Direct solar transmittance and irradiance predictions with broadband models. Part I: detailed theoretical performance assessment

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Received 9 December 2002; accepted 13 May 2003

Abstract

A thorough investigation on the performance of broadband direct irradiance predictions using solar radiation models is detailed here. Nineteen models were selected from an extensive literature survey. In addition, two new models were specifically developed for this study to provide state-of-the-art modelling of the broadband transmittances associated with the most important extinction processes in the atmosphere. The SMARTS spectral radiative code has been selected to provide 2064 reference transmittance and irradiance values, corresponding to as many combinations of solar position and varied atmospheric conditions. Inconsistencies or errors in the modelling of different transmittance functions from existing models were found, and could be corrected in some cases. As a result of this theoretical assessment, it is concluded that detailed transmittance models normally perform better than bulk models, and that models using Linke’s turbidity coefficient in intermediate calculations performed poorly. Four high-performance models can be recommended as a result of this detailed investigation: CPCR2, MLWT2, REST and Yang (in alphabetical order). The new MLWT2 model provides the best performance in all tests, thanks to its elaborate multi-layer spectral weighting scheme.

1. Introduction

Solar energy systems are more efficient under cloudless conditions than under cloudy conditions. Moreover, most solar concentrators can operate properly under cloudless conditions only. Hence the primordial importance of evaluating the maximum solar resource i.e., the incident cloudless-sky (or ‘clear-sky’) direct, diffuse, and global irradiance. Under cloudless skies, solar radiation is essentially in the form of direct beam radiation. The accurate prediction of direct irradiance is therefore essential in many solar energy applications, particularly those relying on concentrators because they use only (or, at least, mainly) direct beam irradiance. This prediction can be performed in various ways, depending on the time scale and availability of necessary data. The present investigation is limited to the prediction of direct irradiance under clear-sky conditions and short time intervals (1 hour or less), from meteorological or atmospheric data only. (In other words, the estimation of direct irradiance from global irradiance using empirical decomposition models is out of the scope of this study.)

The type of radiation models considered here is of much importance because of the need for detailed space and time coverage expressed by a variety of users, such as scientists and designers of solar energy systems. Furthermore, the dire paucity of measured and reliable data is such that the development of solar radiation atlases or Typical Meteorological Years (TMYs) is based primarily on modelled data. For instance, the US National Solar Radiation Data Base provides hourly radiation data and TMYs at 239 US sites, but 93% of these ‘data’ had to be modelled (Maxwell, 1998; Maxwell et al., 1991) — using one of the models reviewed here for clear skies. And because these datasets condition the correct design, performance and economic viability of many solar-related projects, it is...
important to assess the quality of all these radiation predictions.

In an initial study (Gueymard, 1993a), it has been shown that the performance of the tested models was largely variable and that some models had serious limitations due to incorrect transmittance equations or simplistic assumptions. This suggested that there was room left for modeling improvement. New broadband models have been proposed since then, while the development of fast computers and fast spectral irradiance models now permits almost as practical — but far more physical — calculations of solar irradiance (Gueymard, 2001b).

For all these reasons, the present study is aimed at a detailed assessment of a large range of direct radiation models, and the selection of the best methods to predict direct beam irradiance — ideally with an accuracy comparable with more sophisticated, spectral models. To achieve this, a complete re-evaluation of the earlier study mentioned above (Gueymard, 1993a) was deemed necessary. However, the present investigation is limited to direct radiation, considers a larger set of models, and uses updated reference ‘benchmark’ data from both theoretical (in this Part 1) and experimental sources (in the separate Part 2 of this paper).

The following assessment methodology is used here: all models are critically reviewed (Section 2), and their predictions subjected to a common dataset of reference theoretical calculations, first for individual transmittances and then for direct irradiance (Section 3).

### Table 1

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All models are presented below in alphabetical order and reviewed for their general features. A comparison of their requirements appears in Table 1, which shows that some models need more inputs than others. The list of possible atmospheric inputs — besides solar position (zenith angle or solar elevation), which is common necessary input to all models — is as follows:

- site pressure, $p$ (in mb or hPa)
- precipitable water in the vertical column, $w$ (in cm or g/cm²)
- reduced vertical ozone column amount, $u_o$ (in atm-cm)
- reduced vertical nitrogen dioxide ($NO_2$) column amount in the stratosphere, $u_{str}$ (in atm-cm)
- reduced vertical $NO_2$ column amount in the troposphere, $u_{trop}$ (in atm-cm)
- Ångström’s spectral turbidity coefficient, $\beta$ (unitless),
- Ångström’s wavelength exponent, $\alpha$. Ångström’s turbidity coefficient; $\alpha$. Ångström’s wavelength exponent.

**2. Direct irradiance models**

Nineteen basic models that predict direct normal irradiance (DNI) using a broadband scheme have been identified after an extensive literature review. Some models that had been analyzed in the previous investigation (Gueymard, 1993a) but did not perform well enough, have been excluded from the present study for conciseness. Moreover, two new models (MLWT2 and REST, see Sections 2.13 and 2.18) have been specially developed for this study in an attempt to obtain the best possible performance, so that a total of 21 models are considered here.

All models are presented below in alphabetical order and reviewed for their general features. A comparison of their requirements appears in Table 1, which shows that some models need more inputs than others. The list of possible atmospheric inputs — besides solar position (zenith angle or solar elevation), which is common necessary input to all models — is as follows:

- site pressure, $p$ (in mb or hPa)
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- reduced vertical nitrogen dioxide ($NO_2$) column amount in the stratosphere, $u_{str}$ (in atm-cm)
- reduced vertical $NO_2$ column amount in the troposphere, $u_{trop}$ (in atm-cm)
- Ångström’s spectral turbidity coefficient, $\beta$ (unitless),
- Ångström’s wavelength exponent, $\alpha$. Ångström’s turbidity coefficient; $\alpha$. Ångström’s wavelength exponent.
• Ångström’s wavelength exponent, \( \alpha \) (unitless), and
• Unsworth–Monteith’s broadband turbidity coefficient, \( \tau_a \) (unitless).

An important criterion for the selection of models was that, for their aerosol calculations, they had to rely on either \( \beta \) or \( \tau_a \). Models depending entirely on Linke’s coefficient, \( T_h \), were excluded because of the limitations inherent to this coefficient, particularly its ambiguous dependence on both turbidity and water vapor, and its virtual daily variations caused by changes in air mass and precipitable water (Gueymard, 1998). The only exceptions were models for which \( T_h \) could be explicitly evaluated from \( \beta \) and \( w \), and was thus only an intermediate calculation step. As will be demonstrated later, the performance of these models is found to be modest anyway, which confirms that whenever accurate predictions of direct irradiance are needed, this type of model should not be used, unless separate data of turbidity and precipitable water are unavailable. Complementary tests have shown that those models that are based entirely on Linke’s coefficient might provide acceptable predictions of direct irradiance under some typical conditions, but also that they cannot compete with most of those that are based on the two other turbidity coefficients (\( \beta \) or \( \tau_a \)).

Because of the performance-driven goal of this study, it must be emphasized that a number of existing models were improved by modifying some of their equations, or replacing tabulated data by appropriate fitting functions. Whenever possible, these modifications were submitted to the original authors for approval, and are detailed below. In only one instance (see Section 2.11), a model was found to be flawed with too many parameterization problems and could not be improved.

Because of the very limited number of models calculating aerosol transmittance as a function of \( \alpha \) (Table 1), and the relative difficulty in obtaining reliable data of \( \alpha \) in most cases, its value has been simply fixed here to the usual, Ångström-recommended 1.3 value, which corresponds well to rural or continental aerosols — but not to other types of aerosols. This conventional value is also used by essentially all methods deriving \( \beta \) from direct irradiance data (e.g. Cañada et al., 1993; Gueymard, 1998; Louche et al., 1987), so that many turbidity datasets are implicitly tied to this specific value if obtained with these methods (e.g. Gueymard and Garrison, 1998; El-Wakil et al., 2001; Pedrós et al., 1999).

In all what follows, \( E_{bn} \) is the direct (beam) normal irradiance, \( E_{in} \) is the extraterrestrial irradiance (i.e. the solar constant times the sun–earth distance correction factor), \( Z \) is the sun’s zenith angle, and \( h \) the sun’s elevation angle (\( h=90°-Z \)).

### 2.1. Bird model

The original Bird model (Bird and Hulstrom, 1980, 1981a,b) formed the basis for Iqbal’s Model C that was reviewed in previous investigations (Gueymard, 1993a; Battles et al., 2000). It is also the basis for the more recent METSTAT model (Maxwell, 1998) that is reviewed here in Section 2.9.

With Bird’s model, DNI is obtained as

\[
E_{bn} = CE_{in}T_hT_rT_aT_s
\]

where \( C=0.9662, T_h \) is the Rayleigh transmittance, \( T_r \) the ozone transmittance, \( T_g \) the uniformly-mixed gas transmittance, \( T_w \) the water vapor transmittance, and \( T_s \) the aerosol transmittance. Expressions for the different transmittances appear in the original publications (Bird and Hulstrom, 1980, 1981a,b). The aerosol transmittance expression is repeated below for the sake of subsequent discussions:

\[
T_a = \exp(-f_a^{0.875}(1 + f_a^{-0.7088})m^{0.9108})
\]

where \( m \) is the relative air mass, and the average aerosol optical depth, \( t_a \), is specifically defined for this model from the spectral optical depths at 0.38 and 0.50 \( \mu \)m as

\[
t_a = 0.2758 \tau_{a(0.38)} + 0.357 \tau_{a(0.51)}.
\]

Because these spectral optical depths are generally not known, Eq. (3) has been modified here through the use of Ångström’s law,

\[
\tau_a = \beta \lambda^{-\alpha}.
\]

For the single value of the wavelength exponent considered here, \( \alpha = 1.3 \), Eq. (3) becomes:

\[
t_a = 1.832 \beta.
\]

### 2.2. CEM model

This model has been named after the affiliation of its authors (Atwater and Ball, 1978, 1979). As originally presented, all the governing equations of this model are straightforward, except for the aerosol transmittance, which is calculated as a function of an unspecified ‘aerosol volume absorption coefficient’ (Atwater and Ball, 1978). A previous publication (Atwater and Brown, 1974) describes the derivation process (based on Mie theory) in general terms, but does not provide any specifics. To circumvent this limitation, the CEM authors’ aerosol volume absorption coefficient has been simply replaced here by the Unsworth–Monteith turbidity coefficient, according to its definition

\[
T_a = \exp(-m\tau_a)
\]

where the relative air mass, \( m \), is defined by CEM’s authors as

\[
m = 35/(1 + 1224 \cos^2 Z)^{0.5}.
\]

There are two other noticeable features in this model: (i) no ozone dependence is considered, so that the ozone transmittance has been set to 1 for this study; (ii) water
vapor absorptance is used by the model rather than transmittance. To compare the water vapor transmittance with that of other models (in Section 3.1), it has been simply evaluated here as the complement of the absorptance. However, DNI's predictions (for Section 3.2) are from the unmodified model.

2.3. Choudhury’s model

This model (Choudhury, 1982) adopts, for its most part, Hoyt’s basic equations proposed earlier (Hoyt, 1978). Another version of Hoyt’s model is Iqbal’s Model B, which has been previously reviewed and tested (Gueymard, 1993a). The specific contribution of Choudhury consisted in his Eqs. (3) and (10)–(12), which are used here. Because this model uses the concept of ‘absorption ratios’ rather than transmittance (as in the CEM model just described), the transmittance for extinction process $i$ has been evaluated as $T_i = 1 - e_i$, where $e_i$ is the corresponding absorption ratio (from Eqs. (1)–(4) in Choudhury’s paper). The aerosol transmittance has been evaluated here from

$$T_s = 1 - e_s - r_a$$

where $e_s$ and $r_a$ are determined by Choudhury’s Eqs. (7) and (8), respectively. As with the CEM model, these transmittance expressions are used for comparative purposes only in Section 3.1.

2.4. CPC2 model

This is the only two-band model in this study, with a separation wavelength at 0.7 μm. It has been found to be a top performer in earlier evaluation studies (Gueymard, 1993a; Battles et al., 2000). It is completely described elsewhere (Gueymard, 1989), publication which the reader should consult for further details. Even though this model depends on $\alpha$ (Table 1), it has been fixed here to 1.3 for the reasons indicated above.

2.5. Dogniaux’s model

This older radiation model (Dogniaux, 1973, 1975; Dogniaux and Lemoine, 1976) first calculates $T_L$ from $h$, $w$ and $\beta$ using Dogniaux’s empirical expression:

$$T_L = 0.1 + [(h + 85)/(39.5 \exp(-w) + 47.4)]$$

$$+ (16 + 0.22w)\beta,$$

and then irradiance from

$$E_{mw} = E_{mc} \exp(-m\tau_c T_L)$$

where the clean-dry atmosphere optical depth, $\tau_c$, is obtained as

$$\tau_c = 1.4899 - 2.1099 \cos h + 0.6322 \cos 2h$$

$$+ 0.0252 \cos 3h - 1.0022 \sin h + 1.0077 \sin 2h$$

$$- 0.2606 \sin 3h.$$  

[Note that the coefficient of $\cos 3h$ is corrected here for a misprint in one of the original publications (Dogniaux and Lemoine, 1976).]

2.6. Iqbal’s model C

This model has been described elsewhere (Iqbal, 1983), and is essentially identical to the original Bird model reviewed in Section 2.1. The only differences are:

- Coefficient C is here 0.9751 in Eq. (1)
- Precipitable water is pressure and temperature corrected (Iqbal, 1983) [but only the pressure correction will be considered here for simplicity].
- The aerosol optical mass is pressure corrected. Results from two previous independent evaluations (Gueymard, 1993a; Battles et al., 2000) showed that this model’s performance was close or equivalent to that of CPC2.

2.7. King and Buckius model

In this bulk transmittance model, turbidity can be specified either by $\beta$ or by ‘visibility’ (in fact, meteorological range). The former option is used here, and the overall direct transmittance of the atmosphere is evaluated according to Eq. (23) in the original publication (King and Buckius, 1979). This clear-sky model has been selected to form the basis for an all-sky model (Ideriah, 1981).

2.8. Modified MAC model: MMAC

The original MAC model (Davies et al., 1975, 1988; Davies and McKay, 1989) has been used in various forms by different authors (Freund, 1983; Uboegbulam and Davies, 1983; Hay and Darby, 1984; McGuffie et al., 1985). It was reviewed and evaluated in a previous study (Gueymard, 1993a), with the conclusion that its simplistic expression for the aerosol transmittance was generally degrading its performance in a significant way. For the present study, therefore, an alternate expression for the aerosol transmittance, $T_a$, has been used, as suggested by some of the MAC model’s proponents:

$$T_a = \exp(-m' \tau_c)$$

where $m'$ is the absolute air mass, i.e. $m' = m (p/1013.25)$ and $m$ is obtained from $Z$ (Kasten, 1965). Note that preliminary tests have shown that MMAC performs globally better with this unusual expression (where $m'$ is used rather than $m$) than with the more physically-sound Eq (6). [The latter equation is used in other variations of MAC as
2.11. Molineaux’s model

This model (Molineaux et al., 1995) is comparable to Dogniaux’s because it, too, does not detail all transmittances, and is based on \( w \) and \( \beta \) via \( T^\prime \). It uses Eq. (10) along with the following expression for \( \tau^\prime \):

\[
\tau^\prime = 0.124 - 0.0285 \ln m^\prime.
\]

where \( m^\prime \) the pressure-corrected (or absolute) air mass (Kasten and Young, 1989). \( T^\prime \) is obtained from \( \beta, w \) and \( m^\prime \) through Eq. (19) in the original publication (Molineaux et al., 1995).

2.11. Meteorological radiation model (MRM)

MRM has been developed to predict irradiance under all possible atmospheric conditions, not only the clear sky case strictly envisioned here. It has been described in different publications (Kambezidis et al., 1993, 1997; Muneer et al., 1996, 1997a,b, 1998, 2000), with some variations. The equations used here are those from the latest Fortran version (4.0) of the model (Personal Communications with Tariq Muneer and Harry Kambez, 2002). This model has been used to evaluate solar radiation for different European projects.

Its equations are summarized below for the sake of subsequent discussions.

\[
E_\text{bs} = E_\text{on} \left( \frac{T^\prime}{T^\prime_0} - a_w \right) T_s
\]

\[
T^\prime = \left[ \frac{1}{1 + \exp(2.12182 - 0.791532 \ln m^\prime)} + 0.024761 \ln^2 m^\prime \right].
\]

All other expressions are common with the references mentioned above. As with CEM and Choudhury’s models, the water vapor transmittance is simply considered here to be \( T_w = 1 - a_w \).

2.9. METSTAT model

Like Iqbal’s Model C, METSTAT (Maxwell, 1998) is based on Bird’s model. The modifications are:

- Coefficient \( C \) is here 0.9751 in Eq. (1), as for Iqbal’s Model C
- A different expression for \( T_w \) is used
- A different expression for the air mass is used (Kasten and Young, 1989)
- Eq. (6) is used for \( T^\prime_w \), rather than Eq. (2).

This model has been used to predict irradiances at 239 locations for the US National Solar Radiation Data Base (NSRDB), as documented elsewhere (Maxwell et al., 1991).

2.10. Molineaux’s model

This model (Molineaux et al., 1995) is comparable to Dogniaux’s because it, too, does not detail all transmittances, and is based on \( w \) and \( \beta \) via \( T^\prime \). It uses Eq. (10) along with the following expression for \( \tau^\prime \):

\[
\tau^\prime = 0.124 - 0.0285 \ln m^\prime.
\]

where \( m^\prime \) the pressure-corrected (or absolute) air mass (Kasten and Young, 1989). \( T^\prime \) is obtained from \( \beta, w \) and \( m^\prime \) through Eq. (19) in the original publication (Molineaux et al., 1995).

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Its equations are summarized below for the sake of subsequent discussions.

\[
E_\text{bs} = E_\text{on} T^\prime \frac{T^\prime_0}{T^\prime} T^\beta \frac{T^\omega}{T_s}
\]

\[
T^\prime = 0.8325 + 0.0216 m^\prime + 0.0174 m^\prime 2 - 0.007 m^3
\]

\[
+ 0.0002 m^4
\]

\[
T_w = 1 - \frac{0.1611 X_w (1 + 139.48 X_w)^{-0.3035} - 0.002715 X_w / (1 + 0.44 X_w + 0.0003 X_w^2)}
\]

\[
T_v = \exp(-0.0123 m^\omega)
\]

\[
T_w = 1 - 3.4462 X_w [(1 + 77.0248 X_w)^{0.414} + 3.3584 X_w]
\]

where \( X_w = m_w \), \( X_w = m_w \), \( m \) and \( m^\prime \) are defined elsewhere (Kasten and Young, 1989), and \( T_v \) is obtained from Eq. (2) but with \( t_v \) fixed at the constant value 0.394 rather than being a turbidity variable. This value is considered by the MRM authors to be representative of aerosol conditions over all the United Kingdom and US (Muneer et al., 1996).

For \( Z > 85^\circ \), Eq. (16) is replaced by

\[
E_\text{bs} = 0.65317 E_\text{on}.
\]
evaluate turbidity from measured irradiance data. For the sake of clarity, DNI is obtained here from:

\[ E_{\text{in}} = E_{\text{irr}} T_{\text{R}} T_{\text{a}} T_{\text{u}} T_{\text{s}} \]

(22)

where \( T_{\text{R}} \) is the transmittance for the combined Rayleigh and mixed-gas effects, and is obtained from \( \exp(-m R f_R (f_2 + f_3)) \), per the Appendix of the latter publication. Similarly, \( T_{\text{a}} \) is obtained as \( \exp(-m_a f_a) \), \( T_{\text{u}} \) as \( \exp(-m_u f_u) \), \( T_{\text{m}} \) as \( \exp(-m_m f_m) \), \( T_{\text{w}} \) as \( \exp(-m_w f_w) \), and \( T_{\text{s}} \) as \( \exp(-m_s f_s) \), where all necessary expressions are also found in the above-mentioned Appendix (Gueymard, 1998).

### 2.13. Multilayer-weighted transmittance model, version 2 (MLWT2)

The slight revision proposed here for the previous model reflects the changes in the predicted DNI that occur when using the latest version of the spectral code SMARTS (2.9.2 as of this writing) rather than the completely different version 2.8 that was used to derive MLWT1. (This spectral code is discussed in Section 3). Corrections are introduced to the original optical depths for each extinction process. The resulting model is detailed in Appendix A.

### 2.14. Parameterized solar irradiance model (PSIM)

This author developed a special kind of radiation model where the irradiance is parameterized as a function of \( \beta \) and of the sine of solar elevation rather than air mass — thus allowing an algebraic integration over time in order to obtain daily irradiations directly, without any recourse to numerical integration (Gueymard, 1993b). The latter reference contains all the necessary equations.

### 2.15. Perrin de Brichambaut’s model

Better known for his empirical models adapted to the French climate, this author also devised a few simple equations (Perrin de Brichambaut and Vague, 1982) that can be combined to form a complete radiation model. Because the Rayleigh and aerosol transmittances were only provided as tabulated data, best fits (Eqs. (24) and (25) below) were obtained in the course of this study for easier computerized use. This adaptation also provides a way of expanding the model’s capabilities to a larger range of conditions. The resulting revised model will be referred to as ‘Perrin’.

Because the original publication is not widely available, all the necessary equations (as implemented here) are provided below.

\[ E_{\text{in}} = E_{\text{irr}} T_{\text{R}} T_{\text{a}} T_{\text{u}} T_{\text{s}} (1 - a_s - a_w - a_5) \]

(23)

where

\[ T_{\text{R}} = \exp(-0.031411 - 0.064331 m^2) \]  
(24)

\[ T_{\text{a}} = \exp(-1.4327 m \beta) \]  
(25)

\[ a_s = 0.015 + 0.024 m \mu_s \]  
(26)

\[ a_w = 0.1 + 0.03 \ln(mw) + 0.002 \ln^2(mw) \]  
(27)

\[ a_5 = 0.013 - 0.0015 \ln(mv) \]  
(28)

where Kasten’s expression is used for \( m \). As with other absorptance-based models (see Sections 2.2 and 2.3), transmittances have been calculated as the complement of absorptance to allow their performance assessment in Section 3.1.

### 2.16. Power’s model

Using the same approach as in PSIM reviewed in Section 2.14, this model predicts irradiance from functions of solar elevation rather than air mass (Power, 2001a). This forms the basis of a daily radiation model (Power, 2001b) that can be purposefully reversed to obtain the daily-average turbidity (Power and Willmott, 2001). Power’s Corrigendum (Power, 2001a) states that her method is not really meant to be used in a predictive mode, i.e. to derive instantaneous irradiance or turbidity values. However, it can be argued that the summation of instantaneous irradiance values over a day cannot produce accurate daily irradiations in all cases if these basic values are not accurate themselves, i.e. that any daily model must be consistent with accurate instantaneous predictions.

### 2.17. Psiloglou’s model

The development of the individual transmittances for this model is described in a number of separate publications (Psiloglou et al., 1994, 1995a,b, 1996, 1997). The resulting complete model is described in a more recent contribution (Psiloglou et al., 2000). A special feature of this model is that the mixed gas transmittance is obtained as the product of five individual transmittances, one for each of the gases that are uniformly mixed in the atmosphere: \( \text{CH}_4 \), \( \text{CO} \), \( \text{CO}_2 \), \( \text{N}_2\text{O} \), and \( \text{O}_3 \). The transmittance for all these gases, as well as for water vapor, ozone, and aerosols, has the general functional form:

\[ T_i = 1 - A_i m U_i / [(1 + B_i m U_i)^{C_i} + D_i m U_i] \]

(29)

where \( A_i \), \( B_i \), \( C_i \), and \( D_i \) are numerical coefficients that depend on each extinction process \( i \), and are given in Table 1 of the latter reference (Psiloglou et al., 2000). According to all the model’s references cited above, the relative air mass is implicitly used for all extinction processes. For a more consistent approach, the relative air mass has been used here only for ozone, water vapor, and aerosols, whereas the absolute air mass has been used for Rayleigh
scattering and mixed gas absorption. This improvement has been accepted by the original author (Personal communications with Dr. Basil Psiloglou, 2002). In Eq (29), $U$ represents the ‘absorber amount’ for each extinction process. This quantity is variable for ozone and water vapor, and equal to $u_o$ and $w$, respectively. It is fixed for both the aerosols (and equal to 1 in this case) and the uniformly mixed gases. The necessary $U$ values, for these gases were omitted in the model’s description (Psiloglou et al., 2000). These missing values were originally obtained from (Pierluissi and Tsai, 1987) and are provided here for the reader’s convenience: 1.60 for CH$_4$, 0.075 for CO, 330.0 for CO$_2$, 0.28 for N$_2$O, and 2.095×10$^3$ for O$_3$.

It is important to note that the aerosol transmittance has been evaluated for fixed ‘typical’ urban conditions of the Mediterranean region, using aerosol profiles from Athens, Greece. Therefore, this aerosol transmittance is not a function of turbidity, with the result that Psiloglou’s model should be regarded more as a site-specific model than a general-purpose model.

2.18. REST model

This newly proposed model called ‘Reference Evaluation of Solar Transmittance’ (hereafter, REST) is a direct spin-off of the intermediate calculations performed for this study. Its basic functional form is similar to other models, except that the total NO$_3$ absorption is taken into account through a specific transmittance, $T_\text{a}$:

$$E_\text{ma} = E_\text{m0} T_\text{k} T_\psi T_\alpha T_\text{w} T_\text{a}$$ (30)

New and highly accurate parameterizations for each of the extinction processes have been obtained here by fitting the reference calculations discussed in Section 3.1 to very efficient functions (mostly polynomial ratios). The resulting equations are detailed in Appendix B. These parameterizations can be regarded as the state-of-the-art in single-band modelling as far as individual transmittances are concerned, i.e. with no provision for the multilayer spectral weighting effect described above. Therefore, this model does not guarantee the best overall performance in irradiance prediction because of the limitations in the Beer–Bouguer–Lambert law when extrapolated to broadband calculations, as mentioned in Section 2.12.

2.19. Rodgers’ model

This parameterized, computer-ready radiation model predicts $E_\text{ma}$ from $\tau_a$, $w$ and $m$ without calculating separate atmospheric transmittances (Rodgers et al., 1978). Because the original publication is not widely available, the necessary equations are reproduced here for the convenience of the reader. $E_\text{ma}$ is obtained as a function of the ideal irradiance below an aerosol-free atmosphere, $E_\text{ma}$, from:

$$E_\text{ma} = T_a E_\text{ma}$$ (31)

where $T_a$ is from Eq. (6) and $E_\text{ma}$ is parameterized as:

$$E_\text{ma} = E_\text{m0} \exp(b_0 + b_1 m' + b_2 m'' + b_3 m''')$$ (32)

The air mass is obtained from $m' = (p/1013.25)/\cos Z$ if $Z < 80^\circ$. Otherwise:

$$m' = (p/1013.25) \exp(3.67985 - 24.4465 \cos Z$$

$$+ 154.017 \cos^2 Z - 742.181 \cos^3 Z + 2263.36 \cos^4 Z - 3804.89 \cos^5 Z + 2661.05 \cos^6 Z).$$ (33)

The coefficients in Eq. (32) are quadratic functions of $w$ through:

$$b_0 = -0.129641 + 0.0412828 w - 0.0112096 w^2$$ (34)

$$b_1 = -0.0642111 - 0.0801046 w + 0.0153069 w^2$$ (35)

$$b_2 = -0.0046883 + 0.0220414 w - 0.00429818 w^2$$ (36)

$$b_3 = 0.000844097 - 0.00191442 w + 0.000374176 w^2.$$ (37)

2.20. Santamouris model

This transmittance model consists in a reevaluation of Bird’s model, with different numerical coefficients, and is fully described in a set of publications (Santamouris et al., 1985; Santamouris and Rigopoulos, 1987; Santamouris, 1991). As implemented here, the absolute air mass — rather than the relative air mass — is used for two extinction effects (Rayleigh scattering and mixed gases absorption). The turbidity-dependent expression for the aerosol transmittance has been used here [per Eq. (8) in the latter reference Santamouris, 1991]. It is a function of an average aerosol optical depth $\tau_a$, itself derived from two spectral aerosol optical depths, like in Bird’s model (see Eq. (2) above). The same reduction process has been used here, providing $\tau_a = 1.92074$. These modifications have been approved by the model’s author (Personal communication with Dr. M. Santamouris, 2002).

2.21. Yang’s model

This recent solar radiation model is based on a product of transmittances exploiting the idea of effective wavelengths, as in CPCPR2 (Yang et al., 2001). Due to some inconsistencies in the air-mass pressure correction in the published equations (Yang et al., 2001), those actually used here take subsequent, unpublished corrections into account (Personal communication with Dr. K. Yang, 2002). For clarity, the revised model is described below:

$$E_\text{ma} = E_\text{m0}(T_a T_\psi T_\alpha T_\text{w} - 0.013)$$ (38)

where
\[ T_R = \exp \left[ -0.008735m' (0.5474 + 0.01424m'^2 - 0.0003834m'^2) + 0.00000459m'^2 \right] \] (39)

\[ T_a = \exp \left[ -0.0365(\mu u)^{0.7136} \right] \] (40)

\[ T_b = \exp \left[ -0.0117m^{0.3139} \right] \] (41)

\[ T_u = \min[1.0, 0.909 - 0.036 \ln(mv)] \] (42)

\[ T_s = \exp \left[ -m\beta(0.6777 + 0.1464m\beta - 0.00626(m\beta)^2)^{1.3} \right]. \] (43)

and \( m' \) the pressure-corrected (or absolute) air mass based on (Kasten, 1965).

3. Theoretical validation

The radiation models reviewed above may only be validated if they can be compared to reference datasets of superior quality and accuracy. These can be either modelled (using state-of-the-art rigorous atmospheric radiative codes) or measured (using first-class instruments). The latter alternative will be detailed separately (in Part 2 of this contribution). The former alternative is described below.

In the previous study (Gueymard, 1993a), published results from three different reference codes were used. Two of them (BRITE and Braslau and Dave’s spherical harmonics code) have not been maintained and are not in use anymore. The third code, SOLTRAN is not in use anymore either, but its basic foundation, LOWTRAN’s code version 3, has been updated up to version 7. It has then been replaced by MODTRAN, which is now at version 4. Over the years, LOWTRAN and then MODTRAN have become the de facto standard codes in atmospheric radiation computation. Fig. 1 illustrates how the historic modelling changes in the successive versions of LOWTRAN and MODTRAN affected the predictions of the total atmospheric transmittance (i.e. \( E_{\text{on}}/E_{\text{off}} \)) for identical atmospheric conditions. These conditions are: Mid-Latitude Summer atmosphere (\( w = 2.922 \text{ cm}, u_r = 0.32 \text{ atm-cm}, Z = 0^\circ \), and standard 5 and 23-km meteorological ranges. Vertical arrows on the plot correspond to the release date of some of the different versions of LOWTRAN and MODTRAN. The transmittance values used here for the LOWTRAN2 version, released in 1972, were obtained from the basic data on which Hottel’s model was founded (Hottel, 1976). (Note that this model has not been examined here because it is not a direct function of precipitable water and turbidity; due to the model’s dependence on LOWTRAN2, which now appears to significantly overestimate direct irradiance, it can be argued that this model’s performance is necessarily limited anyway). The transmittance predictions from LOWTRAN3, released in 1975, were calculated from the direct irradiance data in Table 3 of the previous investigation (Gueymard, 1993a), as initially calculated by Bird (Bird and Hulstrom, 1981a).

All the other results — obtained with subsequent versions of LOWTRAN and MODTRAN — have been obtained by this author. Fig. 1 shows that a considerable decrease in transmittance occurred between versions 2 and 7 of LOWTRAN, followed by a very slight increase as the MODTRAN family evolved. It can be argued that, for these atmospheric conditions at least, the direct transmittance predictions have stabilized and are most probably within 1% of the truth.

MODTRAN could have been the reference of choice in the present study but some of its features prevented its use for the kind of routine calculations needed here, and more generally in most solar radiation applications. For instance, MODTRAN (i) does not accept turbidity coefficients such as \( \beta \) or \( \tau \) — it only uses meteorological range, which can be related to turbidity in theory, but not easily or precisely
in practice; (ii) limits the possibilities to modify other important input variables, such as pressure, precipitable water, and ozone column amount — they cannot be given arbitrary values; (iii) does not evaluate the circumsolar irradiance that is sensed by pyrheliometers in addition to direct beam irradiance; (iv) does not integrate each individual transmittance over the spectrum; (v) outputs its results in equal steps of wavenumbers rather than wavelengths; (vi) is not fast nor easy to use; and (vii) is not in the public domain.

For these reasons, a ‘secondary standard’ spectral radiative code, SMARTS (Gueymard, 1995, 2001a, 2003; Gueymard et al., 2002) will rather be used here. In its latest version (2.9.2, released April 2003), this code benefits from improved compatibility with MODTRAN, without the latter’s limitations. Different tests, both at the spectral and broadband levels, have shown their respective predictions to be in excellent agreement. An example of such comparison is shown in Table 2, which groups one set of predictions with MODTRAN (version 4.2.1, released March 2002) and three sets with SMARTS 2.9.2. All sets are relative to the same basic atmospheric conditions: sea-level pressure \( p = 1013.25 \) mb, US Standard Atmosphere (USSA) resulting in 1.416 cm of precipitable water, 0.3438 atm-cm of ozone, 0.204 matm-cm of stratospheric \( NO_2 \), and no tropospheric \( NO_2 \). A humidity-dependent rural aerosol sub-model (Shettle and Fenn, 1979) is used in both MODTRAN and SMARTS, with a meteorological range of 124.5 km in MODTRAN, which corresponds to a spectral aerosol optical depth of 0.0840 at 500 nm, or \( \beta = 0.0314 \) in SMARTS. MODTRAN was run with its basic Wehrli extraterrestrial spectrum (which was adopted by the World Meteorological Organization (WMO), and integrates to a solar constant of 1367 W/m²), at its maximum effective resolution (2 reciprocal centimetres, cm \(^{-1}\)) over the interval 0.28–4.00 \( \mu \)m. The same spectral range was used for SMARTS in all cases, at a resolution that increases between 0.5 \( \mu \)m in the UV, 1 \( \mu \)m in the visible and near infrared, and 5 \( \mu \)m in the 1.7–4 \( \mu \)m interval. The extraterrestrial spectrum used with SMARTS was either the same Wehrli/WMO spectrum as in MODTRAN, or the standard SMARTS spectrum recently synthesized by this author, that also integrates to a solar constant of 1367 W/m². Finally, the last column in Table 2 provides the simulated ‘experimental’ direct normal irradiance, including the circumsolar contribution within a cone of 5.8° (full aperture), which is typical of field pyrheliometers.

These atmospheric conditions have been recently selected for use with SMARTS in order to obtain reference spectra — now standardized (ASTM, 2003) — for photovoltaic and other solar energy applications (Gueymard et al., 2002). Solar zenith angles between 0 and 87° are used here, corresponding to optical air masses between 1.0 and 5.5.

<table>
<thead>
<tr>
<th>( Z )</th>
<th>( m )</th>
<th>Model E.T. spectrum</th>
<th>Circumsolar</th>
<th>MODTRAN Wehrli</th>
<th>SMARTS Wehrli</th>
<th>SMARTS Gueymard</th>
<th>SMARTS Gueymard</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>No</td>
<td>No</td>
<td>989.1</td>
<td>987.9</td>
<td>988.7</td>
<td>990.4</td>
</tr>
<tr>
<td>15</td>
<td>1.035</td>
<td>No</td>
<td>No</td>
<td>982.1</td>
<td>981.3</td>
<td>982.2</td>
<td>983.9</td>
</tr>
<tr>
<td>30</td>
<td>1.154</td>
<td>No</td>
<td>No</td>
<td>959.5</td>
<td>958.1</td>
<td>959.0</td>
<td>960.9</td>
</tr>
<tr>
<td>37</td>
<td>1.286</td>
<td>No</td>
<td>No</td>
<td>941.9</td>
<td>940.0</td>
<td>941.0</td>
<td>942.9</td>
</tr>
<tr>
<td>40</td>
<td>1.304</td>
<td>No</td>
<td>No</td>
<td>932.6</td>
<td>930.5</td>
<td>931.4</td>
<td>933.4</td>
</tr>
<tr>
<td>48.236</td>
<td>1.500</td>
<td>No</td>
<td>No</td>
<td>899.9</td>
<td>896.9</td>
<td>897.9</td>
<td>900.2</td>
</tr>
<tr>
<td>54</td>
<td>1.698</td>
<td>No</td>
<td>No</td>
<td>869.0</td>
<td>865.1</td>
<td>866.2</td>
<td>868.6</td>
</tr>
<tr>
<td>60.085</td>
<td>2.000</td>
<td>No</td>
<td>No</td>
<td>825.9</td>
<td>820.9</td>
<td>822.1</td>
<td>824.7</td>
</tr>
<tr>
<td>66.536</td>
<td>2.500</td>
<td>No</td>
<td>No</td>
<td>762.8</td>
<td>756.1</td>
<td>757.4</td>
<td>760.3</td>
</tr>
<tr>
<td>70.671</td>
<td>3.000</td>
<td>No</td>
<td>No</td>
<td>708.0</td>
<td>699.9</td>
<td>701.3</td>
<td>704.5</td>
</tr>
<tr>
<td>73.573</td>
<td>3.500</td>
<td>No</td>
<td>No</td>
<td>659.8</td>
<td>650.5</td>
<td>652.0</td>
<td>655.3</td>
</tr>
<tr>
<td>75.730</td>
<td>4.000</td>
<td>No</td>
<td>No</td>
<td>617.0</td>
<td>606.7</td>
<td>608.2</td>
<td>611.7</td>
</tr>
<tr>
<td>77.399</td>
<td>4.500</td>
<td>No</td>
<td>No</td>
<td>578.7</td>
<td>567.6</td>
<td>569.1</td>
<td>572.7</td>
</tr>
<tr>
<td>78.733</td>
<td>5.000</td>
<td>No</td>
<td>No</td>
<td>544.2</td>
<td>532.5</td>
<td>534.0</td>
<td>537.7</td>
</tr>
<tr>
<td>80</td>
<td>5.590</td>
<td>No</td>
<td>No</td>
<td>507.7</td>
<td>495.3</td>
<td>496.8</td>
<td>500.5</td>
</tr>
<tr>
<td>85</td>
<td>10.310</td>
<td>No</td>
<td>No</td>
<td>315.5</td>
<td>301.1</td>
<td>302.5</td>
<td>306.1</td>
</tr>
<tr>
<td>87</td>
<td>15.146</td>
<td>No</td>
<td>Yes</td>
<td>211.4</td>
<td>196.5</td>
<td>197.7</td>
<td>200.7</td>
</tr>
</tbody>
</table>
Table 3
Cumulative statistics for the reference dataset (N=2064). Nomenclature: m, optical air mass; \( E_{\text{dn}} \), direct normal irradiance including circumsolar contribution (W/m²); all other variables as in Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Z</th>
<th>m</th>
<th>p</th>
<th>w</th>
<th>( a_{\text{n}} )</th>
<th>( u_{\text{sn}} )</th>
<th>( u_{\text{vn}} )</th>
<th>( \beta )</th>
<th>( E_{\text{dn}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>71.19</td>
<td>5.90</td>
<td>952.9</td>
<td>1.43</td>
<td>0.339</td>
<td>0.187</td>
<td>1.07</td>
<td>0.155</td>
<td>423.4</td>
</tr>
<tr>
<td>Median</td>
<td>78.07</td>
<td>4.75</td>
<td>1013.2</td>
<td>1.00</td>
<td>0.350</td>
<td>0.200</td>
<td>0</td>
<td>0.100</td>
<td>386.6</td>
</tr>
<tr>
<td>S.D.</td>
<td>20.98</td>
<td>3.94</td>
<td>114.9</td>
<td>1.40</td>
<td>0.048</td>
<td>0.034</td>
<td>3.86</td>
<td>0.143</td>
<td>302.8</td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
<td>1.00</td>
<td>616.6</td>
<td>0</td>
<td>0.100</td>
<td>0.100</td>
<td>0</td>
<td>0</td>
<td>1.2</td>
</tr>
<tr>
<td>Maximum</td>
<td>86.96</td>
<td>14.99</td>
<td>1013.3</td>
<td>5.00</td>
<td>0.500</td>
<td>0.220</td>
<td>20.00</td>
<td>0.500</td>
<td>1223.5</td>
</tr>
</tbody>
</table>

15.15. Incidentally, some of the zenith angles considered here are common to that in Table 3 of the previous study (Gueymard, 1993a), so that a few intercomparisons are possible.

The following observations can be drawn from the results summarized in Table 2:
1. The predictions by MODTRAN and SMARTS are in excellent agreement (better than 1%) up to a zenith angle of 66.5°, or air mass 2.50.
2. The difference between MODTRAN and SMARTS increases slightly with Z, but reaches only 2.4% for Z=80°, or air mass 5.59.
3. The change induced by differing versions of the extraterrestrial spectrum results in a variation of only about 1 W/m² at all zenith angles.
4. The circumsolar contribution adds from 1.7 W/m² (or 0.2%) for an overhead sun to 3.7 W/m² (or 0.7%) for an air-mass-5 sun. The circumsolar contribution is small in this case because of the selected low-turbidity conditions. It would reach 1–10% under more typical turbidity conditions (Gueymard, 1998).

It can be concluded from the above that, with state-of-the-art calculations, the ‘true’ direct normal irradiance under ideal conditions can be estimated to within about 1% for 0°≤Z≤66°.

The discussion above justifies that all reference calculations be done with SMARTS. A dataset obtained with the inputs and outputs of 2064 SMARTS parametric runs has been constructed. This dataset covers a large number of combinations between all atmospheric variables and thus encompasses nearly all possible field conditions, including exceptionally rare or ideal ones. (See Table 3 for some cumulative statistics about this dataset). The outputs are (i) the individual broadband transmittances for each extinction process (to be used in Section 3.1), and (ii) the direct normal irradiance (to be used in Section 3.2). It should be noted that this reference dataset is totally independent of all models reviewed here, except MLWT2 and REST.

3.1. Individual transmittances

In this section, only those models that use a detailed formulation (i.e. individual transmittances for each of the main extinction processes) are subject to a complete analysis. The performance of each transmittance prediction is obtained by comparison with the reference dataset explained just above. It must be stressed that the reference transmittances used here are either the usual extraterrestrial spectrum-weighted broadband values (hereafter, ‘ESW scheme’) or the multilayered spectrum-weighted values (hereafter, ‘MSW scheme’). In the former scheme, the broadband transmittance \( T_i \) for extinction process \( i \) is obtained as

\[
T_i = \int E_{\text{on},\lambda} T_{i,\lambda} \, d\lambda / \int E_{\text{on},\lambda} \, d\lambda
\]

(44)

where \( E_{\text{on},\lambda} \) is the spectral extraterrestrial irradiance, \( T_{i,\lambda} \) is the spectral transmittance for wavelength \( \lambda \), and the summation is extended over all the spectral domain of interest here (0.28–4 \( \mu \)m). In the MSW scheme, the transmittance for layer \( i \) is rather evaluated from

\[
T_i^* = \int E_{\text{on},\lambda} T_{i,\lambda} \, d\lambda / \int E_{\text{on},\lambda} \, d\lambda
\]

(45)

where \( E_{\text{on},\lambda} \) is the spectral irradiance incident on top of layer \( i \), which is thus dependent on the transmittance of all other layers above. [The reader is referred to Gueymard, 1996, 1998 for a more detailed discussion, and to Fig. 1 of the latter reference (Gueymard, 1998) for a visual explanation of the different layers considered here]. As mentioned above, MLWT1 and MLWT2 are the only known operational models using this type of derivation.

The overall distortion effect due to deviations from the Beer–Bouguer–Lambert law will be also evaluated from the accuracy in the end result, \( E_{\text{dn}} \) (Section 3.2).

Cumulative statistics on the performance of each transmittance are displayed in Tables 4–8. These statistics are the Mean Bias Error (MBE) and the Root Mean Square Error (RMSE), both expressed in percent of the average reference transmittance. These statistics have been calculated for both the ESW and MSW reference transmittances.

3.1.1. Rayleigh and mixed-gas transmittance

Most models consider separate Rayleigh and uniformly mixed gas transmittances. There are some exceptions: CEM and MMAC, which have no provision for the latter, and MLWT1 and MLWT2, which do not consider them separately but rather evaluate a combined transmittance.
Table 4
Performance statistics for combined Rayleigh/mixed-gas transmittance relative to the reference dataset based on SMARTS. Mean Bias Errors (MBE) and Root Mean Square Errors (RMSE) are expressed in % of the average SMARTS results, whose values are 0.7133 for the ESW integration scheme (extraterrestrial-spectrum weighting) and 0.7294 for the MSW scheme (multilayered-spectrum weighting), respectively. Numbers in bold indicate best results for each scheme.

<table>
<thead>
<tr>
<th>Model</th>
<th>Min</th>
<th>Max</th>
<th>ESW scheme</th>
<th>RMSE (%)</th>
<th>MSW scheme</th>
<th>RMSE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>MBE (%)</td>
<td></td>
<td>MBE (%)</td>
<td></td>
</tr>
<tr>
<td>SMARTS (ESW)</td>
<td>0.5124</td>
<td>0.9313</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMARTS (MSW)</td>
<td>0.5302</td>
<td>0.9371</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bird</td>
<td>0.5815</td>
<td>0.9316</td>
<td>2.3</td>
<td>3.0</td>
<td>0.0</td>
<td>1.3</td>
</tr>
<tr>
<td>CEM</td>
<td>0.3862</td>
<td>0.9136</td>
<td>-5.0</td>
<td>6.6</td>
<td>-7.1</td>
<td>8.7</td>
</tr>
<tr>
<td>Choudhury</td>
<td>0.5116</td>
<td>0.9368</td>
<td>0.4</td>
<td>0.7</td>
<td>-1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>CPCR2</td>
<td>0.5288</td>
<td>0.9325</td>
<td>1.6</td>
<td>1.6</td>
<td>-0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>Iqbal C</td>
<td>0.5815</td>
<td>0.9316</td>
<td>2.3</td>
<td>3.0</td>
<td>0.0</td>
<td>1.3</td>
</tr>
<tr>
<td>METSTAT</td>
<td>0.5814</td>
<td>0.9316</td>
<td>2.3</td>
<td>3.0</td>
<td>0.0</td>
<td>1.3</td>
</tr>
<tr>
<td>MLWT1</td>
<td>0.5333</td>
<td>0.9384</td>
<td>2.7</td>
<td>2.8</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>MLWT2</td>
<td>0.5301</td>
<td>0.9369</td>
<td>2.3</td>
<td>2.4</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>MMAC</td>
<td>0.5391</td>
<td>0.9256</td>
<td>1.6</td>
<td>2.0</td>
<td>-0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>MRM</td>
<td>0.8428</td>
<td>2.0328</td>
<td>96.0</td>
<td>123.5</td>
<td>92.2</td>
<td>119.5</td>
</tr>
<tr>
<td>Perrin</td>
<td>0.3628</td>
<td>0.9198</td>
<td>-3.4</td>
<td>7.3</td>
<td>-5.6</td>
<td>9.1</td>
</tr>
<tr>
<td>Psiloglou</td>
<td>0.5350</td>
<td>0.9211</td>
<td>1.0</td>
<td>1.5</td>
<td>-1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>REST</td>
<td>0.5126</td>
<td>0.9313</td>
<td><strong>0.0</strong></td>
<td><strong>0.0</strong></td>
<td>-2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>Santamouris</td>
<td>0.5754</td>
<td>0.9306</td>
<td>1.8</td>
<td>2.5</td>
<td>-0.5</td>
<td>1.3</td>
</tr>
<tr>
<td>Yang</td>
<td>0.5361</td>
<td>0.9340</td>
<td>1.2</td>
<td>1.4</td>
<td>-1.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

For the sake of conciseness, this procedure (combining calculations have been performed for altitudes from sea level (\(p=1013.25\) mb) to 4 km (\(p=616.6\) mb, from the USSR). For the range of zenith angles considered here (Z=0–87°), the optical air mass used by each model — with the exception of CEM and Choudhury’s — differ only little from that of SMARTS/REST, Eq. (B8), which is the reference optical air mass used here (Fig. 2).

Results for the combined Rayleigh/mixed-gas transmittance, \(T_{\text{Rg}}\), shown in Fig. 3, demonstrate excellent agree-

Table 5
Performance statistics for ozone transmittance. The reference transmittance average is here 0.9506. The ESW and MSW integration schemes are equivalent in this case.

<table>
<thead>
<tr>
<th>Model</th>
<th>Min</th>
<th>Max</th>
<th>ESW and MSW schemes</th>
<th>RMSE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>MBE (%)</td>
<td></td>
</tr>
<tr>
<td>SMARTS (ESW and MSW)</td>
<td>0.9094</td>
<td>0.9898</td>
<td>-1.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Bird</td>
<td>0.8379</td>
<td>0.9926</td>
<td>-5.1</td>
<td>5.4</td>
</tr>
<tr>
<td>CEM</td>
<td>1.0000</td>
<td>1.0000</td>
<td>-0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Choudhury</td>
<td>0.8904</td>
<td>0.9843</td>
<td>-0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>CPCR2</td>
<td>0.8856</td>
<td>0.9922</td>
<td>-1.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Iqbal C</td>
<td>0.8379</td>
<td>0.9926</td>
<td>-1.2</td>
<td>2.1</td>
</tr>
<tr>
<td>METSTAT</td>
<td>0.8385</td>
<td>0.9926</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>MLWT1</td>
<td>0.8944</td>
<td>0.9895</td>
<td><strong>0.0</strong></td>
<td><strong>0.0</strong></td>
</tr>
<tr>
<td>MLWT2</td>
<td>0.9089</td>
<td>0.9898</td>
<td>-1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>MMAC</td>
<td>0.8810</td>
<td>0.9763</td>
<td>-1.3</td>
<td>2.0</td>
</tr>
<tr>
<td>MRM</td>
<td>0.9222</td>
<td>0.9932</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Perrin</td>
<td>0.8042</td>
<td>0.9826</td>
<td>-1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>Psiloglou</td>
<td>0.8463</td>
<td>0.9932</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>REST</td>
<td>0.9068</td>
<td>0.9897</td>
<td>-1.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Santamouris</td>
<td>0.8379</td>
<td>0.9926</td>
<td>-0.7</td>
<td>1.5</td>
</tr>
<tr>
<td>Yang</td>
<td>0.8571</td>
<td>0.9930</td>
<td>0.0</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Table 6
Performance statistics for water vapor transmittance. The reference average transmittances are 0.8512 for the ESW scheme and 0.7965 for the MSW scheme

<table>
<thead>
<tr>
<th>Model</th>
<th>Min</th>
<th>Max</th>
<th>ESW scheme</th>
<th>MSW scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>MBE (%)</td>
<td>RMSE (%)</td>
</tr>
<tr>
<td>SMARTS (ESW)</td>
<td>0.6769</td>
<td>1.0000</td>
<td>3.2</td>
<td>4.3</td>
</tr>
<tr>
<td>SMARTS (MSW)</td>
<td>0.4807</td>
<td>1.0000</td>
<td>3.8</td>
<td>4.1</td>
</tr>
<tr>
<td>Bird</td>
<td>0.7811</td>
<td>1.0000</td>
<td>0.3</td>
<td>1.3</td>
</tr>
<tr>
<td>CEM</td>
<td>0.7102</td>
<td>1.0000</td>
<td>2.4</td>
<td>3.4</td>
</tr>
<tr>
<td>Choudhury</td>
<td>0.6300</td>
<td>1.0000</td>
<td>3.3</td>
<td>4.3</td>
</tr>
<tr>
<td>CPCR2</td>
<td>0.7513</td>
<td>1.0000</td>
<td>1.8</td>
<td>2.7</td>
</tr>
<tr>
<td>Iqbal C</td>
<td>0.7811</td>
<td>1.0000</td>
<td>4.6</td>
<td>6.1</td>
</tr>
<tr>
<td>METSTAT</td>
<td>0.7514</td>
<td>1.0000</td>
<td>2.6</td>
<td>8.6</td>
</tr>
<tr>
<td>MLWT1</td>
<td>0.5214</td>
<td>1.0000</td>
<td>3.2</td>
<td>1.2</td>
</tr>
<tr>
<td>MLWT2</td>
<td>0.4839</td>
<td>1.0000</td>
<td>1.4</td>
<td>1.9</td>
</tr>
<tr>
<td>MRM</td>
<td>0.1317</td>
<td>1.0000</td>
<td>45.9</td>
<td>48.8</td>
</tr>
<tr>
<td>Perrin</td>
<td>0.7330</td>
<td>0.9585</td>
<td>0.9</td>
<td>3.1</td>
</tr>
<tr>
<td>Psiloglou</td>
<td>0.7127</td>
<td>1.0000</td>
<td>0.2</td>
<td>1.2</td>
</tr>
<tr>
<td>REST</td>
<td>0.6580</td>
<td>1.0000</td>
<td>1.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Santamouris</td>
<td>0.7563</td>
<td>1.0000</td>
<td>2.2</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table 7
Performance statistics for aerosol transmittance. The reference average transmittances are 0.5390 for the ESW scheme and 0.5575 for the MSW scheme

<table>
<thead>
<tr>
<th>Model</th>
<th>Min</th>
<th>Max</th>
<th>ESW scheme</th>
<th>MSW scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>MBE (%)</td>
<td>RMSE (%)</td>
</tr>
<tr>
<td>SMARTS (ESW)</td>
<td>0.5124</td>
<td>0.9313</td>
<td>3.2</td>
<td>4.3</td>
</tr>
<tr>
<td>SMARTS (MSW)</td>
<td>0.5302</td>
<td>0.9371</td>
<td>3.8</td>
<td>4.1</td>
</tr>
<tr>
<td>Bird</td>
<td>0.0000</td>
<td>1.0000</td>
<td>9.9</td>
<td>10.6</td>
</tr>
<tr>
<td>CEM</td>
<td>0.0000</td>
<td>1.0000</td>
<td>2.8</td>
<td>8.7</td>
</tr>
<tr>
<td>Choudhury</td>
<td>−0.0500</td>
<td>1.0000</td>
<td>3.4</td>
<td>13.0</td>
</tr>
<tr>
<td>CPCR2</td>
<td>0.0031</td>
<td>1.0000</td>
<td>1.4</td>
<td>1.7</td>
</tr>
<tr>
<td>Iqbal C</td>
<td>0.0000</td>
<td>1.0000</td>
<td>6.9</td>
<td>9.8</td>
</tr>
<tr>
<td>METSTAT</td>
<td>0.0000</td>
<td>1.0000</td>
<td>4.8</td>
<td>6.8</td>
</tr>
<tr>
<td>MLWT1</td>
<td>0.0050</td>
<td>1.0000</td>
<td>4.0</td>
<td>6.9</td>
</tr>
<tr>
<td>MLWT2</td>
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<td>1.0000</td>
<td>3.7</td>
<td>5.8</td>
</tr>
<tr>
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<td>0.3401</td>
<td>0.9459</td>
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<td>56.9</td>
</tr>
<tr>
<td>Perrin</td>
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<td>1.0000</td>
<td>0.7</td>
<td>5.8</td>
</tr>
<tr>
<td>Psiloglou</td>
<td>0.1177</td>
<td>0.7795</td>
<td>24.0</td>
<td>59.0</td>
</tr>
<tr>
<td>REST</td>
<td>0.0069</td>
<td>1.0000</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Santamouris</td>
<td>0.0000</td>
<td>1.0000</td>
<td>16.0</td>
<td>17.5</td>
</tr>
<tr>
<td>Yang</td>
<td>0.0086</td>
<td>1.0000</td>
<td>1.1</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Table 8
Performance statistics for nitrogen dioxide transmittance under either unpolluted or polluted conditions. The reference average transmittances are 0.9929 in the unpolluted case and 0.9328 in the polluted case.

<table>
<thead>
<tr>
<th>Model</th>
<th>Unpolluted Min (%)</th>
<th>Unpolluted Max (%)</th>
<th>Unpolluted MBE (%)</th>
<th>Unpolluted RMSE (%)</th>
<th>Polluted Min (%)</th>
<th>Polluted Max (%)</th>
<th>Polluted MBE (%)</th>
<th>Polluted RMSE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMARTS (ESW)</td>
<td>0.5124</td>
<td>0.9313</td>
<td></td>
<td></td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>SMARTS (MSW)</td>
<td>0.5302</td>
<td>0.9371</td>
<td></td>
<td></td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Bird</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>CEM</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Choudhury</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>CPCR2</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Iqbal C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>METSTAT</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>MLWT1</td>
<td>0.9939</td>
<td>0.9987</td>
<td>0.5</td>
<td>0.5</td>
<td>0.9984</td>
<td>0.9986</td>
<td>-0.1</td>
<td>5.7</td>
</tr>
<tr>
<td>MLWT2</td>
<td>0.9979</td>
<td>0.9986</td>
<td>0.0</td>
<td>0.0</td>
<td>0.7564</td>
<td>0.9966</td>
<td>6.5</td>
<td>2.9</td>
</tr>
<tr>
<td>MMAC</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>MRM</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Perrin</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Psiloglou</td>
<td>0.9859</td>
<td>0.9989</td>
<td>0.1</td>
<td>0.1</td>
<td>0.7656</td>
<td>0.9973</td>
<td>1.3</td>
<td>3.2</td>
</tr>
<tr>
<td>REST</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Santamouris</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
<tr>
<td>Yang</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.7</td>
<td>0.8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>7.2</td>
<td>10.2</td>
</tr>
</tbody>
</table>

REST formula, which provides an almost perfect fit with the ESW scheme, underestimates by about 2.2% in the MSW scheme. These results are reversed when considering the MLWT2 formula.

3.1.2. Ozone Transmittance
Most of ozone absorption occurs in the stratosphere, above the layers where other molecular, gaseous, or aerosol extinction processes take place. Because ozone constitutes the top atmospheric layer, the ESW and MSW...
schemes provide identical results for the ozone transmittance. All formulas (except CEM, whose ozone transmittance is fixed at 1) provide results in close agreement (better than 2%) with those of the best performers, MLWT2 and REST (Fig. 4 and Table 5). Notice, however, that MRM shows an unusual transmittance curve shape (constant value at high zenith angles), and that the MMAC results could be better if its formula was not based on a fixed value of ozone amount (0.35 atm-cm). A part of the differences in the results, as well as the unusual shapes of some transmittance curves, can be attributed to the difference between the optical mass used by some models and the reference optical ozone mass used here, from SMARTS/REST, Eq. (B9). Most models use the air mass to account for ozone effects, but in reality the optical ozone mass is significantly smaller than air mass because of the height of the ozone layer (Fig. 5).

3.1.3. Water vapor transmittance

Extinction by water vapor is considerably more intense than that of ozone. Because the spectroscopy of water vapor absorption went through significant qualitative and quantitative changes in the last decades and is still in a state of flux, it is not surprising to find larger differences in water vapor transmittance predictions than with the previous effects. Furthermore, like in the ozone case, most models use the air mass for convenience rather than the more correct optical water vapor mass, $m_w$. The latter is significantly larger than the former because water vapor is concentrated in the lower troposphere, resulting in sizeable differences at large zenith angles (Fig. 6). Reference values for $m_w$ are obtained here with Eq. (B.11).

Fig. 7 shows a comparison between the transmittances predicted by all models, for common conditions (3 cm of precipitable water at sea level). Because the irradiance on top of the water vapor layer is attenuated by ozone, mixed gases as well as molecules (Rayleigh scattering), the difference between the results obtained with the ESW and MSW schemes is noticeable, in the order of 7% (Table 6). This difference increases with optical mass and reaches 12% for $m_w = 15$, when considering the prediction difference between MLWT2 and REST (Fig. 7). All models but one predict within about 10% of REST and MLWT2, the best performers with the ESW and MSW schemes, respectively. The exception is MRM, which considerably underestimates, by about 40%. This is relatively fortunate in its particular case because this compensates in part for its considerable overestimation in Rayleigh transmittance.

3.1.4. Aerosol transmittance

Aerosol scattering and absorption have a considerable effect on DNI (and on diffuse irradiance as well). The aerosol burden of the atmosphere varies rapidly over time and space, causing large variations in DNI that must be accounted for as accurately as possible. Despite the predominant role of aerosol extinction on clear-sky DNI, its transmittance appears to be the least well modelled of
all processes. This constitutes a major cause of inaccuracy in the prediction of both DNI and diffuse irradiance (Gueymard, 1993a).

Like water vapor, aerosols are normally concentrated in the lower layers of the troposphere. Hence, their optical mass is close to that of water vapor, and significantly larger than the air mass. This constitutes a first cause of modelling error (Fig. 8).

Another issue is related to the selected turbidity coefficient. Whereas most of the models reviewed here rely on Angström’s $\beta$, CEM, MMAC, METSTAT and Rodgers rely on $\tau_s$. Because $\tau_s$ is not a perfectly pure turbidity coefficient, but slightly depends also on zenith angle and water vapor (Gueymard, 1998), a fixed representative value was calculated from REST for each fixed values of $\beta$ and $w$, at an average air mass of 1.5 (or $Z=48.24\degree$). The latter value can be considered an ‘effective’ value for a large range of mid-latitudes (Gonzalez and Ross, 1980).

A special case is MRM, which relies on a fixed value of $\tau_s$, equal to 0.394 in Eq. (2). This ‘freezing’ of the aerosol transmittance is unfortunate because of its disastrous effect on the model’s performance (Table 7). Whereas the average RMS error on $T_e$ for the tested models is rarely $>10\%$, that of MRM exceeds 50%. A similar problem occurs with Psiloglou’s model for reasons detailed in Section 2.17. Its fixed — but unspecified — turbidity considerably degrades the model’s performance when the actual turbidity is different than its assumed value. When
comparing the aerosol transmittance predicted for different zenith angles by SMARTS and Psiloglou’s model, it can be inferred that the latter’s equivalent $\beta$ is about 0.15.

The results in Table 7 also show that the changes in aerosol transmittance modelling that marked the passage from Bird’s model to Iqbal C and then to METSTAT are clearly to the advantage of the latter.

Fig. 9 illustrates the effect of zenith angle, altitude and turbidity on aerosol transmittance. The case of a very low turbidity ($\beta=0.02$), typical of high-altitude sites (in this case, 4 km amsl), is shown in Fig. 9a. The difference between the ESW and MSW schemes appears noticeable when comparing the transmittances predicted by MLWT1 and REST. This difference is only a small fraction of the range of transmittances predicted by all models, however.

A moderately high turbidity ($\beta=0.10$) at a 3-km high site is assumed in Fig. 9b. The scatter in transmittance predictions appears relatively less here. Finally, Fig. 9c addresses the case of a sea-level site subjected to very hazy conditions ($\beta=0.50$, the maximum turbidity considered in this study). The scatter is important here in relative terms, but the difference between the ESW and MSW schemes is negligible.

From these plots, it appears that the function chosen for MRM’s aerosol transmittance is inadequate: like in the case of ozone, it becomes unphysically constant above a certain fixed value of zenith angle. It is also apparent, particularly from Fig. 9c, that the transmittance predicted by some models decreases too rapidly with the optical mass, to the point of becoming zero — or even negative in the case of Choudhury’s model.

### 3.1.5. Nitrogen dioxide transmittance

Only four of the selected models analysed here take the effect of nitrogen dioxide (NO$_2$) into account: MLWT1, MLWT2, Power, and REST. Even though the effect of NO$_2$ can be considered small or negligible in most cases, it becomes noticeable under polluted conditions. [NO$_2$ is an important constituent of ‘brown clouds’ and photochemical smog]. Of the four models listed above, two (MLWT1 and MLWT2) request distinct inputs of $u_n$ and $u_m$, the stratospheric and tropospheric NO$_2$ columns, respectively. The other two models (Power and REST) only request $u_\nu$, the total NO$_2$ vertical column (i.e. $u_n = u_{n\nu} + u_m$). Because the former (Power) is not a transmittance model, it will not be discussed further in this Section.

A comparison of the NO$_2$ transmittance, $T_n$, predicted by the three remaining models appears in Fig. 10. The top curves show that $T_n$ is generally of the order of 0.99 when no pollution exists ($u_{n\nu} = 0$). This finding implies that a fixed representative value for $u_\nu$ (e.g. 1.5×10$^{-4}$ atm·cm) can be used to represent all unpolluted cases. However, under heavy pollution, $T_n$ can be much lower, to the point of becoming quantitatively less than the ozone transmittance, and comparable in magnitude to the water vapor transmittance.

The MLWT2 and REST models predict similar results with a slight advantage to the former because of its consideration for $u_{n\nu}$ and $u_m$ separately rather than for just their sum (Table 8).

### 3.2. Direct irradiance

DNI predictions from all the selected models are compared here to the reference ‘benchmark’ dataset mentioned above. In preliminary tests, it has been discovered that the minimum DNI can be negative for two models, those by King and Buckius and Yang. For the former, the DNI underestimation is considerable, with an absolute minimum DNI of −221.4 W/m$^2$. In the case of Yang’s model, the negative value problem is not as pronounced (with an absolute minimum of −13.5 W/m$^2$), and is clearly caused by the presence of a negative correction coefficient in Eq. (38). Users of these models need to be aware of this issue and should zero out negative irradiance values.

Other problems were found with Rodgers’ model, which may unphysically predict DNI values larger than the solar constant. It can be conjectured that this problem comes from inconsistencies in the derivation of Eq. (32). As a solution, this model’s predictions have been capped at a limit value of 1250 W/m$^2$. The same correction has been applied to Power’s model for similar reasons.

All tests have been rerun with these improvements. Cumulative statistical results of this global performance assessment are detailed in Table 9. The minimum and maximum values of the predicted DNI can be compared to those of the reference model as given in Table 3. Besides MBE, RMSE and the Min and Max values, the series of predicted DNI values from each model has been linearly correlated with the reference series (taken as the independent variable), using an equation of the form $Y = kX$. The values of $k$ and of the associated regression coefficient, $R$, are displayed in Table 9. As could be expected, $k$ is close to (1 + MBE) for most models. The best models are characterized by $k$=1 and $R$>0.99.

The cumulative MBE and RMSE statistics given in Table 9 can be interpreted as follows:

- Detailed transmittance models (as critically reviewed in Section 3.1) systematically provide the best performance, with the exception of MRM and Psiloglou’s. The latter’s disappointing results can be completely explained by its lack of dependence on turbidity due to its embedded site-specificity. MRM’s poor results are caused by a combination of various modelling flaws, as detailed above.
- The only two models (Dogniaux and Molineaux) that use the Linke turbidity coefficient for intermediate calculations deliver only modest performance, with RMSEs around 20%.
- The only two models (Power and PSIM) that do not use air-mass-dependent formulas are performing very dif-

Fig. 9. Transmittance for aerosol extinction predicted by all models. (a) $\beta = 0.02$, $w = 0.2$ cm, 4 km altitude; (b) $\beta = 0.10$, $w = 0.2$ cm, 3 km altitude; (c) $\beta = 0.50$, $w = 3$ cm, sea level.
ferently. PSIM’s performance compares well to many air-mass-based models. Conversely, Power’s model performs poorly, even though it was apparently meant as an improvement over PSIM.

- Iqbal’s model C appears to offer modelling improvements over its parent, Bird, but the more recent model in this family, METSTAT, performs even better.
- The MMAC model performs significantly better than the original MAC, following the introduction of Eq. (12) and other modelling improvements.
- Out of the 21 models tested here, a leading group of seven (CPCR2, MMAC, METSTAT, MLWT1, MLWT2, REST, and Yang) can be pre-selected on the basis of their lower RMSE (<8%) and small MBE. However, some of the MMAC and METSTAT transmittances show noticeable deviations from reference values at large zenith angles. Also, because MLWT2 is a slightly revised version of MLWT1 and performs better than its predecessor, only the most recent model needs to be considered further. Finally, the four ‘top’ models that can be recommended on a theoretical basis for their high performance under all possible conditions are: CPCR2, MLWT2, REST, and Yang.

- The intricate MSW integration scheme that has been used to derive the transmittances in MLWT2 — which obtains the lowest RMSE of the whole set of models —

![Fig. 10. Transmittance for nitrogen dioxide absorption predicted by all models.](image)

Table 9
Cumulative statistics for the prediction of DNI with all models, with reference to SMARTS. Numbers between brackets correspond to improved modelled values where all irradiances <0 or >1250 W/m² have been corrected (see text). The MBE and RMSE statistics are expressed in percent of the average SMARTS value, 423.4 W/m². k and R are the slope and regression coefficient of the linear fit between modelled and reference values, respectively

<table>
<thead>
<tr>
<th>Model</th>
<th>MBE (%)</th>
<th>RMSE (%)</th>
<th>k</th>
<th>R</th>
<th>Min DNI</th>
<th>Max DNI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMARTS</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>1223</td>
</tr>
<tr>
<td>Bird</td>
<td>–7.6</td>
<td>11.2</td>
<td>0.950</td>
<td>0.994</td>
<td>0</td>
<td>1183</td>
</tr>
<tr>
<td>CEM</td>
<td>–1.0</td>
<td>10.3</td>
<td>1.025</td>
<td>0.995</td>
<td>0</td>
<td>1236</td>
</tr>
<tr>
<td>Choudhury</td>
<td>1.3</td>
<td>9.8</td>
<td>1.037</td>
<td>0.995</td>
<td>0</td>
<td>1245</td>
</tr>
<tr>
<td>CPCR2</td>
<td>0.7</td>
<td>3.9</td>
<td>0.997</td>
<td>0.999</td>
<td>2</td>
<td>1199</td>
</tr>
<tr>
<td>Dogniaux</td>
<td>11.7</td>
<td>21.3</td>
<td>1.079</td>
<td>0.971</td>
<td>1</td>
<td>1180</td>
</tr>
<tr>
<td>Iqbal C</td>
<td>–3.7</td>
<td>9.5</td>
<td>0.984</td>
<td>0.993</td>
<td>0</td>
<td>1207</td>
</tr>
<tr>
<td>King and Buckius</td>
<td>(7.7)</td>
<td>(9.5)</td>
<td>(1.061)</td>
<td>(0.998)</td>
<td>(0)</td>
<td>1186</td>
</tr>
<tr>
<td>METSTAT</td>
<td>–2.4</td>
<td>7.9</td>
<td>0.998</td>
<td>0.994</td>
<td>0</td>
<td>1209</td>
</tr>
<tr>
<td>MLWT1</td>
<td>4.4</td>
<td>6.4</td>
<td>1.040</td>
<td>0.998</td>
<td>2</td>
<td>1245</td>
</tr>
<tr>
<td>MLWT2</td>
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<td>2.8</td>
<td>1.015</td>
<td>1.000</td>
<td>2</td>
<td>1241</td>
</tr>
<tr>
<td>MMAC</td>
<td>–3.8</td>
<td>7.9</td>
<td>0.991</td>
<td>0.997</td>
<td>0</td>
<td>1223</td>
</tr>
<tr>
<td>Molineaux</td>
<td>8.7</td>
<td>18.9</td>
<td>1.071</td>
<td>0.976</td>
<td>0</td>
<td>1175</td>
</tr>
<tr>
<td>MRM</td>
<td>(34.4)</td>
<td>(84.8)</td>
<td>(0.958)</td>
<td>(0.266)</td>
<td>200</td>
<td>(1250)</td>
</tr>
<tr>
<td>Perrin</td>
<td>–1.5</td>
<td>9.7</td>
<td>1.007</td>
<td>0.993</td>
<td>0</td>
<td>1153</td>
</tr>
<tr>
<td>Power</td>
<td>(–0.4)</td>
<td>(21.7)</td>
<td>(0.951)</td>
<td>(0.954)</td>
<td>29</td>
<td>(1250)</td>
</tr>
<tr>
<td>Psiloglou</td>
<td>–21.5</td>
<td>53.8</td>
<td>0.704</td>
<td>0.725</td>
<td>53</td>
<td>972</td>
</tr>
<tr>
<td>PSIM</td>
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<td>9.0</td>
<td>0.975</td>
<td>0.993</td>
<td>8</td>
<td>1239</td>
</tr>
<tr>
<td>REST</td>
<td>1.0</td>
<td>4.8</td>
<td>1.010</td>
<td>0.998</td>
<td>3</td>
<td>1234</td>
</tr>
<tr>
<td>Rodgers</td>
<td>(6.4)</td>
<td>(20.1)</td>
<td>(1.069)</td>
<td>(0.973)</td>
<td>0</td>
<td>(1250)</td>
</tr>
<tr>
<td>Santamouris</td>
<td>–11.3</td>
<td>15.3</td>
<td>0.937</td>
<td>0.991</td>
<td>0</td>
<td>1221</td>
</tr>
<tr>
<td>Yang</td>
<td>(1.1)</td>
<td>(5.7)</td>
<td>(1.016)</td>
<td>(0.997)</td>
<td>(0)</td>
<td>1226</td>
</tr>
</tbody>
</table>
is apparently more efficient than the simpler ESW scheme that has been used to derive REST, the third-best performer from an RMSE perspective. (Note, however, that neither MLWT2 nor REST are independent from the reference dataset).

- The two-band scheme used in CPCR2 (the second-best performer here) proves to be a robust derivation method and a simpler alternative to the MSW scheme, especially considering that this model is based on relatively outdated spectral data and is completely independent from the reference dataset.

- Yang’s model, the fourth-best performer, is based on the same ESW derivation scheme as REST and shows comparable performance, even though, unlike REST, it is completely independent from the reference dataset. Sample scatterplots of modelled results vs. reference data are shown in Figs. 11 and 12. Figure 11 compares the Bird and METSTAT outputs to their common reference counterpart. The reduced scatter in METSTAT confirms that this model successfully implemented some modelling improvements over Bird, its base model. However, these two models (as well as the third model of this family, Iqbal C, and the MMAC model, not shown for clarity) significantly underestimate DNI below about 400 W/m². This trend is apparently caused by their respective aerosol transmittance functions, Eqs. (2) and (6), which are both predicting too low at large zenith angles and high turbidity (Fig. 9c).

Figure 12 compares the minimal scatter of the best performing model (MLWT2) to the large scatter of a recent Linke-type empirical model (Molineaux). Considering also the modest improvement that Molineaux’s model brings over Dogniaux’s despite the 20-year period that separates them, this finding supports the above statement.

![Calculated vs Reference Direct Normal Irradiance](image.png)

**Fig. 11.** Scatterplot of DNI predictions by the Bird and METSTAT models vs. reference predictions by the SMARTS code.
(Section 2) that Linke-type models are not suitable for accurate DNI predictions, at least under known or ideal aerosol and water vapor conditions.

4. Conclusion

Twenty-one irradiance models have been identified for this thorough investigation on the accuracies achievable in predicting clear-sky direct normal irradiance (DNI) only from the sun’s position and a few atmospheric variables. The main objectives of this study were to evaluate the performance of these models under a large range of carefully selected conditions.

To achieve the best possible results, some of the models of the literature received specific improvements, and two models, MLWT1 and REST, were, respectively, revised (into MLWT2) and developed.

A spectral radiative code, SMARTS, has been selected to provide reference broadband transmittance and DNI values. Its predictions are shown to be in close agreement with the best radiative transfer calculations currently available.

Two series of theoretical tests were conducted. The first series involved the majority of models that rely on individual transmittance calculations to simulate the different extinction processes in the atmosphere. Each individual transmittance of each of these models was calculated for a large range of atmospheric conditions, and compared to a set of reference transmittance values obtained by integrating the spectral transmittance predictions of the SMARTS code in two different ways, the simpler ESW scheme and the more elaborate and physical MSW scheme.

Significant differences are apparent between the transmittance predictions from these models. Serious transmittance modelling problems were found for one model (MRM), and could not be resolved. In the case of Psioglu’s model, good to excellent results are obtained...
for all transmittances, except that for aerosols. This is caused by the particular site-specific approach used in this model, where the average aerosol conditions of a Mediterranean city are considered, rather than all possible turbidity conditions. The large range of turbidity conditions encompassed by the tests implemented here seriously hinders the global performance of this model. For general use, a beneficial alternative would be to replace its aerosol transmittance by one of the best performers in Table 7.

Detailed and accurate transmittance functions have been obtained by fitting SMARTS results and are proposed for high-performance transmittance predictions with the MLWT2 and REST models, detailed in Appendices A and B, respectively.

In the second series of theoretical tests, the DNI predictions from all 21 models were compared to those of SMARTS. It is found that models using individual transmittances are normally performing better than ‘bulk’ models, and that the two bulk models using Linke’s turbidity factor for intermediate calculations demonstrate only modest performance. Seven models are found to have low RMS errors (<8%).

In summary, four models (CPCR2, MLWT2, REST and Yang, in alphabetical order; including the corrections implemented here for the latter) can be recommended because of their consistently high performance in all theoretical tests. Based on the high accuracies achievable with these models, it is concluded that DNI can be determined by broadband modelling with a high level of performance that compares well to other theoretical (spectral modelling) methods of determination.

The four best models have something in common: they consider individual transmittances for each important extinction process in the atmosphere. However, they differ on the implementation of this method. Best results (with MLWT2) are achieved through the sophisticated ‘Multi-layered-Spectrum Weighting’ (MSW) scheme, to the expense of added complexity compared to most other models. Excellent results are also achieved with a two-band modelling scheme, represented by CPCR2, which implies intermediate complexity. Even if these models perform significantly better than the majority of other transmittance models, their prediction improvement over the most recent ‘single-layer’ transmittance models (REST and Yang) is small. No further improvements in current high-performance models will therefore be necessary until more accurate fundamental data become available.

Acknowledgements

This contribution is the author’s homage to the dedicated work that the late Drs. Richard E. Bird, Anna Mani and Christian Perrin de Brichambaut devoted to the fields of solar radiation modelling and resource assessment.

Appendix A. Equations for the MLWT2 model

\[ E_{nn} = E_{oo}T_{wq}T_rT_{wa}T_{na}T_{oa}T_{oa} \]
\[ T_{wo} = \exp(-m_w f_w \delta_{wo}) \]
\[ T_o = \exp(-m_w f_1 \delta_o) \]
\[ T_{wo} = \exp(-m_w f_w \delta_{wo}) \]
\[ T_n = \exp(-m_w f_n \delta_n) \]
\[ f_k = (a_0 + a_1m_w + a_2m_w^2)(b_0 + b_1m_w + b_2m_w^2)/(1 + a_1m_w + a_2m_w^2)(1 + b_1m_w + b_2m_w^2) \]
\[ f_o = (c_0 + c_1m_w + c_2m_w^2)/(1 + c_1m_w) \]
\[ f_w = (d_0 + d_1m_w + d_2m_w^2)/(1 + d_1m_w) \]
\[ \delta_{wo} = f_k(f_2 + f_1) \]
\[ \delta_o = f_4 \]
\[ f_{wo} = \{ b_0 + b_1m_w + b_2m_w^2)/(1 + b_2m_w^2) \}
\[ \delta_{wo} = [0.001084 + 2.978u_{wq} + (8.623e - 6 + 0.01478u_{wq})m_w]/[1 + (0.005851 - 0.566u_{wq})m_w^2] \]
\[ \delta_n = u_{an}[3.2678 + 1.1414u_{an} + (-0.0086885 + 0.24185u_{an})m_w]/[1 + (0.013358 + 4.5658u_{an} + 18.487u_{an})m_w] \]
\[ m_w = \cos Z + 1.1212Z^{1.6132}/(111.55 - Z)^{3.2629} \]
\[ a_0 = 0.97839 + 0.02981u_{ao} \]
\[ a_1 = (0.39774 - 1.2806u_{ao})/(1 + 3.2629u_{ao}) \]
\[ a_2 = 0.4783 - 0.035928u_{ao}(1 + 7.1644u_{ao}) \]
\[ a_3 = 0.35565 - 1.2345u_{ao}(1 + 1.1414u_{ao}) \]
\[ a_4 = (0.47482 - 0.02786u_{ao})/(1 + 7.081u_{ao}) \]
\[ b_0 = 1 + 0.041199q \]
\[ b_1 = 0.3259 - 1.0635q \]  \hspace{1cm} (A.23)
\[ b_2 = 0.034958 - 0.056654q \]  \hspace{1cm} (A.24)
\[ b_3 = 0.3259 - 1.0619q \]  \hspace{1cm} (A.25)
\[ b_4 = 0.034958 - 0.057423q \]  \hspace{1cm} (A.26)
\[ c_0 = u_0(4.2478 + 296.3u_0)/(1 + 299.31u_0^2) \]  \hspace{1cm} (A.27)
\[ c_1 = (1.6597 - 8.7307u_0 + 25.425u_0^2)/(1 + 23.415u_0^2) \]  \hspace{1cm} (A.28)
\[ c_2 = (-0.034944 + 0.18451u_0 - 0.59576u_0^2)/(1 + 21.542u_0^2) \]  \hspace{1cm} (A.29)
\[ c_3 = (1.5397 - 8.4341u_0 + 24.392u_0^2)/(1 + 31.081u_0^2) \]  \hspace{1cm} (A.30)
\[ d_0 = (0.82912 + 7.7049w - 0.082654w^2)/(1 + 7.3015w) \]  \hspace{1cm} (A.31)
\[ d_1 = (0.17391 + 0.10888w - 0.049887w^2)/(1 + 5.2052w) \]  \hspace{1cm} (A.32)
\[ d_2 = 1 + 0.048643q + 0.52674q^2 \]  \hspace{1cm} (A.33)
\[ d_3 = -0.21312q^3/(1 + 1.1966q) \]  \hspace{1cm} (A.34)
\[ d_4 = (0.10505 + 0.10032w - 0.055816w^2)/(1 + 6.4284w) \]  \hspace{1cm} (A.35)
\[ e_0 = h_0 + j_1\beta + h_2\beta^2 \]  \hspace{1cm} (A.36)
\[ e_1 = j_0 + j_1\beta + j_2\beta^2 \]  \hspace{1cm} (A.37)
\[ e_2 = k_0 + k_1\beta + k_2\beta^2 \]  \hspace{1cm} (A.38)
\[ h_0 = 0.99498 - 0.004206w + 0.00074035w^2 \]  \hspace{1cm} (A.39)
\[ h_1 = 0.081335 - 0.038255w + 0.008651w^2 \]  \hspace{1cm} (A.40)
\[ h_2 = -0.14102 + 0.039367w - 0.009852w^2 \]  \hspace{1cm} (A.41)
\[ j_0 = 0.04653 - 0.031369w)/(1 + 0.60021w) \]  \hspace{1cm} (A.42)
\[ j_1 = (-0.17365 + 0.26361w)/(1 + 0.42941w) \]  \hspace{1cm} (A.43)
\[ j_2 = (0.25679 - 0.439w)/(1 + 0.37068w) \]  \hspace{1cm} (A.44)
\[ k_0 = (0.03224 - 0.029128w)/(1 + 0.48108w) \]  \hspace{1cm} (A.45)
\[ k_1 = (-0.089516 + 0.27787w)/(1 + 0.41995w) \]  \hspace{1cm} (A.46)
\[ k_2 = (0.10778 - 0.47418w)/(1 + 0.3792w) \]  \hspace{1cm} (A.47)

Expressions for \( f_s - f_0 \) and \( \delta_0, \delta_1, \delta_2, \delta_3, \) and \( q \) are the same as in the MLWT1 model (Gueymard, 1998).

**Appendix B. Equations for the REST model**

\[ E_{wa} = E_{wo}T_wT_oT_sT_rT_a \]  \hspace{1cm} (B.1)
\[ T_k = \exp(-m_k\tau_k) \]  \hspace{1cm} (B.2)
\[ T_v = \exp(-m_v\tau_v) \]  \hspace{1cm} (B.3)
\[ T_o = \exp(-m_o\tau_o) \]  \hspace{1cm} (B.4)
\[ T_a = \exp(-m_a\tau_a) \]  \hspace{1cm} (B.5)
\[ T_s = \exp(-m_s\tau_s) \]  \hspace{1cm} (B.6)
\[ m_k = (p/1013.25)\exp(0.4835Z^0.095846/(96.741 - Z)^{1.754})^{-1} \]  \hspace{1cm} (B.7)
\[ m_v = [\cos Z + 1.0651Z^{0.6379}/(101.8 - Z)^{2.2604}]^{-1} \]  \hspace{1cm} (B.8)
\[ m_o = [\cos Z + 1.1212Z^{1.6132}/(111.55 - Z)^{3.2629}]^{-1} \]  \hspace{1cm} (B.9)
\[ m_a = [\cos Z + 0.10648Z^{0.11423}/(93.781 - Z)^{1.9203}]^{-1} \]  \hspace{1cm} (B.10)
\[ m_s = [\cos Z + 0.16851Z^{0.18199}/(95.318 - Z)^{1.9342}]^{-1} \]  \hspace{1cm} (B.11)
\[ \tau_k = (0.11005 + 0.014758m_k^2 + 0.000051409m_k^4)/(1 + 0.3269m_k^2 + 0.012374m_k^4) \]  \hspace{1cm} (B.12)
\[ \tau_v = 0.028786 + 0.019308m_v^2 + 0.00004277m_v^4(1 + 1.9068m_v^2 + 0.23897m_v^4) \]  \hspace{1cm} (B.13)
\[ \tau_o = u_o(c_0 + c_1m_o + c_2m_o^3)/(1 + c_1m_o) \]  \hspace{1cm} (B.14)
\[ c_0 = 0.21877 + 0.1757u_o \]  \hspace{1cm} (B.15)
\[ c_1 = 0.0035648 + 0.7597u_o)/(1 - 0.048034u_o) \]  \hspace{1cm} (B.16)
\[ c_2 = -0.0063843 + 0.03094u_o)/(1 - 0.17989u_o) \]  \hspace{1cm} (B.17)

\[ c_s = \left( -0.34886 + 23.624u_n - 15.024u_n^2 \right) / (1 - 0.77644u_n) \]  
\[ \tau_n = \left[ 0.0014037 + 3.2468u_n + 1.1068u_n^2 
+ m_wu_n(-0.01002 + 0.27959u_n) \right] / \left[ 1 + m_w(0.01974 + 3.8668u_n + 23.153u_n^2) \right] \] 
\[ q = 1 - p / 1013.25 \] 
\[ d_0 = (1.3613 + 0.91385w) / (1 + 5.9651w + 0.99609w^2) \] 
\[ d_1 = (0.049719 + 0.014125w) / (1 + 4.1818w + 0.29987w^2) \] 
\[ d_2 = w(29.588 + 3.3427w) / (1 + 16.414w + 1.1646w^2) \] 
\[ \tau_e = \beta (e_0 + e_1m_w) / (1 + e_2m_w) \] 
\[ e_0 = 1.6933 \] 
\[ e_1 = (-0.013029 + 0.13126\beta) / (1 + 0.42003\beta) \] 
\[ e_2 = (-0.0083581 + 0.40323\beta + 0.123\beta^2) / (1 + 0.42003\beta) \]  

References


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