STOCHASTIC AND DETERMINISTIC ATTENUATION OF ONE BEAM IN THE NEUTRON INTERFEROMETER

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Experiments are reported, where one of the beams inside the neutron interferometer was attenuated in three different ways: by partially absorbing gold or indium foils, by a slow beam chopper and by a fine cadmium lattice. When the attenuation was the same the different methods still had a different effect on the amplitude of the interference pattern. This counter intuitive result can be formally explained in a quantum mechanical analysis which nicely illuminates how the history of a neutron affects its wave function.

Neutron interferometry experiments can with good justification be considered one particle experiments: the probability of having more than one particle in the apparatus at a given time is very small at all available neutron sources. Therefore one experiment consists in one particle passing through the interferometer (IFM) and being detected at one of the two detectors (fig. 1). The experiment has to be repeated sufficiently often to determine the relative counts at the two detectors, which as a function of the phase shift shows the interference pattern.

Now suppose we perform an experiment with the usual aluminum phase shifter and count a total of 1000 neutrons in O and H detectors at each phase shifter position. This is a typical situation and we will obtain a certain amplitude of the interference pattern at the O detector. It will be the same at the H detector, but with opposite modulation.

What if we perform an experiment where we again count 1000 neutrons in O and H but insert a 100% absorbing material in the left path on and off, such that the left path was open only for a certain fraction of time $\alpha$? (Here and in all subsequent experiments we assume a stationary incident beam.) We expect that the amplitude of the interference pattern will be smaller.

Now consider the same experiment where instead of inserting and withdrawing a perfect absorber we leave one with transmission probability $\alpha$ permanently installed in the left path. Again we expect a reduced interference pattern. And we might even think that it should be more or less the same in both experiments, because all we have done is to take away about the same percentage of particles in the left beam. And since we are dealing with many repetitions of a one particle experiment it should not matter how once in a while a particle is absorbed in the left path. But quantum theory tells us that there is a significant difference in the amplitudes of the resulting interference patterns [1]. We have performed experiments to demonstrate this difference quantitatively.

The experiments with the permanently installed absorber with a certain transmission probability were performed by inserting between 1 and 5 sheets of 1 mm thick slabs of gold or indium into the left path (fig. 1a). In this manner values of the transmission probability between 0.9% and 48% could be achieved. (Details are given in [2] and [3].) We call this kind of absorption stochastic, because there exists only a probability for the absorption when a neutron passes the slabs. What results does quantum theory predict? Let us only consider the O beam. Its wave function $\psi_0$ is a sum of the wave functions corresponding to the left and the right paths inside the IFM. We have

$$\psi_0(\chi, \alpha) = \sqrt{\alpha}\psi_{0L} + \psi_{0R} e^{i\chi}. \quad (1)$$

Here, $\chi$ denotes the relative phase shift caused
Fig. 1. Experiments with absorbers in the left path of a single crystal silicon interferometer which uses Bragg reflection in Laue geometry at the (220)-planes for beam splitting. The beams widen at each plate due to the Bormann fan. Monochromacy of the incident beam in these experiments is $\Delta \lambda / \lambda_0 \leq 1\%$. (a) Slabs of gold or indium as absorbers. Central wavelength of incident neutrons is $\lambda_0 = 1.974(6)$ Å. (b) Appropriately cut rotating cadmium disk of 1 mm thickness as beam chopper. The absorbing sections were virtually black with an efficiency better than 99.99%. Disks with ~25%, ~50% and ~75% transmissions were used. Central wavelength: 1.974(6) Å. (c) A one-dimensional cadmium lattice as absorber. The absorbing sections were 50 μm thick and had a length along the beam of 2 mm. The fully transparent space between them was 20 μm wide. Central wavelength: 1.924(6) Å.

by the usual phase plate between the wave functions of the left and the right path of the empty IFM, $\psi_{0L}$ and $\psi_{0R}$, respectively. And $\alpha$ represents the neutron transmission probability through the left path. It is normalized to 1 when no absorber is present in the left path. We neglected the real phase shift due to the absorber. The observed intensity will then be

$$I_0(\chi, \alpha) = |\psi_0(\chi, \alpha)|^2 = |\psi_{0R}|^2 (1 + \alpha + 2\sqrt{\alpha} \cos \chi).$$  \hspace{1cm} (2)

Here, we made use of the theoretical result that for an ideal IFM with equal thicknesses of the three crystal plates we have $\psi_{0L} = \psi_{0R}$ [4]. The important point is that the amplitude of the interference pattern varies as the square root of the transmission probability $\alpha$.

Experiments with insertion and withdrawal of a black absorber were done by chopping the left beam path (fig. 1b). The chopper was made of an appropriately cut disk of 1 mm thick cadmium. Experiments with three different choppers with effective transmissions of around 25%, 50% and 75% were performed. The open-close frequency was either 8 Hz or 16 Hz. We call this kind of absorption deterministic, because it is certain what will happen to a neutron when it is in the region of the chopper: depending on the position of the absorbing sections at that moment it will either be absorbed or be able to pass. What is the prediction for the interference pattern?

If before the chopper the wave function is given by a plane wave (which we can assume, since we are not interested in the transverse probability density of the neutron beam), then behind the chopper it will be given by a superposition of plane waves with different energies resulting in localized traveling packets. They are originally of rectangular shape if we assume instantaneous opening and closing. We get
\[ \psi_{OL}(\alpha, t) \sim \psi_{OL} \left\{ \sum_{n=-M}^{\infty} c_n \exp[i k_n x - i(\omega_0 + n\omega_c) t] \right\}, \] (3)

with \( k_n = \sqrt{2m(\omega_0 + n\omega_c)}/\hbar \) and \( M \geq \omega_0/\omega_c \), where \( \omega_0/2\pi \) is the open-close frequency of the chopper and \( \hbar k_0 \) and \( \hbar \omega_0 \) are the momentum and the energy of the incident neutrons, respectively. The approximation is due to the fact that the sum is only over positive energy states, while the Fourier transformation for obtaining the \( c_n \) starts from \( n = -\infty \). In our case it is well justified, because we have \( \omega_0/\omega_c \sim 0 \). The probability amplitudes of the different energy states are

\[ c_n \sim \sin(n \pi \alpha)/n \pi. \] (4)

In the experiment only the time-averaged interference pattern was measured:

\[ I_0(\chi, \alpha) = (\omega_c/2\pi) \int_0^{2\pi/\omega_c} |\psi_{OL}(\alpha, t) + \psi_{OR} e^{i\chi}|^2 dt = |\psi_{OR}|^2(1 + \alpha + 2\alpha \cos \chi). \] (5)

We note that the amplitude of the interference pattern is proportional to the transmission probability \( \alpha \). Thus, although the transmission probability through the left path is the same in both experiments, there is an essential difference in the amplitude of the interference pattern. The results of the experiments are shown in fig. 2a. An interesting investigation of the implications of the square root behavior has been done by Wootters and Zurek [5]. Zeilinger [6] analysed how the result is correlated to the information the observer has about a particle passing the IFM, and Mittelstaedt et al. [7] applied the same approach to a similar experiment with photons. The question that immediately arises when look-

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Fig. 2. Amplitude of the interference pattern as measured with absorber in the left path and normalized to that without absorber, drawn as a function of transmission probability through the left beam, which is normalized to 1 for the empty interferometer. (a) Experimental results are in agreement with prediction (fully drawn lines). The dashed line indicates where possible experimental results could lie when for a certain transmission of a deterministic absorber neutrons with changed energy or momentum are filtered out before reaching the detectors. (b) Results at the H detector as obtained with cadmium lattice as absorber. Point “d” obtained in the experiment as drawn in fig. 1c. A maximum of neutrons with changed momentum is filtered out by the second crystal slab. Point “a”: cadmium lattice rotated around beam axis by 90° as compared to fig. 1c. Neutrons with changed momentum are not filtered out by the second crystal plate and reach the detectors. Points “b” and “c”: cadmium lattice rotated to intermediate settings.
ing at the experimental confirmation of eqs. (2) and (5) is whether situations are conceivable where the amplitude of the interference pattern is neither proportional to the transmission $\alpha$ nor to its square root, but lies somewhere in between. In fact this is possible.

The difference between the two experiments is that we have more detailed knowledge about the possible history of a neutron in the IFM in the case of deterministic absorption. This is reflected in the wave function $\psi_0$. It contains energy states caused by the chopping, which are not present in the incident beam. Therefore, if we perform an energy measurement on the O or H beam and the result differs from $\hbar \omega_0$ then we know that this particle has gone through the left path. The energy measurement is compatible with observation of the time-independent interference pattern if the energy state is $\hbar \omega_0$. If we scan for other energy states, no interference pattern can simultaneously be observed. But this means, that the amplitude of the interference pattern is independent of whether we single out particles with energies different from $\hbar \omega_0$ at any point behind the chopper. On the other hand, the mean intensity will drop, because the effective transmission through the left path will have become smaller. (Although, of course, the transmission through the chopper itself is unchanged.) Depending on the width of the energy selection mechanism around $\hbar \omega_0$ the effective transmission through the left path can therefore vary between $\alpha^2$ and $\alpha$, so that with the unchanged amplitude of the interference pattern the points in fig. 2a could come to lie anywhere on a horizontal line between the square root and the linear behavior. Going along that line from the linear to the square root behavior means loosing information about the path of the particle inside the IFM.

We performed experiments to show this effect by using a one-dimensional cadmium lattice instead of a chopper (fig. 1c). The lattice had a periodicity of 70 $\mu$m with completely absorbing sections of width 50 $\mu$m and fully transparent sections of 20 $\mu$m. It represents a deterministic absorber for the left beam, which is a few mm wide. Any plane wave incident on the lattice will be partially absorbed, but the transmitted wave function will be a superposition of plane waves with different directions of momentum but equal energy due to diffraction on the lattice. The information about the path is now carried by those particles which behind the lattice are in a momentum state different from before the lattice. If these particles remain in the beams until detection, then the linear dependence of the interference pattern on the transmission through the left path is found. In the experiment this was tested by turning the lattice by 90° around the axis of the beam, so that the diffracted neutrons went in directions above and below the plane determined by the left and the right path inside the IFM. Then they were reflected at the second and the third crystal plate just as if their momentum had not been changed, because the crystal plates are only sensitive to changes of momentum normal to the (220)-planes which are used for Bragg reflection in these IFMs. The result of this experiment is represented by the point labelled “d” in fig. 2b.

Filtering out neutrons whose momentum changed when passing the cadmium lattice was partially achieved in the experiment as sketched in fig. 1c. Now a fraction of the neutrons in the left path no longer fulfilled the Bragg condition at the second crystal plate and thus was lost before reaching the O or the H detector. The effective transmission through the left path thus decreased, while the interference pattern should not be influenced. The result is shown as point “a” in fig. 2b. The points “b” and “c” were obtained by rotating the cadmium lattice to intermediate positions. More details can be found in ref. [3].

Finally we should mention how the transmission through the left path was measured in the three experiments. In all of them this was done by blocking the right path and measuring the intensity at the O and the H detector with and without the absorber in the left beam path. From the ratio the transmission through the left path was obtained by applying a small correction for background.

In conclusion we can say that deterministic absorption of neutrons in one path inside the IFM always has two effects: some neutrons are
absorbed, but of those which pass the absorber there are some which change their momentum or energy state. (Depending on whether the absorber has spatial or temporal structure.) For these the path can in principle be determined behind the IFM. In stochastic absorption, however, neutrons are either absorbed or transmitted. All the transmitted neutrons are in the same state as before the absorber. Behind the IFM there are then no neutrons for which the path could be determined. Therefore the interference pattern is larger in the case of stochastic absorption.

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References