Velocity Multiplexing for Precision Atom Interferometry

presented by

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Submitted in partial fulfillment of the Requirements for the Degree of

1. Bachelor of Science
   at the
   MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2. Master of Science
   at the
   ROYAL INSTITUTE OF TECHNOLOGY

June, 2001

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TRITA-FYS-1078
ISSN 0280-316X
Abstract

Velocity multiplexing is a new tool of atom interferometry, which allows large dispersive interactions to be studied without total loss of contrast. This thesis describes a new velocity multiplexing scheme based on electrostatics, which achieves a factor of four improvement in intensity and a factor of two improvement in the Signal to Noise Ratio compared to previously proposed schemes. For the first time, velocity multiplexing has also been implemented and tested. The data is in good agreement with the new theory developed in this thesis.
Preface

This thesis describes and evaluates a new experimental method for realizing velocity multiplexing in an atom beam interferometer. A background knowledge similar to that of a new graduate student in physics is assumed. The thesis is based upon laboratory work in Professor David E. Pritchard's Atom Interferometry group at MIT between April 2000 and May 2001. Many times during this period, I have struggled to understand many of the basic concepts of the MIT setup, which are already assumed in all available references. This work aims to bridge the divide between these references and people new in the field. I hope it will be a useful resource for understanding atom interferometry in general, and velocity multiplexing in particular.
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1 Introduction

This thesis describes a new experimental method of velocity multiplexing in an atom interferometer and demonstrates its feasibility. The basic utility of velocity multiplexing for studying velocity dependent phase shifts in an atom interferometer has been discussed previously [HPC95,HAM97]. However, the analysis of this thesis extends further. The electronic chopping scheme outlined herein has advantages in terms of efficiency and flexibility relative to previously suggested schemes. For the first time ever, a velocity multiplexing scheme has also been implemented and tested in an atom interferometer. The results are presented in this thesis.

The first chapter starts by explaining the intuitive picture of what velocity multiplexing is. The following sections provide a brief historical background of atom interferometry and describe the apparatus used in the MIT experiments. In the end of the first chapter, the interference pattern and the velocity distribution are discussed, and the fundamental equations that are used in later chapters are explained.

Chapter two highlights some shortcomings of previous atom interferometry experiments, and explains the benefits of the velocity multiplexing scheme. Chapter three describes several possible implementations. The theory is found in chapter four, and experimental results are presented at the end of the thesis.

1.1 Intuitive picture

Velocity multiplexing is an experimental technique of atom interferometry, which aims to reduce the effects of the velocity spread that naturally exist in all atom beams. If a velocity dependent interaction, e.g. one with an electric or magnetic field, is studied with an atom interferometer, then the variations in velocity will reduce the observed interference. If the velocity variations are too large, there might not be any interference at all. Velocity multiplexing overcomes this velocity smearing by selecting velocities which all contribute constructively in phase to form the final interference pattern.

Atom interferometry is based on interference between two components of an atom's wave function which have a relative phase between them. Velocity multiplexing relies on the periodicity of this phase, i.e. that phase shifts of 0, 2\pi, 4\pi, 6\pi, … constitute the same wave function. The width of the velocity peaks used in velocity multiplexing can be adjusted by the
frequency of a pair of choppers, such that the velocity dependent interaction that is studied changes the phase of each adjacent peak by exactly $2\pi$. Then all the atoms in the center of the peaks will have the same phase, and the 'smear' caused by the variations in velocity is reduced.

Fig. 1-1A shows how the velocity distribution of an atom beam is changed by the velocity multiplexing scheme. The original velocity distribution provides an envelope for the chopped distribution, which has alternating valleys and peaks that extend to one half of the envelope. The name "velocity multiplexing" comes from the fact that multiple velocities or peaks are selected, rather than one. The spacing of these peaks in velocity space is such that they all contribute with the same phase to form a high contrast interference pattern.

The workings of two square wave choppers are intuitively described in the distance vs. time diagram in fig. 1-1B. The horizontal blocks indicate when the choppers are on as they move synchronously in time, and the slanted lines represent individual atoms moving at different velocities. The velocity of an atom is equal to the distance from one chopper to the other divided by the flight time between the choppers. Thus, the velocity is just the slope of the line itself. The spread of the velocity distribution is represented by the angular spread of the atom trajectories.
Fig 1-1A and 1-1B are intimately related. The velocity peaks in fig. 1-1A correspond to the lines in fig. 1-1B with a slope, such that they intersect the two choppers exactly an integral number of periods apart. Atoms with these velocities are guaranteed to pass through the second chopper once they have passed the first. Analogously, the lines that intersect the two choppers an odd number of half periods apart will never pass through both of them. The choppers constitute a total blockage for these lines, which correspond to velocities at the bottom of the valleys in fig. 1-1A.

In experiments it is usually desirable to study as large interactions as possible, so that the phase error is dominated by a large phase shift producing a small relative error. To achieve the required \(2\pi\) phase difference between adjacent peaks, the velocity spacing between the peaks must be roughly inversely proportional to the strength of the interaction. The velocity spacing is also inversely proportional to the applied chopping frequency. Thus, the study of a strong interaction requires a fast chopping frequency to achieve the \(2\pi\) phase difference between adjacent peaks.

### 1.2 Introduction to atom interferometry

Matter waves have been studied ever since 1924, when de Broglie proposed that every particle has wavelike properties and is associated with a characteristic wavelength. The first experimental evidence of matter waves, based on electron diffraction, was obtained in 1927 [DAG27, THR27]. Matter-wave interferometry was first manifested with electrons in the 1950's [MAR 52] and with neutrons in the 1960's [MAS62]. Not until the development of nanofabricated atom optics in the mid 1980's did the first atom interferometers take form [KEI91].

An interferometer of neutral atoms, as opposed to electrons or neutrons, has several advantages. It allows for precision studies of an atom's rich internal structure and strong interaction with the environment. In addition, the short deBroglie wavelength combined with a large beam intensity allows for high precision measurements.

The high precision experiments that have been done in Professor David E. Pritchard's group at MIT include the measurements of collision induced phase shifts in atom-atom scattering [SCE95, KOK01]. These works provide an improved picture of the mid- to long-range atom interaction potentials. In 1995, the polarizability of sodium was measured with a precision of 0.35%, which was a factor of 20 improvement over previous measurements [ESC95]. This thesis describes a method to further improve the results of that experiment, which is discussed in more detail in chapter 6.
The MIT interferometry group has additionally demonstrated previously untested quantum phenomena. The interference of sodium molecules was first seen in 1995 [CEH95]. The same year, the loss of coherence between interfering atoms, due to single photon-atom scattering was studied [CHL95]. This study has lately been generalized to multiple photon-atom scattering [KCR01].


1.3 The MIT interferometer

The MIT atom interferometer has been in operation since 1991, and its current configuration is drawn to scale in fig. 1-2. Its assembly and operation are described in detail in several early theses of the MIT group [KEI91, EKS93, CHA95], and later modifications are described in more recent works [HAM97, KOK01].

The apparatus consists of three different regions. In the source region, sodium is heated in an oven to 850 K, such that it evaporates and forms the atom beam. This formation process is further described in Sec. 1.5. After the beam escapes the oven, it is collimated by transverse and vertical slits, before entering the main region. The main region contains the actual interferometer where outside interactions are measured and studied. In order to reduce the interaction between the beam and the background gas, the region must be kept at a vacuum of less than $3 \times 10^{-7}$ Torr. Besides the interferometer, which is described in Sec. 1.4, the main region contains a Stern-Gerlach magnet, a 2$^{nd}$ transverse slit, and, in the polarizability measurements discussed in chapter two, a thin foil is inserted between the interfering arms, which allows for an electric interaction in only one of them.

The third and last region contains the detector. The detector is a 50 $\mu$m wide rhenium wire kept at 800 K. Sodium atoms that hit the wire are ionized as their valence electron tunnels into the wire. An electric field forces the ions into a high-gain channel electron multiplier, which outputs digital pulses corresponding to the detected ions. The position of the detector is controlled with a motor through a computer interface with micrometer precision, which also controls the collimating slits and the diffraction gratings that constitute the atom interferometer.
1.4 The interference pattern

The MIT atom interferometer is composed of three nanofabricated gratings, which coherently split and recombine the matter waves by means of diffraction. The 1\textsuperscript{st} order diffraction angle is given by
\[ \theta_{\text{diff}} = \frac{\lambda_{dB}}{\lambda_g}, \]
where \( \lambda_{dB} = \frac{h}{mv} \) is the deBroglie wavelength of each atom, and \( \lambda_g \) is the period of the gratings.

Recently, new gratings with a 100 nm period, as opposed to the 200 nm period gratings used before, have been installed, allowing for a greater diffraction angle and more sensitive interferometry. This thesis presents the first experimental results with these new gratings installed.

The MIT interferometer has a Mach-Zehnder geometry, where the incoming beam is split and recombined to interfere with itself, as shown in fig. 1-3. The first grating in the interferometer splits the incident atom wave into many diffracted orders. Two of these orders are directed back
towards each other by the second grating, and they form a spatial interference pattern at the location of the third grating. As long as the gratings have the same period and are equally spaced, the resulting interference pattern is independent of the individual atoms' velocities and corresponding wavelengths, and it has the same period as the gratings themselves.

The interference pattern could theoretically be detected at the location of the third grating, but it would require a very small detector sensitive to the 100 nm period of the interference fringes. Instead, the third grating filters or masks the fringes. Since the fringe and the grating periods are identical, the grating allows the same part of every fringe to proceed to the detector. If the grating is positioned to block the deconstructive interference as in the inset of fig. 1-3, a maximum intensity is transmitted, and vice versa. The interference pattern is measured as the grating is moved back and forth, while the different parts of the interference fringes are transmitted.

The intensity of the interference pattern is a function of the phase difference $\phi$ between the interfering plane waves $\psi_1$ and $\psi_2$ and it is proportional to $|\psi_1 + \psi_2|^2$, the square of the sum of the plane wave amplitudes. The standard intensity equation is derived as

$$I_0 = |\psi_1 + \psi_2|^2$$
\[ \psi_1 = A_1 e^{i \phi_1} \]
\[ \psi_2 = A_2 e^{i \phi_2} \]
\[ I_0 = |\psi_1 + \psi_2|^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\phi_2 - \phi_1) \]
\[ = N[1 + C \cos \phi] \]  
(1.2)

where \( N \) is the mean intensity and \( C \) is the contrast

\[ N = A_1^2 + A_2^2 \]
\[ C = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{2A_1A_2}{A_1^2 + A_2^2} \]  
(1.3)

Interference data obtained with the MIT apparatus is shown in fig. 1-4.

1.5 *The velocity distribution*

There would be no need for velocity multiplexing if all atoms in an atomic beam traveled at the same velocity. Unfortunately, as explained by the fundamental laws of thermodynamics, at finite temperature there will always be a certain spread in internal energy of the atoms leading to a distribution of velocities. The width and shape of this distribution is highly dependent on the source of the atom beam. Below, the simple effusive source is contrasted to the supersonic one, which is used in the MIT setup. A discussion of why the supersonic source is more suited in atom
interferometry experiments is complemented by the justification of the Gaussian shaped velocity
distribution, which is assumed in the rest of this thesis.

1.5.1 The Effusive source

The effusive source, also called a thermal beam source or ideal-hole source, gives the most
intuitive picture of gas kinetics. It consists of a heated chamber containing low pressure gas in
thermal equilibrium, where the mean free path of the atoms is equal to or greater than the
diameter of the hole through which the atoms escape to form the beam. Under these conditions
the atoms in the chamber obeys the Maxwell-Boltzmann distribution, and the probability to find
an atom in the velocity interval \( v \) to \( v + dv \) is

\[
P(v) dv = \frac{dN}{N} = \frac{4}{\sqrt{\pi}} \frac{1}{\beta^3} v^2 e^{-\frac{v^2}{\beta^2}} dv,
\]

where for any given volume \( dN \) is the number of atoms in the \([v, v+dv]\) interval, \( N \) is the total
number of atoms,

\[
\beta = \sqrt{\frac{2k_b T}{m}}
\]

is the most probable velocity, \( T \) is the absolute temperature, \( m \) is the mass of the atoms, and \( k_b \) is
Boltzmann's constant [RAM56].

The velocity distribution of the atoms in the beam looks slightly different, because the probability
of an atom escaping from the source is proportional to the atom's velocity. Thus, Eqn. 1.2 must be
weighted with an additional \( v \), and renormalized such that \( P(v) \) integrated over all velocities
equals unity. The end result is the velocity distribution

\[
P(v) dv = \frac{2}{\beta^4} v^3 e^{-\frac{v^2}{\beta^2}} dv.
\]

Eqn. 1.6 represents a Gaussian weighted by velocity cubed. The standard deviation of \( P(v) \) is 48% of
the most probable velocity \( \beta \), or 64% of the mean velocity. This spread is too large for precise
atom interferometry measurements, and a different 'beam machine' is needed.
1.5.2 The Supersonic source

The atomic beam used in the MIT experiment is supersonic, which creates a narrow velocity distribution. In addition, the pressure in the supersonic source is much higher than in the effusive source, and the mean free path is short compared to the exit hole diameter. Thus, a beam intensity gain, compared to the effusive source, of a factor of 1,000 is easily realized.

In the source, sodium is evaporated in a mixture of noble gases. The gases mix with the sodium and determine the velocity of the sodium beam, without reacting with the sodium, and without being recorded by the detector. The MIT apparatus can use an arbitrary mixture of noble gases to achieve continuous variation of the mean velocity $v_0$ in the 500-3000 m/s range, according to

Figure 1-5. The velocity distributions of an effusive source and a supersonic source with a 5% standard deviation. Both have a mean velocity of 1,000 m/s.
where \( T \) is the gas temperature in the source and \( m' \) is the average mass of all particles in the gas mixture weighted by their molar concentration[HAM97, KOK01].

The physics of supersonic beams is complicated, and involves two processes that the atoms undergo after leaving the source. First, there is an adiabatic expansion where the atoms accelerate to supersonic speeds. The thermal energy of the statistically distributed velocities inside the source is almost completely converted into kinetic energy of the directed flow [CAR92]. The supersonic state is reached because of the increased overall beam velocity, and because of the decreased internal beam temperature, which in turn lowers the speed of sound. The second process is the transition to free molecular flow. The number of collisions between atoms decreases rapidly, as the atoms reach close to their final speeds by the end of the expansion region. Thereafter, the expansion is just in the plane perpendicular to the beam direction [HAB77]. A skimmer is used to remove unwanted atoms from the expansion as the beam leaves the source region. The result of these processes is a very narrow velocity distribution around a mean velocity \( v_0 \).

Carnal [CAR92] shows that under assumptions satisfied by the supersonic expansion, the narrow velocity distribution is given by

\[
P(v) dv = \text{Norm} v^3 \exp \left[ -\frac{1}{2} \left( \frac{v - v_0}{\sigma_v} \right)^2 \right] dv ,
\]

where \( \text{Norm} \) is the normalization constant. In the MIT setup, it has been found experimentally that the standard deviation \( \sigma_v \) varies in 3-7% range of the mean velocity. In such a case, the \( v^3 \) factor of Eqn. 1.8 is approximately constant throughout the center velocity region of interest, and it can be factored into the normalization constant. Thus, to simplify the analysis, a pure Gaussian description will be used throughout this thesis

\[
P(v) dv = \frac{1}{\sqrt{2\pi} \sigma_v} \exp \left[ -\frac{1}{2} \left( \frac{v - v_0}{\sigma_v} \right)^2 \right] dv ,
\]

without any significant loss of qualitative or quantitative analysis.
2 Motivation for a velocity multiplexing scheme

The primary use of atom interferometry is the measurement of quantum-mechanical phases, where the phase differences between the paths of the interferometer are measured versus an applied interaction. If the phases are dispersive, i.e. they depend on velocity, then the spread in the velocity distribution of the atom beam blurs the interference pattern and the measurements will have reduced contrast. This chapter describes the physics and math underlying this concept in the case of polarizability measurements, and it discusses what properties a velocity multiplexing scheme must have to overcome part of the contrast reduction.

2.1 Polarizability

A static external electric field $E$ applied to a neutral atom will distort the atom's electronic charge distribution, and the atom will acquire a dipole moment $p$. If higher order corrections are ignored, then the induced dipole moment is proportional to the electric field

$$ p = \alpha E, \quad (2.1) $$

where the proportionality constant $\alpha$ is the polarizability of the atom.

The change in the atom's energy due to the induced dipole moment, i.e. the Stark effect, may be viewed in either a macroscopic classical or a microscopic quantum mechanical framework. According to classical electrostatics the energy of an induced dipole is

$$ U_{dipole} = -\frac{p \cdot E}{2} = -\alpha \frac{E^2}{2}, \quad (2.2) $$

whereas second order perturbation theory asserts that the ground state energy change due to the electric field is

$$ U_0^{(2)} = -e^2 E^2 \sum_{f \neq 0} \frac{\langle 0 | z | f \rangle \langle f | z | 0 \rangle}{U_f^{(0)} - U_0^{(0)}}, \quad (2.3) $$

where $e$ is the electron charge, $U_f^{(0)}$ is the energy of an unperturbed eigenstate, $|0\rangle$ and $|f\rangle$ are the ground state and an excited state, respectively.

The polarizability of an atom in the ground state is found by equating the classical and quantum mechanical results of Eqn. 2.2 and 2.3, which yields
\[ \alpha = 2e^3 \sum_{f \neq 0} \frac{\langle 0 \mid \hat{f} \mid f \rangle \langle f \mid \hat{f} \mid 0 \rangle}{U_f^{(0)} - U_0^{(0)}}. \] (2.4)

Measurements of atomic polarizability are compared to the matrix elements in Eqn. 2.4 as a test of atomic theory. Knowledge of the polarizability also enables predictions of many properties of atomic systems including the dielectric constant and index of refraction, the van der Waals constant between two polarizable systems, and the Rayleigh scattering cross section. A direct experimental determination of atomic polarizability constitutes a very sensitive check on the accuracy of electronic wave functions used in the calculations of these quantities. The polarizability of Na is \( 24.11(6) \) Å\(^3\) [ESC95].

2.2 Experimental methods to determine the polarizability

There are at least three fundamentally different experimental approaches to measure atomic polarizability. The first technique below does not rely on interferometry, and it can be fully explained classically. The latter two, however, rely on quantum mechanics and the wave properties of matter.

A. Electric field gradient deflection.

This was for a long time the most accurate technique known. In the 70s, a few high precision experiments were performed, which yielded the most accurate polarizability measurements for 20 years [MSM74, HAZ74]. These experiment relied on methods of deflecting an atomic beam in an electric field gradient as illustrated in figure 2-1A. However, the experiments were limited by both uncertainties in the characterization of the applied gradient and the velocity distribution of the atomic beam.

B. Homogeneous electric field in an atom interferometer with physically separated arms.

The MIT atom interferometer was the first interferometer with separated arms, which allows for an interaction in one arm only. This feature has been used to perform high-accuracy measurements of the polarizability of sodium [EKS93, ESC95]. As shown in fig. 2-1B, a homogeneous electric field was applied to one of the arms only, which changed the phase of this arm relative to the other. This technique has allowed much improved results in these works compared to the previous experiments. However, imperfect knowledge and control of the velocity distribution was a limiting factor in these experiments. Sec. 2.3 elaborates further on these limitations and motivates the need for velocity multiplexing.
C. Electric field gradient in an atom interferometer.

A new proposed method of measuring polarizability, which does not require a separated arm interferometer, is outlined in fig. 2-1C. Though this method suffers from some of the same uncertainties as method A, it has potential to accurately measure the relative polarizability between atoms, where the uncertainties due to geometry cancel. The method relies on a geometry where an electric potential is applied between a wire and a ground plate (or some similar geometry), which results in an electric field gradient across the path of the atoms. The atoms in the different interfering arms will acquire different phase shifts, because the electric field is stronger closer to the gradient wire. This gives rise to a relative phase difference between the atoms, which is readily deduced from the shifts in the detected interference pattern. This method forms the basis for the electrical choppers discussed in later chapters.

2.3 Analysis of the experimental methods

Because method A of the previous section is the most inaccurate one, and because it does not involve atom interferometry, it will not be further analyzed in this thesis. Method B and C, however, are of great interest both practically and theoretically in polarizability precision measurements. In both methods, an electric field interaction causes a relative phase shift between the interfering arms. This section introduces the common theory, before describing what differentiate the two methods from each other.
2.3.1 Common theory

When an electric field $E$ is applied across the path of the atoms, there is a shift in each atom's energy equal to the Stark potential $U$, which is given by Eqn. 2.2. This potential leads to a phase shift of each atom's wave function. The difference between the phases an atom accumulates along a path of the interferometer with the interaction potential on and off, can be written, in the WKB approximation, as

$$\Delta \phi = \int (k(z) - k_0) dz,$$

where $k(z)$ and $k_0$ are the perturbed and unperturbed wave vectors, respectively, and $z$ is the spatial direction, along which the atom travels. The WKB approximation assumes that the amplitude of the atom's wave function varies slowly, such that the second spatial derivative of the amplitude can be neglected when solving the Schrödinger equation. This assumption is valid when the kinetic energy $e$ of the atom is much greater than the interaction potential $U$. When the beam velocity is equal to a few hundred meters per second or more, then $e$ is several orders of magnitude greater than $U$, and Eqn. 2.5 may be expanded to the first order in $U/e$:

$$\Delta \phi = \frac{1}{\hbar} \int \sqrt{2m(E + U(z))} \, dz - \frac{1}{\hbar} \int \sqrt{2mE} \, dz$$

$$= \frac{1}{\hbar} \int \sqrt{2mE} \left(1 + \frac{U(z)}{2E}\right) \, dz - \frac{1}{\hbar} \int \sqrt{2mE} \, dz$$

$$= \frac{1}{\hbar} \int \sqrt{\frac{m}{2E}} U(z) \, dz$$

$$= \frac{1}{\hbar \nu} \int U(z) \, dz$$

where $\nu$ is the velocity of the atom. Eqn. 2.6 is one of the basic relationships in atom interferometry, which generalize to any interaction potential $U$.

2.3.2 Homogeneous electric field – method B

The MIT polarizability experiments in 1993 and 1995 used the experimental technique B of Sec. 2.2. An electric potential $V$ was applied between a flat conductor and a thin foil separating the arms of the interferometer. This procedure results in a uniform electric field, in one arm only, throughout the interaction region.
When \( E \) is constant throughout the interaction region, Eqn. 2.2 and 2.6 combined with \( E = \frac{V}{D} \), yield

\[
\alpha = \left( \frac{\Delta \phi}{V^2} \right) \left( \frac{D^2}{L_{\text{eff}}} \right) (2hV),
\]

where \( L_{\text{eff}} \) is the effective length of the interaction region, \( V \) is the voltage applied to one side of the interaction region across the distance \( D \).

The 1993 and 1995 MIT experiments measured \( \alpha \) to a 0.35% accuracy by separately determining the three factors of Eqn. 2.7. To determine the first factor, several data points were used to fit the phase shift \( \Delta \phi \) to a quadratic function of voltage with a statistical uncertainty of 0.2%. The second factor was determined by numerical calculations based on the geometry of the interaction region. The error was estimated to 0.1%. The third factor was the problematic one. It contained the most error, due to the 0.15% uncertainty in the mean velocity, and 10% uncertainty in the rms width of the velocity distribution.

The difficulty of accurately determining the third factor had to do with the coupling between the phase shift \( \Delta \phi \) and the velocity of each individual atom. Eqn. 2.6 shows that the acquired phase shift, due to the electric field interaction of each atom, is inversely proportional to its velocity, i.e. it is proportional to the time it spends in the interaction region. This interaction phase affects the interference pattern described in Sec 1.4, and Eqn. 1.2 must be written as the velocity-dependent interference pattern

\[
I(v) = N[1 + C \cos(\phi_0 + 2\pi \frac{v_{\text{int}}}{v})]
\]

\[
v_{\text{int}} = \frac{\alpha E^2}{h} \frac{L_{\text{eff}}}{2}
\]

where \( v_{\text{int}} \) is defined as the velocity for which the phase shift is \( 2\pi \). The total interference pattern is obtained by integrating the contribution of each atom (Eqn. 2.8) over the velocity distribution of the atom beam (Eqn. 1.9). The resulting integral is

\[
I = \int_{0}^{\infty} P(v) I(v) \, dv = N + \frac{NC}{\sigma_v \sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{1}{2} \left( \frac{v-v_0}{\sigma_v} \right)^2} \cos(\phi_0 + 2\pi \frac{v_{\text{int}}}{v}) \, dv.
\]
Two approximations will allow the integral to be evaluated analytically. First, change its lower bound from 0 to -8, to get a symmetric integration interval. The approximation is valid, because the tail of the Gaussian velocity distribution is negligible for $v < 0$. Second, expand the $1/v$ dependent term of Eqn. 2.9 around the mean velocity $v_0$ of the velocity distribution, using

$$v_0 = \frac{1}{v} - \frac{v - v_0}{v_0} + \mathcal{O}\left(\left(\frac{v - v_0}{v_0}\right)^2\right), \quad (2.10)$$

and ignore 2nd and higher order terms. This approximation is valid when the width of the velocity distribution $s_v$ divided by $v_0$ is much smaller than unity. In the MIT experiments this ratio is in the range 3-7%.

With the above approximations, Eqn. 2.9 transforms into an integral which is found in standard lookup tables:

$$I = N(1 + C' \cos(\phi_0 + 2\pi \frac{v_{\text{int}}}{v_0}))$$

$$C' = C \exp\left[-\frac{1}{2}\left(2\pi \frac{v_{\text{int}}}{v_0} \frac{\sigma_v}{v_0}\right)^2\right]. \quad (2.11)$$

Eqn. 2.11 shows that the observed contrast $C'$ decreases fast with increasing $v_{\text{int}}$, which is proportional to the electric field squared. In fig 2-2, the decay is plotted versus the phase shift applied to the mean velocity $\Delta\phi = 2\pi v_{\text{int}}/v_0$. The figure indicates that there is an upper limit on the
strength of the applied interaction, for which a contrast can be observed. One of the goals of the velocity multiplexing scheme is to remove this limit.

### 2.3.3 Electric field gradient – method C

Method C of Sec. 2.2 involves an electric field gradient across the two interfering paths, which gives rise to a relative phase difference between them. The electrical choppers, which are discussed throughout the rest of this thesis, rely on this method.

In the gradient field method, the relative phase difference $\Delta \phi$ acquired when the interaction is turned on, is proportional to $1/v^2$. The $1/v^2$-dependence results from two physically distinct sources that each contributes $1/v$. One of these velocity factors in the denominator is related to the time that each atom spends in the interaction region, which shows up in Eqn. 2.6. The other velocity factor is related to the separation of the paths. The absolute phase that each atom acquires is approximately linear in the distance to the gradient wire. Thus, $\Delta \phi$ is proportional to the path separation, which is proportional to the diffraction angle. Eqn. 1.1 shows that this angle is inversely proportional to the velocity.

A formal derivation of the $1/v^2$-dependence is provided in Sec. 4.3. For now, it suffices to know that it exists. Analogously to Eqn. 2.8, the interaction defines a velocity $v'_{\text{int}}$, for which the phase shift is $2\pi$,

$$I(v) = N[1 + C \cos(\phi_0 + 2\pi \frac{v'_{\text{int}}}{v^2})]. \quad (2.12)$$

The same approximations as in Sec. 2.3.2 can be used to obtain a closed formula for the interference integrated over the velocity distribution, after the series expansion

$$\frac{v_0^2}{v^2} = 1 - 2 \frac{v - v_0}{v_0} + O\left(\frac{(v - v_0)^2}{v_0^2}\right) \quad (2.13)$$

is applied to the last term of Eqn. 2.12. The end result,

$$I = N(1 + C' \cos(\phi_0 + 2\pi \frac{v'_{\text{int}}}{v_0^2}))$$

$$C' = C \exp\left[-2\left(2\pi \frac{v'_{\text{int}}^2}{v_0^2} \frac{\sigma_v}{v_0}\right)^2\right] \quad (2.14)$$
is similar to the case of a \(1/v\)-dependent phase shift. However, with the \(1/v^2\)-dependence the observed contrast \(C'\) is washed out even faster, due to the four times greater exponent. Consequently, in this case the limit on the strength of the applied interaction is even lower. In order to eliminate this shortcoming, which ultimately limits the accuracy of any method B or C polarizability measurement, a new improved experimental technique is needed.

## 2.4 Velocity multiplexing – the fix

In this chapter, two general problems of precision polarizability measurements have been identified. The velocity multiplexing scheme analyzed in this thesis addresses them both, and has the potential to significantly improve the experimental results. As a matter of fact, the scheme can improve any atom interferometry experiment which involves measurements of a dispersive phase. Such experiments include gravity and rotation phase measurements, as well as magnetic interaction.

The improvements of using the velocity multiplexing scheme and the problems it addresses can be summarized in two points.

1. It eliminates the need for precise knowledge of the mean and the width of the velocity distribution. Chapter 6 describes an experimental method, which does not require precise velocity knowledge.

2. It removes the upper limit on the strength of the applied phase, i.e. it eliminates the exponential decay of the contrast. Consequently, a greater range of data points may be acquired, and the relative phase error is reduced.

A quantitative analysis of these two improvements is provided in chapter 4-6.
3 The velocity multiplexing scheme candidates

The velocity multiplexing scheme can be implemented in several different ways. This chapter describes the methods which were considered in the MIT experiment for measuring the atom polarizability. In addition to describing the pros and cons of each method, section 3.2 and 3.3 explain why the mechanical and electrical chopper schemes were selected for implementation and how the schemes were in the end implemented.

Table 3-1. Properties of the various velocity multiplexing schemes that were considered for implementation. All the schemes but the single-velocity selector have contrast reductions up to 50%. The other properties are discussed throughout chapter 3.

<table>
<thead>
<tr>
<th>Velocity multiplexing scheme</th>
<th>Beam intensity after the choppers</th>
<th>Species independent</th>
<th>Position in the apparatus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-velocity selector</td>
<td>very weak</td>
<td>Yes</td>
<td>Anywhere</td>
</tr>
<tr>
<td>Mechanical choppers</td>
<td>25 %</td>
<td>Yes</td>
<td>Anywhere</td>
</tr>
<tr>
<td>Laser choppers</td>
<td>25 %</td>
<td>No</td>
<td>Before collimator</td>
</tr>
<tr>
<td>Deflection choppers</td>
<td>25 %</td>
<td>Possibly</td>
<td>Before collimator</td>
</tr>
<tr>
<td>Electrical choppers</td>
<td>100 %</td>
<td>Species adjustable</td>
<td>Inside the interferometer</td>
</tr>
</tbody>
</table>

3.1 Single-velocity selector

A single-velocity selector allows a small range of the velocity distribution to pass through. Eldridge [ELD27] performed the first implementation in 1927, when he used multiple spinning disks mounted on a single rod to block all velocities but the ones that managed to pass through the open slots in all the disks. By varying the slot sizes, the distances between the disks, and the rotation frequency of the rod Eldridge employed the selector to measure the shape of various velocity distributions. A single-velocity selector may also be implemented by using, in series, any of the methods for blocking atoms that are described later in this chapter.

The problem with a single-velocity selector is not its implementation, but its efficiency. High atom flux is needed in precision measurements in order to reduce statistical errors. Only a small fraction of the atoms are allowed to pass through the single-velocity selector, and hence, it is not ideal for precision measurements. If the velocity range of the selector is widened, then more atoms can pass through. However, a wide velocity range is just what the velocity selector is meant to prevent in the first place. The velocity multiplexing scheme is designed to circumvent this tradeoff between strong signal of atoms and well-defined velocity.
3.2 Mechanical choppers

The velocity multiplexing scheme based on mechanical chopping is the most intuitive and straight-forward one – it relies only on two mechanical obstacles that periodically block the atom beam. Initially, the mechanical chopping scheme was selected as the implementation of choice in the MIT polarizability experiment.

The implementation of the mechanical choppers that I have done in the fall of 2000 uses two spinning disks with numerous slots in them separated a certain distance apart. The disks have equally much open and closed space – the open fraction is 50%. Thus, half of the atom beam is blocked by each chopper, and the detected signal is one forth of the original. That is a much stronger signal than for the single-velocity selector, but it is weaker than for the electrical choppers described in Sec. 3.5.

The original velocity distribution is approximately Gaussian (Sec. 1.5). After the atoms have passed through the two choppers, the velocity distribution looks like a saw blade under the Gaussian envelope with many velocity peaks crammed next to one another (fig. 1-1A). When a dispersive interaction is measured, as in the electric polarizability experiment, the chopping frequency may be adjusted, such that the all the peaks add constructively to the interference pattern and most of the contrast is preserved. Illustrative figures and the governing equations are found in chapter 4.

3.2.1 Implementation details of the mechanical choppers

The chopper disks are produced by Scitec Instruments Ltd in the UK and they are purchased through Boston Electronics Corp in the US. The chopper setup works with 3 mm thick 445 slot steel disks, as well as with 5 mm thick 200, 30 or 10 slot brass disks. More slots per disk boosts the maximum chopping frequency, but it also increases the phase jitter. Infrared optoswitches record the state of the chopper, and enable the frequency and the phase jitter to be measured on an oscilloscope.

Each chopper is powered by a Brushless DC Motor 2444, which is a three-phased stepper motor produced by Minimotor SA, Switzerland, and provided through MicroMo Electronics, Inc. in the US. Each motor uses three Hall sensors for continuous feedback of its speed and position, which enables Minimotor's PWM Servo Amplifier BLD 5604-SH2P to synchronously control the stepper drive. However, the BLD 5604 is intended to control one motor only, and hence, certain
circuitry and substantial testing was required to enable the synchronous use of two motors. The additional circuitry has a push-pull design, and consists of six NPN and six PNP transistors, one for each phase of the two motors, which amplify the BLD 5604 control signal. Additionally, one capacitor per phase was included to filter out high frequency noise. The additional circuitry, the BLD 5604 circuit board along with connectors and switches are all mounted in the aluminum box seen in Fig. 3-1. The current implementation of the mechanical chopping scheme has demonstrated satisfactory performance in tests outside the MIT apparatus. However, it was abandoned in favor of the electrical scheme (Sec. 3.5) before tests with the atom beam were performed.

Figure 3-1. The implementation of the mechanical choppers. The rotating disks have 200 slots each and the box in the background controls their synchronous motion.
3.2.2 Implementation difficulties of the mechanical choppers

The implementation of a vacuum compatible system with precise synchronous motion of the two choppers is non-trivial. Many of the difficulties could have been avoided, if both the slotted disks would have been mounted on a single axis. However, due to the requirement of a wide separation between the spinning disks, such an implementation was not feasible in the MIT apparatus. The problems and imperfections of the selected implementation are that

- the manufacturing process of the Scitec Instruments' disks uses an imperfect light interference template. Thus, the slots in the disks vary slightly in size, which results in phase jitter. The phase jitter, in turn, causes imperfections in the chopped velocity distribution.
- in a typical setup the normal of the disk is tilted 2º-7º relative to the axis of rotation, due to imperfections in the Scitec Instruments' mounting piece.
- the phases of the choppers oscillate somewhat. The oscillations are damped over time and the problem would hopefully vanish in a vacuum environment.
- the MicroMo motors must be heat sunk to limit their operating temperature in the vacuum environment.
- the additional amplifying circuitry generates a lot of heat and might require cooling.
- if either motor gets stalled, the BLD 5604 control stops both of them, and it will not start until the stalled motor has been rotated. The problem is solved by permuting the feedback phases from the motor, which changes the state of the feedback system as if the motor had been rotated.

3.3 Laser choppers

The laser chopping scheme uses two lasers to resonantly push atoms out of the atom beam path, such that these deflected atoms do not strike the detector at the end of their flight. The scheme differs from the mechanical one only in the way the atoms are removed from the beam, but is otherwise analogous.

In the MIT apparatus, a 0.03 mrad deflection is enough to remove an atom from the highly collimated atom beam. This deflection corresponds to the momentum transfer of a few absorbed photons, which is easily achieved by a laser in resonance with one of the major transitions of sodium.
Though lasers work well in vacuum, and the synchronous control of two laser choppers is easier than the synchronous control of two spinning disks, the laser chopping scheme was considered less desirable than the mechanical one, due to two factors. First, there are no semi-conductor lasers in resonance with the major sodium transmissions, and the existing in-resonance dye lasers are expensive, bulky and cumbersome to work with. Second, there is an interest in high precision relative polarizability measurements of two different elements, e.g. sodium and cesium. However, unlike the mechanical choppers, the laser choppers are not species independent, and could not be used in such an experiment without duplicating the cumbersome and expensive laser system, and applying a synchronous control. For these two reasons, the laser chopping scheme was abandoned.

### 3.4 Deflection choppers

Similar to the laser choppers, the deflection choppers force the atoms out of the beam, such that they get blocked by collimating slits. The deflection can be caused by an electric field gradient, which induces an electric dipole moment in the atoms, and thereafter forces the atoms along the direction of the increasing field.

The problem with the electrical deflection choppers is that available electronics can not handle the electric field strength required to fully deflect the atoms. Using a gradient wire with a 0.5 mm radius and a 1 mm separation to the ground plate, as described in the next section, requires a voltage of 10 kV for an 800 m/s sodium beam to achieve the required 0.03 mrad deflection angle. The required voltage raises proportional to velocity, and for a 3000 m/s beam the voltage must be at least 40 kV. For a cesium beam these voltage requirements increase further with a factor of 2.3. These values are all based on the equations in Sec. 4.3. Experimentally, it has been found that sparks ignite in the vacuum when 18 kV is applied to a 1 mm radius wire with a 2 mm spacing to the ground plate. Although this breakdown voltage may increase if the wire and ground plate materials and the finish of the surfaces are altered, it will not increase enough to meet the above requirements. Thus, even if sufficiently fast and powerful electronics become available, the deflection chopping scheme is not suited for current and future MIT experiments.

### 3.5 Electrical choppers

The electrical chopping scheme is less intuitive and more complicated than the other ones. The electrical choppers block none of the atoms, and hence, the intensity of the beam is 100% – four times higher than for the mechanical scheme. Once this superiority became clear and the
theoretical calculations were verified, the electrical chopping scheme, rather than the mechanical one, became the prime choice, and the implementation started in January 2001.

Instead of blocking atoms from certain parts of the velocity distribution, as the other choppers do, electrical choppers change the phase of these atoms. An electrical chopper applies an electric field gradient inside the interferometer across the two paths that the atoms travel. The electric field is weaker than for the deflection choppers, and it shifts the relative phase of the atoms in the two arms, rather than deflecting them out of the beam. An electrical chopper induces a $\pi$ relative phase shift to the atoms when the chopper is on (class A atoms), and no phase shift to the atoms when it is off (class B atoms). Additionally, the class A and B atoms are half a chopping period out of phase. This phase difference cancels the applied $\pi$ phase shift, and in the end all atoms add constructively to the interference pattern. In chapter 4, the obtained interference pattern is analyzed in detail.

The magnitude of the applied voltage required to achieve a $\pi$ phase shift is proportional to the inverse of the square root of the atomic polarizability (Eqn. 4.27). Thus, the electrical chopping scheme is not species independent, but rather it depends on the atoms in the beam. However, the scheme is extremely easy to adjust for different species – all that is required is a change in the applied voltage.
3.5.1 Implementation details of the electrical choppers

The electrical choppers are home-made with a 0.5 mm radius copper wire, separated by a 1 mm glass spacer from a copper plate. The copper plate is grounded to the rest of the apparatus, whereas the copper wire is connected through standard hook-up wire and a high-voltage vacuum-to-air feed-through to a high-voltage square wave PVX-430 ±6,000 V Pulse Generator manufactured by Directed Energy, Inc. The DEI pulse generator requires two inputs. A frequency generator feeds it a 5 V square wave, which regulates the DEI's output frequency, and a Bertan 825-5N high voltage supply, determines the amplitude of the output pulses. Both the frequency generator and the high voltage supply can be computer controlled to enable automatic data collection.

The choppers' positions are also controlled through the same computer interface. Each chopper is mounted on a movable stage with two degrees of freedom. One motor can translate each chopper in and out of the beam, and one motor can tilt it, in order to line up the charged chopper wire in parallel with the rectangularly collimated atom beam. All motors have a precision of 1 μm.

3.5.2 Implementation difficulties of the electrical choppers

The implementation of the electrical chopping scheme has been relatively straight-forward in comparison to the mechanical one. The main reason is that electrical devices allow for better control than mechanical ones. The electrical choppers are based mostly on standard electronics, which has made implementation and subsequent modification easier, and operation of the choppers more flexible. Yet, there are several difficulties and limitations of the implementation, which must be recognized:

- Operation of the choppers at several kilovolts requires precautions. Cables must be well isolated and connectorized.
- It is crucial that both choppers are in phase. Thus, the cables leading to the two choppers must have the same length and capacitance, and they must be hooked up to the same square wave.
- High chopping frequencies require a lot of power and current. The DEI switches the chopper's copper wires alternatively between the high voltage $V$ and ground at a chopping frequency $f$. There are three constraints that limit the applied voltage as a function of frequency. First, the DEI has a power limit $P_{\text{max}} = 100$ W. Second and third, the Bertan
825-5N has a current and voltage limit of $I_{\text{max}}=40$ mA and $V_{\text{max}}=5$ kV, respectively. These limits can be expressed as three upper bounds on the applied voltage:

$$V_1 = \sqrt{\frac{P_{\text{max}}}{cf}}$$

$$V_2 = \frac{I_{\text{max}}}{cf}$$

$$V_3 = V_{\text{max}}$$

(3.1)

where the total capacitance $c=200$ pF is the sum of 100 pF each from the choppers and the DEI. When the choppers are used in polarizability experiments, it is desirable to have chopping frequencies as high as possible, while maintaining a voltage of 2-4 kV. The restrictions in voltage-frequency space imposed by Eqn. 4.1 are summarized in fig. 3-3.
4 Theory

The aims of the velocity multiplexing scheme have been defined in section 2.4 and chapter 3. In summary they are

- to select atoms of certain velocities to avoid the exponential decay of contrast described in Eqn. 2.11 and 2.14, which result from the spread of the velocity distribution.
- to allow for a new experimental method that does not require knowledge of the precise width and shape of the velocity distribution.
- to rephase multiple velocities to achieve greater contrast and Signal to Noise Ratio (SNR) than the single-velocity selector.

The key to analyzing the effect of the chopper is to study how the chopper’s transmission function affects the velocity distribution of the atom beam. In this chapter, calculations of the transmission function and the resulting signal and contrast are performed first for the general case, and then the result is applied to the special cases of mechanical and electrical choppers. At the end of the chapter, various imperfections are discussed, which reduce the choppers' performance relative to the ideal case.

4.1 Ideal Mechanical choppers

4.1.1 Transmission of mechanical choppers

Suppose the time dependent transmission function $z$ of a single chopper is periodic with the period $T=2\pi/\omega$. Then it can be expanded in a cosine Fourier series:

$$z(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t), \quad \text{where } 0 \leq z(t) \leq 1.$$ (4.1)

The coefficients $a_n$ defines the transmission function. For a square wave chopper symmetric around time $t=0$,

$$a_0 = \gamma, \quad a_n = 2 \frac{\sin(n\pi \gamma)}{n\pi},$$ (4.2)

where $\gamma$ is the fraction of time the chopper is open. Consider two identical choppers that chop in phase with one another. If they are separated by a distance $l$, then an incident atom reaches the second chopper a time $l/v$ after the first, where $v$ is the velocity of the atom. The total transmission function $Z$ is the product of the transmission of each chopper:
\[ Z(t, v) = z(t) z(t + \frac{l}{v}) = \sum_{m,n} a_m a_n \cos(m\omega t) \cos(n\omega (t + \frac{l}{v})). \]  \tag{4.3} 

If the interval, in which atoms are detected and counted, is much greater than the period of the choppers, Eqn. 4.3 reduces to the time-independent transmission averaged over a period \( T \)

\[ Z(v) = \frac{1}{T} \int_{0}^{T} Z(t, v) dt \]

\[ = \sum_{m,n} a_m a_n \frac{1}{T} \int_{0}^{T} \cos(m\omega t) [\cos(n\omega t) \cos(n\omega \frac{l}{v}) - \sin(n\omega t) \sin(n\omega \frac{l}{v})] dt, \]  \tag{4.4} 

\[ = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2 \cos(n\omega \frac{l}{v}) \]

where in the last step the orthogonality of sine and cosine functions

\[ \frac{1}{T} \int_{0}^{T} \cos(m\omega t) \cos(n\omega t) dt = \begin{cases} 0, & m \neq n \\ 1, & m = n = 0 \\ \frac{1}{2}, & m = n > 0 \end{cases} \]  \tag{4.5} 

\[ \frac{1}{T} \int_{0}^{T} \sin(m\omega t) \cos(n\omega t) dt = 0 \]

has been used.

### 4.1.2 Rephased contrast with mechanical choppers

Once the time-averaged transmission function \( Z(v) \) of the choppers is known, the chopped velocity distribution of the atom beam

\[ P'(v) = P(v) Z(v) \]  \tag{4.6} 

can be calculated, where the unchopped distribution \( P(v) \) is given by Eqn. 1.9. A square wave chopper produces the distribution \( P'(v) \) that is shown in fig 1-1A. Under the right conditions, this chopped velocity distribution allows increases in an applied dispersive interaction without exponentially reducing the contrast, counter to what was possible with the old velocity distribution, as discussed in section 2.3 (Eqn. 2.11 and 2.14). The calculations below demonstrate this property.

The calculations are performed assuming that the interaction is dispersive with a phase shift of each atom inversely proportional to its speed, as in the case of the MIT polarizability
measurements discussed in Sec. 2.3.2. Under this assumption the velocity-dependent interference pattern is given by Eqn. 2.8, which is restated here for convenience:

\[ I(v) = N[1 + C \cos(\phi_0 + 2\pi \frac{v_{int}}{v})], \quad (4.7) \]

where the strength of the interaction defines the velocity \( v_{int} \), for which the phase shift is \( 2\pi \). To obtain the total interference pattern, Eqn. 4.7 must be integrated over the velocity distribution \( P' \) in Eqn. 4.6. The resulting integrand can be expanded using Eqn. 4.4:

\[ I_{obs} = \int_0^\infty P'(v) I(v) \, dv = a_o^2 \int_0^\infty P(v) N[1 + C \cos(\phi_0 + 2\pi \frac{v_{int}}{v})] \, dv \\
+ \frac{a_i^2}{2} \int_0^\infty P(v) N[1 + C \cos(\phi_0 + 2\pi \frac{v_{int}}{v})] \cos(\omega l) \, dv \\
+ \frac{a_s^2}{2} \int_0^\infty P(v) N[1 + C \cos(\phi_0 + 2\pi \frac{v_{int}}{v})] \cos(2 \omega l) \, dv \]

(4.8)

If the interaction is strong, and if the angular chopping frequency \( \omega \) is adjusted correctly, such that

\[ \begin{align*}
  v_{int} &> v_0 \\
  2\pi v_{int} &= j\omega l
\end{align*} \]

(4.9)

where \( j \) is a positive integer, then most of the terms in Eqn. 4.8 vanish. The \( n^{th} \) term

\[ \begin{align*}
  n^{th} \text{ term} &= \frac{a_o^2}{2} N \int_0^\infty P(v) \cos(n \frac{\omega l}{v}) \, dv \\
  &= \frac{a_o^2}{2} N \int_0^\infty P(v) \left[ \cos(n \frac{\omega l}{v}) + P(v) C \cos(\phi_0 + j \frac{\omega l}{v}) \cos(n \frac{\omega l}{v}) \right] \, dv \\
  &= \frac{a_o^2}{2} N \int_0^\infty P(v) \cos(n \frac{\omega l}{v}) \, dv + \frac{C}{2} \int_0^\infty P(v) \cos(\phi_0 + (j - n) \frac{\omega l}{v}) \, dv \\
  &\qquad + \frac{C}{2} \int_0^\infty P(v) \cos(\phi_0 + (j + n) \frac{\omega l}{v}) \, dv
\end{align*} \]

(4.10)

can be separated into three integrals, where the standard trigonometric identity for multiplying cosine factors has been used in the last step. All the integrals but two are on the same form as Eqn. 2.9, and, hence, they decay exponentially and they are negligible for sufficiently large \( v_{int} \).
The only surviving integrals of Eqn. 4.10 are the ones with a non-dispersive phase. Specifically, those integrals are the first kind for \( n=0 \) and the second kind for \( n=j \). Thus, under the conditions described in Eqn. 4.9, Eqn. 4.8 reduces to

\[
I_{obs} = a_0^{-2}N + \frac{a_j^{-2}N}{2} C \cos \phi_0 = a_0^{-2}N \left( 1 + \frac{a_j^{-2}}{4a_0^{-2}} C \cos \phi_0 \right). \tag{4.11}
\]

For a square wave chopper, Eqn. 4.2 yields

\[
I_{obs} = N \left( 1 + C' \cos \phi_0 \right)
\]

\[
N' = C \frac{\sin^2(j\pi \gamma)}{(j\pi \gamma)^2}
\]

In order to determine the best integer value of \( j \), or put differently, to determine the right angular chopping frequency \( \omega \), let us look at the interferometer Signal to Noise Ratio, SNR. Assuming that the noise is accurately described by Poisson statistics, the SNR is

\[
SNR = \frac{C \cdot N}{\sqrt{N}} = C \sqrt{N} . \tag{4.13}
\]

Eqn. 4.12 and 4.13 combined yield that the maximum SNR for the mechanical choppers is 23% of the SNR of a non-dispersive interaction. This maximum is achieved for \( j=1 \) and using an open fraction \( \gamma=0.371 \). The contrast and SNR is plotted versus open fraction in fig. 4-2.

### 4.2 Ideal electrical choppers

Instead of blocking atoms from certain parts of the velocity distribution, as mechanical choppers do, electrical choppers change the phase of these atoms. An electrical chopper applies an electric field gradient across the path that the atoms travel, and induces a \( \pi \) phase shift to the atoms' wave function when the chopper is on, and no phase shift when it is off. Because no atoms are blocked, the signal is four times as great for the ideal electrical choppers compared to the ideal mechanical ones.

Consider two square wave electrical choppers, where the first and the second chopper apply a positive and negative \( \pi \) phase shift, respectively. The choppers divide the velocity distribution into four velocity classes corresponding to the combination of the choppers' states as indicated by fig. 4-1. Analogously to Eqn. 4.3, the total transmission function for each class is
where the state of the choppers for each class is indicated in parentheses. When the electrical chopper is on corresponds to when the mechanical chopper blocks the beam.

Analogous to Eqn. 4.4, the transmission functions time-averaged over one period for the different classes are

\[
Z_1(t, v) = z(t) z(t + l/\nu) \quad (\text{off, off})
\]

\[
Z_2(t, v) = z(t) [1 - z(t + l/\nu)] \quad (\text{off, on})
\]

\[
Z_3(t, v) = [1 - z(t)] z(t + l/\nu) \quad (\text{on, off})
\]

\[
Z_4(t, v) = [1 - z(t)] [1 - z(t + l/\nu)] \quad (\text{on, on})
\]

\[ (4.14) \]

where \( Z \) is the same transmission function as for the mechanical chopping scheme.

The electrical choppers shift the interference patterns of the four velocity classes relative to one another. The atoms of each class accumulate a phase shift \( \phi_{\text{chop},i} \), and, Eqn. 4.7 must be rewritten as

\[
I_i(v) = N[1 + C \cos(\phi_0 + 2\pi \frac{\nu_{\text{int}}}{\nu} + \phi_{\text{chop},i})]
\]

\[
\phi_{\text{chop},1} = 0 \quad \phi_{\text{chop},2} = +\pi \quad (i=1,2,3,4)
\]

\[
\phi_{\text{chop},3} = -\pi \quad \phi_{\text{chop},4} = 0
\]

\[ (4.16) \]
The idea behind the electrical choppers is that each velocity class is temporally the same phase \( \phi_{chop,i} \) apart as they acquire from the choppers. In the end, the accumulated phases cancel out and all four velocity classes add in phase constructively to the same interference pattern.

The key is to observe that when the open fraction \( \gamma=1/2 \), the square waves \( z(t) \) and \( 1-z(t) \) are the same waves, but half a period out of phase. Consequently, Eqn. 4.14 simplifies to the same form as Eqn. 4.4, but with the additional phase \( \phi_{chop,i} \):\

\[
Z_i(v) = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2 \cos(n[\omega \frac{l}{v} + \phi_{chop,i}]) , \quad (i=1,2,3,4), \tag{4.17}
\]

where \( \phi_{chop,i} \) is the same as before. The effect of the additional phase \( \phi_{chop,i} \) is that all the terms of velocity class 2 and 3, besides the first \( a_0^2 \) term, changes sign.

Consider how the transmission functions \( Z_i \) affect the interference pattern. Analogous to Eqn. 4.8, the observed interference pattern is

\[
I_{obs} = \int_0^{\infty} P(v) [Z_1(v)I_1(v) + Z_2(v)I_2(v) + Z_3(v)I_3(v) + Z_4(v)I_4(v)] dv . \tag{4.18}
\]

This equation can be split into four parts \( I_{obs,i} \) corresponding to each velocity class, and remarkably, they all contribute in phase to the interference pattern. Treating each part separately and substituting Eqn. 4.16 and 4.17 for Eqn. 4.4 and 4.7, when rederiving Eqn. 4.11, introduces an extra phase:

\[
I_{obs,i} = a_0^2 N \left( 1 + \frac{a_j^2}{4a_0^2} C \cos[\phi_0 + \phi_{chop,i}(1 - j)] \right) , \quad (i=1,2,3,4). \tag{4.19}
\]

As for the mechanical choppers, the maximum Signal to Noise Ratio is achieved for \( j=1 \), which also eliminates the extra phase. Thus, all four velocity classes interfere constructively, and the signal \( N \) is four times greater for the electrical choppers than for the mechanical ones. Eqn. 4.2 with \( \gamma=1/2 \) divulges the observed interference pattern for the ideal electrical choppers:

\[
I_{obs} = N \left( 1 + C' \cos \phi_0 \right) \\
C' = C \frac{\sin^2(\pi / 2)}{(\pi / 2)^2} = 0.405 C' . \tag{4.20}
\]
The SNR for mechanical and electrical choppers are summarized in table 4-1. Based on Eqn. 4.12, 4.13 and a generalization of Eqn. 4.20, the contrast and SNR is plotted versus the open fraction for the mechanical and electrical chopping schemes in fig. 4-2.

Table 4-1. Summary of the Signal to Noise Ratio corresponding to the open fractions $\gamma$ of 50% as well as the optimum value for the mechanical choppers. By definition, $\text{SNR}=100\%$ for an unchopped beam seeing a non-dispersive interaction.

<table>
<thead>
<tr>
<th>Type</th>
<th>$\gamma$</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical choppers (optimum)</td>
<td>37.1 %</td>
<td>23.1 %</td>
</tr>
<tr>
<td>Mechanical choppers</td>
<td>50 %</td>
<td>20.3 %</td>
</tr>
<tr>
<td>Ideal electrical choppers (optimum)</td>
<td>50 %</td>
<td>40.5 %</td>
</tr>
</tbody>
</table>

4.3 Real electrical choppers

The ideal electrical chopping scheme described above relies on the ability to apply a $\pi$ phase shift to the atoms in one of the interfering arms relative to the other. In the MIT apparatus, the choppers induce the relative $\pi$ phase shift, by producing an electric field gradient across the two arms of the interferometer as shown in fig. 4-3. All the atoms in both the arms of the interferometer acquire a phase shift, but the electric field is adjusted, such that the phase applied to the atoms in one arm is $\pi$ greater than the phase applied to the atoms in the other. Thus, the relative phase shift between the arms is $\pi$.

The electric field gradient is produced by a charged wire on one side of the interferometer and a ground plate a distance $a$ away on the other side. The wire is modeled as a perfect cylinder oriented perpendicular to the plane of the interferometer with radius $r_0$ and at a potential $V$. The
potential $\Phi$ at any point $(x, z)$ in the interferometry plane with the origin located at the center of the wire is found by using an image charge method. The cylinder is replaced by a line charge at a point $(a-b, 0)$, and an image charge at $(a+b, 0)$ with charge density $\rho$ and $-\rho$, respectively.

The potential at a point the distance $r_+$ away from the line charge and the distance $r_-$ away from its image is

$$\Phi(r_+ , r_-) = \frac{\rho}{2\pi\varepsilon_0} \ln \frac{r_+}{r_-}.$$  \hspace{1cm} (4.21)

If the x-coordinates of the line charge and its image is adjusted such that

$$b = \sqrt{a^2 - r_0^2},$$ \hspace{1cm} (4.22)

then the cylindrical surface $(x^2 + z^2)^{1/2} = r_0$ becomes equipotential, and the boundary conditions are satisfied. The uniqueness theorem of the Laplace equation guarantees that the potential $\Phi$ in the interferometer can be uniquely determined by the line charge and its image, which yields
\[ \Phi(x, z) = \frac{V}{\ln \frac{a+b}{a-b}} \ln \frac{(a+b-x)^2 + z^2}{(a-b-x)^2 + z^2}. \] (4.23)

Once the potential is known, it is straightforward to calculate the electric field squared

\[ E^2(x, z) = \left(-\nabla \Phi\right)^2 = \left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2 \]
\[ = \frac{V^2}{\left(\ln \frac{a+b}{a-b}\right)^2} \left[ \frac{16b^2}{(a+b-x)^2 + z^2} \right]. \] (4.24)

and we know from Eqn. 2.2 and 2.6 that the applied phase is

\[ \phi(x) = \frac{\alpha}{2hv} \int E^2(x, z) \, dz = \frac{4\pi\alpha}{\hbar v} \frac{V^2}{\left(\ln \frac{a+b}{a-b}\right)^2} \frac{b}{b^2 - (a-x)^2}. \] (4.25)

For the choppers, the interesting phase is the relative change \( \Delta \phi \) between the atoms in the two interfering arms separated by a distance \( d \). This distance is given by the length from the diffraction grating to the chopper \( L_{\text{chop}} \) multiplied by the diffraction angle \( \theta_{\text{diff}} \), which can be expressed in terms of \( \lambda_g \) and \( \lambda_{\text{dB}} \), the grating period and the deBroglie wavelength of the interfering atoms, respectively. Eqn. 1.1 yields

\[ d = L_{\text{chop}} \theta_{\text{diff}} = L_{\text{chop}} \frac{\lambda_{\text{dB}}}{\lambda_g} = \frac{L_{\text{chop}}}{\lambda_g} \frac{h}{mv}. \] (4.26)

In an atom interferometer \( d \ll x \), and with Eqn. 4.25 and 4.26 the interesting phase difference \( \Delta \phi \) can be accurately approximated as

\[ \Delta \phi = \frac{\partial \phi}{\partial x} \, d = \frac{16\pi^2\alpha}{m} \frac{L_{\text{chop}}}{\lambda_g} V^2 \left( \frac{1}{\ln \frac{a+b}{a-b}} \right)^2 \frac{(a-x)b}{b^2 - (a-x)^2}. \] (4.27)

The electrical choppers require that \( \Delta \phi = \pi \). This relative \( \pi \) phase shift can be achieved between atoms traveling at the mean velocity \( v = v_0 \) in the center of the interfering arms where \( x = x_0 \), by adjusting the applied voltage \( V \). In the MIT setup, the radius of the gradient wire \( r_0 = 0.5 \) mm, and the distance between its center and the ground plate \( a=1.5 \) mm. Thus, in a typical MIT
experiment where $L_{chop}=42 \text{ cm}$, $\lambda_s=100 \text{ nm}$, $v_0=3,000 \text{ m/s}$ and $d=24 \text{ µm}$, a sodium atom with $\alpha/m=7.04 \times 10^{-14} \text{ m}^4\text{s}^{-2}\text{V}^{-2}$ passing 20 µm from the gradient wire requires 2.5 kV to acquire a $\pi$ phase shift. The required voltage for a $\pi$ phase shift versus the distance between the beam and the wire ($x-r_0$) is plotted in fig. 4-4.

### 4.4 Imperfections

The electrostatic theory above, culminating in Eqn. 4.27, indicates how to apply a relative $\pi$ phase shift to atoms traveling at the mean velocity in the center of the interfering arms. However, the velocity distribution has a certain width, which means that there are faster atoms that receive slightly less than a $\pi$ phase shift, and there are slower atoms that receive slightly more than a $\pi$ phase shift. Similarly, there are spatial imperfections associated with the width and shape of the atom beam. A third imperfection is related to the finite length of the electrical interaction region. The atoms that are in this region when the chopper is turned on or off will only receive a fraction of the applied $\pi$ phase shift. Mathematically, these three imperfections show up as three extra error terms $-\phi_{err}^{(1)}(v), \phi_{err}^{(2)}(x)$ and $\phi_{err}^{(3)}(z)$ – in the phase expression of Eqn. 4.16 which then carry through to Eqn. 4.19. The following sections analyze the three imperfections in detail by showing how to correctly average the error terms over the width of the velocity distribution, the width of the beam, and a chopping period, respectively.

![Image](image.png)
4.4.1 Imperfection due to the dispersion of the applied $\pi$ pulse

Atoms traveling with a velocity different than the mean velocity will obtain a phase shift that is different than $\pi$, due to the $1/v^2$-dependency of $\Delta \phi$ as shown in Eqn. 4.27. If the velocity distribution is Gaussian with a small width $\sigma_v$, then by differentiation the relative spread of the phase shift is found to be

$$\frac{\sigma_{\Delta \phi}}{\Delta \phi_0} = -\frac{2 \sigma_v}{v_0},$$

(4.28)

where $\Delta \phi_0 = \pi$.

To understand how this imperfection affects the total interference pattern, consider first the interference patterns of the velocity classes from Sec. 4.2 separately. The classes of atoms that receive the $\pi$ phase shift, i.e. class 2 and 3, have a lower contrast than the unaffected atoms in class 1, due to their spread in phase space indicated by Eqn. 4.28. Analogous to Eqn. 2.14, the gradient field gives rise to the reduced contrast

$$C' = C \exp \left[ -2 \left( \frac{\Delta \phi_0 \sigma_v}{v_0} \right)^2 \right],$$

(4.29)

where $C$ is the contrast of class 1 atoms. Class 4 atoms, which pass through both the choppers when they are on, have no reduction of contrast due to dispersion, if the gradient wires of the two choppers are mounted on separate sides of the beam. In that case, the applied phases are $+\pi$ and $-\pi$, respectively, and the two dispersive phase shifts cancel.
The total interference is the sum of the interference of all the four classes. At non-negligible chopping frequencies, the sizes of the velocity classes are approximately equal, and the observed contrast is the average of the four $C_{obs}^{(1)}=(C+C'+C'+C)/4$. Based on Eqn. 4.29, for a velocity spread of $\sigma_v/v_0=5\%$ this imperfection reduces the observed contrast to 98% of the original, when the electrical choppers apply a $\Delta \phi_0=\pi$ phase shift.

**4.4.2 Imperfection due to the finite width of the beam**

Because of the non-linearity of the gradient electric field, the finite width of the atom beam causes imperfections in the applied $\pi$ phase shift. The atoms near the gradient wire acquire a larger phase shift than the atoms closer to the ground plate, because the electric field changes faster in the region close to the wire as dictated by Eqn. 4.27.

The relative difference of the applied phase over the $2\sigma_x$ wide beam is

$$Q_x = \frac{\Delta \phi(x_0 + \sigma_x) - \Delta \phi(x_0 - \sigma_x)}{\Delta \phi(x_0)}.$$

(4.30)

Based on the dimensions in the MIT setup, it turns out that the relative difference $Q_x$ evaluates in percent to the value of $\sigma_x$ as measured in microns, if the beam is in the proximity of the gradient wire where $x_0 = r_0$. If the beam is half way in between the gradient wire and the ground plate, where $x_0 = (r_0 + a)/2$, $Q_x$ is reduced to two thirds of this value.

To evaluate the reduced contrast due to this imperfection, the beam width and the spatial distribution of the beam cross section $P(x)$ must be known. The width of the atom beam increases as it travels through the interferometer due to the spreading that occurs after it is collimated by two slits with the width $D_s$ which are the distance $L_s$ apart. Based on geometric ray tracing, at the first chopper, which is a distance $L_{c1}$ away from the second slit, the width of the beam is
\[2\sigma_s = D_s \left( 2 \frac{L_{cl}}{L_s} + 1 \right).\] (4.31)

For \(P(x)\) the center two standard deviations of a Gaussian distribution have been used, in order to achieve an upper bound in the calculations below.

When assessing the reduction in contrast due to this imperfection, consider each velocity class independently. Atoms of velocity class 1 pass through both the choppers when they are off and they are not affected by this imperfection. Class 2 and 3 atoms pass through one chopper when it is on and one when it is off. To account for the imperfection the error between the actual and perfect \(\pi\) phase shift \(\phi_{err}^{(2)}(x)\) must be included and averaged over the width of the atom beam.

Since the beam is wider at the second chopper than at the first, \(\phi_{err}^{(2)}(x)\) must be averaged over a wider beam for class 2 than for class 3. Alternatively, the linear transform between \(x\) and \(x'\) introduced in Eqn. 4.32 below, changes the integration bounds of class 2 to the same as those of class 3. This approach also provides for a convenient treatment of velocity class 4, whose atoms pass through both the choppers when they are on. Justified by ray tracing, these atoms have the sum of the two choppers' phase error.

Based on the above discussion the reduced contrast for each velocity class is

\[I'_i = N \int_{x_i - \sigma_{x_i}}^{x_i + \sigma_{x_i}} P(x) \left[ 1 + C \cos(\phi_0 + \phi_{err,i}^{(2)}(x)) \right] dx = N \left[ 1 + C'_i \cos \left( \phi_0 - \tan^{-1} \frac{B_i}{A_i} \right) \right]
\]

\[\phi_{err,i}^{(2)}(x) = \begin{cases} 0 & i = 1 \\ \Delta \phi(x') - \pi & i = 2 \\ \Delta \phi(x) - \pi & i = 3 \\ \Delta \phi(x') - \pi + \Delta \phi(x) - \pi & i = 4 \end{cases}, \quad (4.32)\]

\[x' = x_0 + \frac{L_s}{L_s + 2L_{c2}} (x - x_0)\]

\[A_i = \int P(x) \cos(\phi_{err,i}^{(2)}(x)) dx\]

\[B_i = \int P(x) \sin(\phi_{err,i}^{(2)}(x)) dx\]

\[C'_i = C \sqrt{A_i^2 + B_i^2}\]

where \(\Delta \phi(x)\) is conveniently computed from Eqn. 4.27 by noting that \(\Delta \phi(x_0) = \pi\), and that \(L_{c2}\) is the distance from the second slit to the second chopper. Standard trigonometric identities have been
used to find the reduced contrast $C'$ based on the integral expression. $C'$ must be evaluated numerically, since $\Delta \phi$ is a complicated function of $x$.

With the dimensions in the MIT setup, where $D_s=10 \mu$m, $L_s=965$ mm, $L_{c1}=473$ mm and $L_{c2}=1459$ mm, the observed average contrast of the four velocity classes

$$C_{\text{obs}}^{(2)} = \left(C_1' + C_2' + C_3' + C_4'/4\right)$$

is 98.9% of the original when the beam is next to the edge of the wire, $x_0 \approx r_0$, and 99.5% of the original when $x_0 \approx (r_0 + a)/2$. These upper bound calculations, which use the center two standard deviation of a Gaussian as the spatial distribution of the beam cross section, show that imperfection #2 is negligible.

![Figure 4-7. Imperfection #3 – the finite length $L_{\text{eff}}$ of the interaction region. The atoms which are in the interaction region of a chopper when it is turned on or off acquire only a portion of the $\pi$ phase shift.](image)

### 4.4.3 Imperfection due to the finite length of the interaction region

The atoms that are in the interaction region of a chopper when it is turned on or off acquire only a portion of the $\pi$ phase shift. The size of this fraction of atoms is equal to the effective length of the interaction region $L_{\text{eff}}$ divided by the distance

$$L = \frac{v_0}{2f}$$

that the atoms travel at the velocity $v_0$ in half a chopping period. $L_{\text{eff}}$ is defined as the length of the region where the atoms acquire 99% of the total $\pi$ phase, and it equals 1.3 mm when the beam passes next to the wire ($x_0 \approx r_0$) and 1.8 mm when the beam passes right in between the wire and the ground plate ($x_0 \approx (r_0 + a)/2$). At slow chopping frequencies $f$, the fraction $L_{\text{eff}}/L$ is negligible.
At high chopping frequencies, it must be considered how much phase the atoms acquire, and where. Fig. 4-8 shows how much relative phase $\Delta \phi(x,z)$ each atom has acquired as a function of the distance $z$ from the center of the wire, where in accordance with Eqn. 4.25

$$\Delta \phi(x, z) = \pi \frac{\int_{-\infty}^{\infty} \left[ E^2(x + d, z') - E^2(x, z') \right] dz'}{\int_{-\infty}^{\infty} \left[ E^2(x + d, z') - E^2(x, z') \right] dz'}$$

(4.35)

with $E^2(x, z)$ and $d$ given by Eqn. 4.24 and 4.26, respectively. Notice in fig. 4-8 that $\Delta \phi$ decreases for small values of $z$, and by symmetry $\Delta \phi$ overshoots the $\pi$ phase shift at high values of $z$. The overshoot, which reaches its maximum when $z^2 = b^2 - (x-a)^2$ and the integrand of Eqn. 4.35 changes sign, is 0.8% and 1.4% for $x_0 \approx r_0$ and $x_0 \approx (r_0 + a)/2$, respectively.

Now consider what happens when the chopper is turned on or off. Just when a chopper is turned off, the atoms in its proximity have the phase distribution $\Delta \phi$ seen in fig. 4-8. Ideally, the distribution would be a step function $\pi \theta(z=0)$. However, due to the finite length $L_{\text{off}}$ of the interaction region, there is a phase error between the ideal and real phase step

$$\phi_{\text{err}}^{(3)}(z) = \Delta \phi(x, z) - \pi \theta(z=0)$$

(4.36)
Table 4-2. Key figures associated with imperfection #3. \( L \) is defined in Eqn. 4.34, \( x \) is the distance from the center of the chopper wire to the atom beam, \( L_{\text{eff}} \) is the effective length of the interaction region where the atoms acquire 99\% of their \( \pi \) phase shift, overshoot is equal to \((\Delta \phi_{\text{max}}/\pi - 1)\), \( C' \) and \( C_{\text{obs}}^{(3)} \) are the reduced contrast after one and two choppers, respectively.

<table>
<thead>
<tr>
<th>( v_0 ) (m/s)</th>
<th>( f ) (kHz)</th>
<th>( L ) (mm)</th>
<th>( x ) (mm)</th>
<th>( L_{\text{eff}} ) (mm)</th>
<th>Overshoot (%)</th>
<th>( C' ) (%)</th>
<th>( C_{\text{obs}}^{(3)} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>20</td>
<td>25</td>
<td>( r_0=0.5 )</td>
<td>1.3</td>
<td>0.8</td>
<td>99.0</td>
<td>98.0</td>
</tr>
<tr>
<td>1,000</td>
<td>80</td>
<td>6.25</td>
<td>( r_0=0.5 )</td>
<td>1.3</td>
<td>0.8</td>
<td>96.0</td>
<td>92.1</td>
</tr>
<tr>
<td>1,000</td>
<td>20</td>
<td>25</td>
<td>( (r_0+a)/2=1 )</td>
<td>1.8</td>
<td>1.4</td>
<td>98.4</td>
<td>96.8</td>
</tr>
<tr>
<td>1,000</td>
<td>80</td>
<td>6.25</td>
<td>( (r_0+a)/2=1 )</td>
<td>1.8</td>
<td>1.4</td>
<td>93.5</td>
<td>87.4</td>
</tr>
</tbody>
</table>

Unlike for imperfection #1 and #2, where the phase error of velocity class 4 at the first and second chopper is linked, there is no correlation between the phase error at the two choppers in this case. Thus, each chopper may be considered independently and there is no need to treat the velocity classes separately. Additionally, the phase error when the chopper is turned on and off is anti-symmetric with respect to \( z \), and as evident from fig. 4-8 the observed phase is left unchanged; while the positive phase error at \( z<0 \) affects the observed phase positively, the negative phase error at \( z>0 \) cancel out this phase shift. Anchored in these properties, the interference pattern and reduced contrast resulting from one chopper when averaged over a chopping period may be expressed as

\[
I' = \frac{N}{2L} \int_{-L}^{L} \left[ 1 + C \cos \left( \phi_0 + \phi_{\text{err}}^{(3)}(z) \right) \right] dz = N \left[ 1 + C' \cos \phi_0 \right]
\]

\[
C' = \frac{C}{2L} \int_{-L}^{L} \cos \phi_{\text{err}} dz = \frac{C}{L/2} \int_{-L/2}^{L/2} \cos \Delta \phi dz
\]

Based on the dimensions in the MIT interferometer, numerical calculations of the reduced contrast \( C' \) after one chopper for various configurations are summarized in table 4-2. The last column \( C_{\text{obs}}^{(3)} \) is the observed contrast with both choppers taken into account. Due to the lack of correlation of the phase error between the two choppers, this quantity is simply the square of \( C' \).

Table 4-2 indicates that imperfection #3 reduces the contrast more than imperfections #1 and #2 which were discussed in sec. 4.4.1 and 4.4.2 above, when the chopping frequency is high and the beam velocity is low. The reduction is noticeable if the length \( L \) defined in Eqn. 4.34 is less than 25 mm. However, for the most part of an experiment \( L \) will be greater than 25 mm and this imperfection may be neglected.
5  Applied theory and experimental results

By the writing of this thesis, two fundamental tests of the choppers have been performed. The first involves a single chopper, which divides the atoms into two classes. The second involves two choppers working synchronously in phase, such that the observed contrast varies with the chopping frequency.

5.1 One single chopper

Just as two electrical choppers divide the atoms into four classes, as shown in fig. 4-1, one single electrical chopper divides the interfering atoms into two classes A and B corresponding to the chopper being on and off, respectively. To predict the resulting interference pattern we can model the interference of each class separately, and then add them together.

The interference pattern resulting from a single electrical chopper can be expressed in terms of the phase $\phi$ applied by the chopper, the class A and B intensities $N_A$ and $N_B$, which with an open fraction $\gamma=1/2$ each is half of the total intensity $N$, and the contrast $C_A$ and $C_B$ for the two classes along with the ratio $Q$ between them:

$$
I = I_A + I_B = N_A \left[ 1 + C_A \cos(\phi_0 + \phi) \right] + N_B \left[ 1 + C_B \cos \phi_0 \right]
$$

$$
= \frac{N}{2} \left[ 1 + C_A \cos \phi_0 \right] + \frac{N}{2} \left[ 1 + C_B \cos \phi_0 \right]
$$

$$
= N \left[ 1 + \frac{C_B}{2} \left( Q \cos \phi + 1 \right) \cos \phi_0 - \frac{C_B}{2} Q \sin \phi \sin \phi_0 \right]
$$

$$
= N \left[ 1 + \frac{C_B}{2} \sqrt{(Q \cos \phi + 1)^2 + (Q \sin \phi)^2} \cos \left( \phi_0 - \tan^{-1} \frac{Q \sin \phi}{Q \cos \phi + 1} \right) \right] \quad (5.1)
$$

$$
Q = \frac{C_A}{C_B}
$$

In the ideal case, when the imperfections described in Sec. 4.4 are ignored, the contrast of class A and B are equally big which result in the contrast ratio and observed contrast

$$
Q = 1
$$

$$
C_{obs} = C_B \sqrt{\frac{1 + \cos \phi}{2}}. \quad (5.2)
$$
A more realistic model takes into account that not all atoms get a perfect $\pi$ phase shift because of the spread of the velocity distribution, as described by imperfection #1 in Sec. 4.4.1. Imperfection #2 can always be neglected, and #3 can be neglected at the moderate chopping frequencies in this experiment. Eqn. 4.29 plugged into Eqn 5.1 results in the contrast ratio and the observed contrast

$$Q = \exp \left[ -2 \left( \frac{\phi \sigma_v}{v_0} \right)^2 \right]$$

and

$$C_{obs} = \frac{C_B}{2} \sqrt{Q^2 + 2Q\cos\phi + 1}$$

In fig 5-1, the obtained data is plotted together with the ideal model expressed in Eqn. 5.2 and the realistic model expressed in Eqn. 5.3. The applied phase $\phi$ in these equations is a function of the distance from the gradient wire to the atom beam according to Eqn. 4.27. The contrast is reduced to a minimum when class A atoms receives a $\pi$ phase shift, and there is a revival when the phase shift is $2\pi$. The exponential decay of the class A contrast $C_A$ in the realistic model prevents the
complete cancellation at $\phi = \pi$, which occurs in the ideal model. Similarly, the exponential decay of the realistic model reduces the height of the revival at $\phi = 2\pi$ and positions the peak at a greater distance from the gradient wire than the ideal model peak.

In general, the realistic model is in good agreement with the data of fig. 5-1. However, there are three small discrepancies. First, the unaffected contrast, when the beam is far away from the gradient wire, is 13-14% and not 11.7% as the fitted line suggests. Second, the contrast decay close to the gradient wire is far greater than what the realistic model proposes. Third, data from single diffraction gratings scans, which was taken a few hours after the data in fig. 5-1, suggests that the mean velocity and the rms width were 2,800 m/s and 233 m/s, respectively, which is notably lower than the values obtained from fits to the data in fig. 5-1, which were 3,210 m/s and 306 m/s, respectively. These three discrepancies suggest that there are some imperfections not accounted for in the models. Possible candidates include imperfect geometry of the chopper region e.g. chopper tilt, misalignment in some part of the interferometer, changing conditions due to thermal drifts and vibrations, part of the beam hitting the gradient wire or other edge effects.

5.2 Two choppers together

The simplest model of two choppers working together in phase, ignores the imperfections discussed in Sec. 4.4 and assumes that there are only two classes of atoms – class A with $\pi$ phase, and the class B with 0 or 2$\pi$ phase. Class A corresponds to the combination of classes 2 and 3 that were discussed in Sec. 4.2, and class B corresponds to the combination of classes 1 and 4. Thus, in analogy with Sec. 4.2 the interference pattern can be written as

$$I = \int_0^\infty \left[ P(v)Z_A(v)I_A + P(v)Z_B(v)I_B \right] dv$$

$$= \frac{N}{2} \left[ 1 + C \cos(\phi_0 + \pi) \right] \int_0^\infty P(v)Z_A(v)dv + \frac{N}{2} \left[ 1 + C \cos(\phi_0) \right] \int_0^\infty P(v)\left[ 1 - Z_A(v) \right] dv$$

$$= N \left[ 1 + \left( 1 - 2 \int_0^\infty P(v)Z_A(v)dv \right) C \cos(\phi_0) \right]$$

$$Z_A = Z_2 + Z_3 = 1 - 2Z, \quad Z_B = Z_1 + Z_4 = 2Z$$

$$Z = \left. \mod_1 \omega t \right| \frac{2\pi}{2\pi v} - \frac{1}{2}$$

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where the expression of $Z$ is Eqn. 4.4 in its non-fourier form with $\gamma=1/2$, and the predicted contrast is

$$C' = \left| 1 - 2 \int_0^{\infty} P(v) Z_{\text{a}}(v) dv \right| C .$$  \hspace{1cm} (5.5)

$C'$ is calculated numerically and plotted together with the experimental data in fig. 5-2. There is a satisfactory agreement between the ideal-model prediction and the data. The main qualitative difference between the two is that the envelope of the contrast decreases slower according to the model. This difference is most likely due to errors in the fit of the rms width to the single grating diffraction patterns. If an rms width of 200 m/s is used in the ideal-model prediction rather than 156 m/s, which the fit suggests, the agreement between the model and the data is improved. A more complicated model which incorporates the imperfections in Sec. 4.4 may improve the model-data agreement further.
6 Polarizability measurements using velocity multiplexing

Using the velocity multiplexing scheme in experiments to determine the ground state polarizability of atoms not only eliminates the upper limit on the strength of the applied interaction. The scheme also allows for a new experimental method that eliminates the need for precise knowledge of the velocity distribution. The basics of the method were first proposed in 1995 [HPC95], but it is yet to be realized.

The method relies on the rephasing condition (Eqn. 4.9) for a $1/v$-dependent interactions that was discovered in chapter 4, and which is restated here,

$$\begin{cases} v_{\text{int}} \gg v_0 \\ 2\pi v_{\text{int}} = j\omega l \end{cases} \quad (6.1)$$

In the derivation of Eqn. 4.11, it is shown that if and only if Eqn. 6.1 is satisfied, then all but two terms in the Fourier expansion of the interference (Eqn. 4.8) may be neglected, and there will be a revival of contrast (Eqn. 4.12). The integer $j$ indicates the order of the revival, and $\omega$ and $l$ are the angular chopping frequency and the length between the choppers, respectively. By definition, $v_{\text{int}}$ is given by Eqn. 2.8 or 4.7:

$$\Delta \phi = 2\pi v_{\text{int}} / v. \quad (6.2)$$

The method is an extension of the experimental method B, which uses Eqn. 2.7 to determine the polarizability $a$. Eqn. 2.7 is derived and discussed in chapter 2, and restated here

$$\alpha = \left( \frac{\Delta \phi}{V^2} \right) \left( \frac{D^2}{L_{\text{eff}}} \right) (2\hbar v). \quad (6.3)$$

From the combination of Eqn. 6.1, 6.2 and 6.3, an expression for the polarizability is obtained,

$$\alpha = 2\hbar \omega \frac{D^2}{V^2} \frac{l}{L_{\text{eff}}} j, \quad (6.4)$$

which does not involve the uncertain velocity factor. Thus, a measurement of $\alpha$ may be obtained by fixing one of the parameters $V$ and $\omega$, while adjusting the other to find the first and greatest contrast revival which occurs when $j=1$. To find the center of the peak, the parameters may be fine-tuned to the point where the phase is zero relative to when no interaction is applied. After the
spatial quantities $D$, $l$, and $L_{\text{eff}}$ has been determined from the geometry of the setup, this procedure allows $\alpha$ to be extracted directly from the displays of a voltmeter and a frequency generator.

The solid line in fig. 6-1 shows the result of numerical calculations of the contrast revivals, where the same parameters as in Sec. 5.1 ($v_0=3.210 \text{ m/s}$, $\sigma_v=316 \text{ m/s}$, $l=98.58 \text{ cm}$) and the geometry of the MIT polarizability experiment in 1995 ($D=2 \text{ mm}$, $L_{\text{eff}}=6 \text{ cm}$) have been employed along with the chopping frequency 20 kHz. The calculations are based on the following framework

$$
I = N \int P(v)Z(v)I(v)dv = N \cdot \text{Norm} + NC_0 \int P(v)Z(v)\cos(\phi_0 + \Delta \phi)dv
$$

$$
= N \cdot \text{Norm} \left[ 1 + \frac{\sqrt{G^2 + H^2}}{\text{Norm}} C_0 \cos\left(\phi_0 - \tan^{-1}\frac{H}{G}\right) \right]
$$

$$
C' = \frac{\sqrt{G^2 + H^2}}{\text{Norm}} C_0
$$

$$
\text{Norm} = \int P(v)Z(v)dv
$$

$$
G = \int P(v)Z(v)\cos \Delta \phi dv
$$

$$
H = \int P(v)Z(v)\sin \Delta \phi dv
$$

(6.5)
where the same trigonometric identities as in Eqn. 5.1 have been used, and $\Delta \phi$ is calculated from Eqn. 6.3. The dotted line in fig. 6-1 is the relative contrast obtained from the theory in chapter 4 by combining Eqn. 4.12 and 6.1. These equations are only valid when $j$ is an integer, which is indicated with circles in fig. 6-1. The circles coincide with the numerically calculated revivals and valleys. This perfect agreement shows that the Fourier analysis of chapter 4 is accurate.

Complementary numerical analysis with studies of the phase at the occurrences of the revivals is found in Hammond's thesis from 1997 [HAM97].

The *Norm* factor of Eqn. 6.5 is not needed when modeling electrical choppers, since the *Norm* factors add up to one, when taking all four velocity classes into account. However, when modeling imperfections or mechanical choppers it might be required. If the chopping frequency is low and/or the velocity distribution is narrow, such that there are only a couple of velocity peaks or less in each velocity class under the central region of the Gaussian velocity envelope (velocity peaks and Gaussian envelope is shown in fig. 1-1A), then the choppers operate similar to a single-velocity selector with a wide velocity gap (discussed in Sec. 3.1), and the number of atoms in a velocity class might deviate from the expected 25%. In such a case, small deviation in the velocity distribution can significantly change the contrast. If the chopping frequency is too low and/or the velocity distribution is too narrow, then the peaks in fig. 6-1 might broaden such that the first revival disappears.

The advantage with the new experimental method is seen in a comparison of Eqn. 6.3 and 6.4. The two quantities in Eqn. 6.3, which are difficult to determine accurately, $v$ and $\Delta \phi$, are replaced by $\omega$ and $l$, which are easy to determine accurately. In the 1995 polarizability measurement at MIT [ESC95] that is discussed in Sec. 2.3.2, the total uncertainty of 0.3% was dominated by a 0.15% uncertainty in the mean velocity $v_0$, and a 10% uncertainty in the rms width $s$, of the velocity distribution, both which were determined from fits to single grating diffraction patterns. The phase shift $\Delta \phi$ has been obtained from fits to many hours of interference data, where each individual data point is subject to a large error.

The new proposed method can utilize contrast interferometry, where $\omega$ or $V$ are adjusted around the contrast rival such that a peak can be fitted to the data, rather than monitoring the phase changes as in usual phase interferometry. Contrast interferometry is less sensitive to
environmental changes, e.g. noise vibration and temperature shifts of the apparatus, than phase interferometry, and it can sometime yield more accurate result within a shorter period of time.

This contrast interferometry method also works with the experimental configuration C discussed in chapter 2, though there are some additional complications. Due to the $1/v^2$-dependent phase shift in the gradient potential, there is no analytical expression like 6.1 that accurately pinpoints the occurrence of contrast revivals, even though their location in frequency space may be calculated numerically. However, since the imperfections associated with the electrical choppers are difficult to model accurately, it is hard to make precise numerical calculations; but to the first order, the $1/v^2$ rephasing condition is half of the phase given by the $1/v$ rephasing condition in Eqn. 6.1 and 6.2

$$\Delta \phi = j \frac{\omega l}{2v}. \quad (6.6)$$

Due to the above complications, polarizability measurements based on a gradient electric field (configuration C) will be less accurate than those based on a homogeneous electric field (configuration B).
7 Conclusion

Atom interferometry has been used in high precision measurements during the last ten years, and
the experimental methods are continually developing. Velocity multiplexing is a new tool with
the potential to boost this development process by allowing stronger dispersive interactions to be
studied. The electrical chopping scheme, which is proposed, analyzed and tested for the first time
in this thesis, achieves a factor of four improvement in intensity and a factor of two improvement
in the Signal to Noise Ratio compared to the mechanical chopping scheme that has been studied
before. Thus, the time to run an experiment is chopped in half.

Three imperfections associated with the electrical choppers have been investigated in this thesis.
These imperfections spread out the phase applied by the choppers, and they can potentially
degrad the precision of experimental measurements. Sec. 4.4 has shown that imperfection #1,
caused by dispersion, is more important to model and monitor than imperfection #2, which is due
to the width of the atom beam. Imperfection #3, due to the length of the interaction region,
dominates over both the other imperfections when the chopping frequency is high and the beam
velocity is low. However, at low chopping frequencies imperfection #3 has a negligible effect.
Common for all imperfections is that a symmetrical spread in phase only reduces the contrast
without changing the phase of the observed interference pattern. This condition is fulfilled if the
velocity distribution and the spatial distribution of the beam are symmetric and narrow. Improved
numerical models are needed to understand the effects of the imperfections under asymmetrical
conditions.

The initial testing of the electrical chopping scheme has shown remarkably good qualitative and
quantitative agreement between the theory and data, and the choppers are now ready to be
employed in precision measurements. Within six months an improved polarizability measurement
of sodium based on the electrical chopping scheme is projected. In the future, the new chopping
scheme might be used in many other experiments involving measurements of gravity or rotation
phase, or a magnetic interaction. Precise phase manipulation might also find new applications in
the future, which could make the method of electrical chopping an important tool across the field
of atom interferometry.
8 References


Acknowledgement

I would like to thank everyone in the MIT Atom Interferometry group, who has made this work possible:

- Dave Pritchard, I am grateful having had the chance to assimilate the knowledge that you radiate. Thanks for your guidance and the entertaining chats that we have had.
- Alex Cronin, your quick responses and fresh outlooks to any possible puzzle I put before you never stop amazing me. I've enjoyed your company both in and outside lab.
- Tony Roberts, your groundwork of chopper theory and chopper assembly has been a great foundation for this thesis. Thanks for your support and tireless effort.
- David Kokorowski, it has been challenging to get one decent page of my thesis done, in the time you completed an excellent chunk of ten. I hereby declare you the winner of our thesis competition and a grand price will be delivered.
- Peter Finin, I have enjoyed your company in both the lab and the classes we have had together. I am sure that the undergraduate influence in the lab can only increase as you take over the baton.

In addition, I want to thank the following foundations for your financial support, which has enabled me to attend MIT these last two years:

- Sven och Dagmar Saléns Stiftelse
- Gunvor och Josef Anérs Stiftelse
- Carl Erik Levins Stiftelse
- John Rettigs Resestipendiefond och Gefle Köpmannaförenings samt Sven, Ernst och Jacob Engwalls stipendiefond
- Prins Carl-Gustafs Stiftelse
- Sveriges Civilingenjörsförbunds Kamrathjälpsfond

Thanks a bunch!