplitude of the fringe, and $k_g = 2\pi/(100 \text{ nm})$ is the grating wavevector. The measured interference pattern is a weighted average over the velocity distribution (which is approximately Gaussian with $\sigma_v/v_0 \approx 0.04$, where v_0 is the average velocity (1-2 km/s) and σ_v is the rms velocity width):

$$\begin{aligned} I(z) &= \int dv P(v) (N + A \cos[k_g z + \phi_{int}(v)]) \\ &= N + AC' \cos[k_g z + \phi'], \end{aligned} \quad (9)$$

where the phase and contrast are

$$\begin{aligned} \phi' &= \phi_{int}(v_0) \\ C' &= \exp\left[-\frac{1}{2}\left(\frac{\sigma_v}{v_0}\right)^2 \phi_{int}^2(v_0)\right], \end{aligned} \quad (10)$$

With the phase shifters and the interaction region both on, the phase and contrast of the resulting interference pattern are:

$$\begin{aligned} \phi' &= \phi_{int}(v_0) - 2\pi f L_{shifters}/v_0 \\ C' &= \exp\left[-\frac{1}{2}\left(\frac{\sigma_v}{v_0}\right)^2 (\phi_{int}(v_0) - 2\pi f L_{shifters}/v_0)^2\right]. \end{aligned} \quad (11)$$

Note this is merely a shifted Gaussian that has the same width independent of ramp frequency. In principle the peak contrast remains $C' = 1$.

Measurements of contrast vs. interaction phase (Fig. 4) were made with the phase shifters ramping at

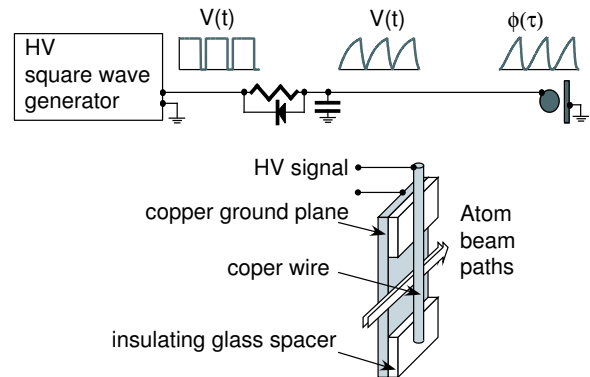


FIG. 3: A schematic of the electronics driving each phase shifter, including a blowup showing how the gradient field region is constructed. Note that $\phi_1 \sim V^2(t)$ is a good approximation to a linear sawtooth.

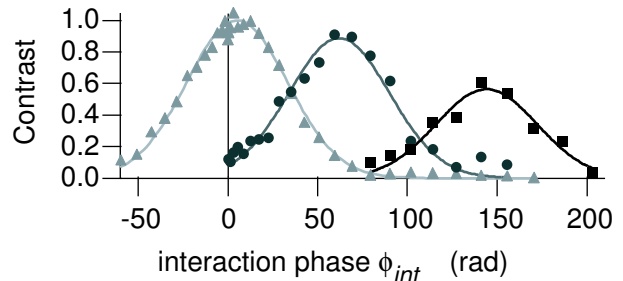


FIG. 4: Contrast of the interference pattern as a function of interaction induced phase, $\phi_{int}(v_0)$. With no dispersion compensation (\blacktriangle) contrast peaks near $\phi_{int}(v_0) = 0$. Using the time-dependent phase shifters, contrast revives at $\phi_{int}(v_0) = 62$ rad for a ramp frequency $f = 17$ kHz (\bullet), and at $\phi_{int}(v_0) = 144$ rad for $f = 40$ kHz (\blacksquare).

fixed frequencies: $f = 0, 17.0,$ and 40.0 kHz. A revival in contrast occurs when $\phi_{int}(v_0) = 2\pi f L_{shifters}/v_0$. However, the maximum rephased contrast is less than the ideal 100% because of imperfections in the phase shifters such as non-linearity of the implemented phase ramp Eq. 6. The rightmost curve (\blacksquare) in Fig. 4 demonstrates that polarizability can now be measured at an interaction phase as large as 150 rad.

Polarizability can be determined using this technique by finding the parameters at which the total phase is exactly zero. Then, there is high contrast, and no explicit dependance on velocity:

$$\omega_{int} L_{int}/v = 2\pi f L_{shifters}/v. \quad (12)$$

Phase measurements are shown in Fig. 5 and are fit to a line whose zero crossing determines V_{reph}^2 . Then the polarizability is:

$$\alpha = \frac{2hf L_{shifters} d^2}{L_{int} V_{reph}^2}. \quad (13)$$

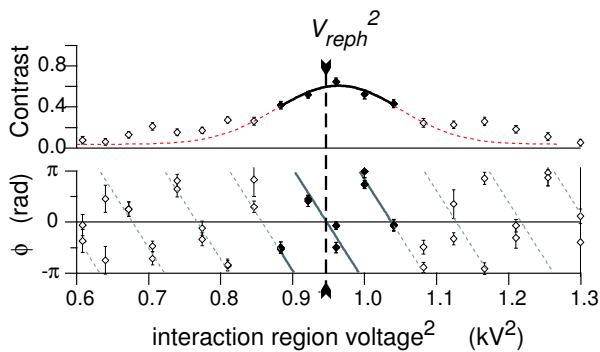


FIG. 5: The relative contrast and phase as a function of the voltage-squared in the interaction region. Atomic polarizability can be determined by finding the voltage, V_{reph}^2 , at which the fitted total phase is zero (and the contrast is maximum). The fit was made only to the 10 central data points (◆). The dashed line is an extrapolation from the fit, shown for comparison with the data outside the fit region (◇).

The precision of the polarizability measurement depends on the precision $\Delta\phi'$ with which the interaction phase can be measured, and also on $\phi_{int}(v_0)$:

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta\phi'}{\phi_{int}(v_0)}. \quad (14)$$

While the statistical uncertainty in phase grows as the contrast is lost, $\Delta\phi' \sim 1/C'$, the fractional error in polarizability is reduced by making the interaction phase $\phi_{int}(v_0)$ larger.

Without the counter phase (e.g. in the leftmost curve (▲) of Fig. 4) the best signal-to-noise ratio occurs at an interaction phase of $\phi_{int}(v_0) = v_0/\sigma_v = 25$ radians. Precision in $\Delta\phi'$ here is ~ 800 mrad/ $\sqrt{\text{sec}}$, which makes us require roughly 300 sec of measurement time to achieve a fractional precision in polarizability of $\Delta\alpha/\alpha = 0.2\%$.

With the counter phase we achieve the same 0.2% statistical precision in α with only 20 sec of measurement. The ten phase measurements used in the linear fit (◆ data points in Fig. 5 constitute a total of 20 sec of measurement, and the resulting uncertainty in the interaction phase is $\Delta\phi' = 130$ mrad, while the interaction phase itself is $\phi_{int}(v_0) = 66$ rad. Thus, the fractional uncertainty in polarizability is $\frac{\Delta\alpha}{\alpha} = \frac{130 \text{ mrad}}{66 \text{ rad}} = 0.2\%$. Using the contrast revival demonstrated at 150 rad, it should be possible to measure the interaction phase to 10^{-4} statistical precision with 50 minutes of data.

Sources of systematic error include uncertainty in $L_{shiffters}$, the calibration of the phase shifts $\phi_{1,2}$, and the dimensions of the interaction region. (The frequency f can be determined essentially without errors.) The distance $L_{shiffters}$ is on the order of 1 m and can be measured to one part in 10^4 with a ruler, or much better if measured interferometrically. The most difficult new systematic errors to analyze stem from deviations in the ramped phase from the ideal form in Eq. 1. If the ramp

maxima deviate from $\pm 2\pi$ by ϵ but are at least symmetric, i.e. $\phi_1(t) = -\phi_2(t)$, the corresponding error in α is smaller than $\epsilon/2\pi$ by a factor equal to $L_{shiffters}v^{-1}f^{-1}$. This is because each time $\phi_2(t)$ returns to zero during an atom's time of flight, it is equivalent to adding a precise integer times -2π to the counter phase. However, if $\phi_1(t) \neq -\phi_2(t)$ the error in phase will be:

$$\phi_{error} = \langle \phi_1(t) + \phi_2(t) \rangle_t \quad (15)$$

where $\langle \dots \rangle_t$ represents an average over one period of the ramp signal [10]. To correct for this, ϕ_{error} was measured in steady state, i.e. with $f = 0$, and both phase shifters on.

The dispersion compensation technique presented here eliminates two of the three major sources of uncertainty that limited the precision in α obtained in [6] (i.e. contrast loss and uncertainty in the velocity distribution). However a measurement of sodium's polarizability at the 10^{-4} level will still be difficult due to uncertainty in the geometry of the interaction region. To attain 10^{-4} precision in the parameter d^2/L_{int} in Eq. 13, the dimensions must be measured to within 10^{-7} meters and the electric field modelled very accurately. New techniques can meet these requirements [10]. However, by measuring the the ratio of polarizability of two alkali atoms using the same interaction region and phase shifters, uncertainties in all the geometrical lengths ($L_{shiffters}$, L_{int} , and d) will be common to both so the resulting ratio of polarizabilities will be limited only by statistical errors if the atoms traverse the same path through the interaction region.

Used as a Sagnac gyroscope the interferometer is also sensitive to rotations which create a $1/v$ -dependent phase shift:

$$\phi_{int}(v) = 2k_g L_g^2 \Omega / v \quad (16)$$

where Ω is the rotation rate and L_g is the distance between the interferometer's gratings, and k_g is the wave number of the gratings [1, 2]. Our technique exactly cancels such dispersion and may therefore be regarded as equivalent to physically rotating the interferometer. Thus, one could servo the engineered phase and use the integral of the rephasing frequency as a measurement of accumulated angular displacement.

The rephasing technique will also work to some extent for interactions with *any* velocity dependence, $\phi_{int}(v) \propto v^n$. For example, the interaction phase with optimum signal to noise can be increased by the factor $v_0/|n+1|\sigma_v$ since

$$\phi_{int}(v_0) < \left| \frac{1}{n(n+1)} \right| \left(\frac{v_0}{\sigma_v} \right)^2 \quad (17)$$

is allowed by using the v^{-1} counter phase presented here.

In conclusion, we have demonstrated a new method to maintain high contrast of the interference pattern while studying large de Broglie wave phase shifts in an atom

interferometer. The method will be of immediate benefit to new precision measurements of atomic polarizability. In addition, the technique should serve to restore contrast in any experiment where the path length difference grows larger than the atomic coherence length.

-
- [1] A. Lenef, T. D. Hammond, E. T. Smith, M. S. Chapman, R. A. Rubenstein, and D. E. Pritchard, Phys. Rev. Lett. **78**, 760 (1997).
- [2] T. Gustavson, P. Bouyer, and M. Kasevich, Phys. Rev. Lett. **78**, 2046 (1997).
- [3] A. Peters, K. Chung, and S. Chu, Nature **400**, 849 (1999).
- [4] A. Peters, K. Chung, and S. Chu, Metrologia **38**, 25 (2001).
- [5] F. J. S. M. McGuirk, J.M. and M. Kasevich, Phys. Rev. A **65**, 033608 (2002).
- [6] C. R. Ekstrom, J. Schmiedmayer, M. S. Chapman, T. D. Hammond, and D. E. Pritchard, Phys. Rev. A **51**, 3883 (1995).
- [7] T. D. Hammond, D. E. Pritchard, M. S. Chapman, A. Lenef, and J. Schmiedmayer, Appl. Phys. B-Lasers Opt. **60**, 193 (1995).
- [8] F. Clauser, Physica B **151**, 262 (1988).
- [9] M. Kasevich and S. Chu, Appl Phys B **54**, 321 (1992).
- [10] T. D. Roberts, PhD thesis, MIT (2002).