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Solar radiation model

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Abstract

Solar radiation models for predicting the average daily and hourly global radiation, beam radiation and diffuse radiation are reviewed in this paper. Seven models using the Ångström–Prescott equation to predict the average daily global radiation with hours of sunshine are considered. The average daily global radiation for Hong Kong (22.3°N latitude, 114.3°E longitude) is predicted. Estimations of monthly average hourly global radiation are discussed. Two parametric models are reviewed and used to predict the hourly irradiance of Hong Kong. Comparisons among model predictions with measured data are made.

1. Introduction

A knowledge of the local solar-radiation is essential for the proper design of building energy systems, solar energy systems and a good evaluation of thermal environment within buildings [1–6]. The best database would be the long-term measured data at the site of the proposed solar system. However, the limited coverage of radiation measuring networks dictates the need for developing solar radiation models.

Since the beam component (direct irradiance) is important in designing systems employing solar energy, such as high-temperature heat engines and high-intensity solar cells, emphasis is often put on modelling the beam component. There are two categories of solar radiation models, available in the literature, that predict the beam component or sky component based on other more readily measured quantities:

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Nomenclature

A	Apparent extraterrestrial irradiance (W m^{-2})
a_1, a_2, a_3	Coefficients defined in Eqs. (33), (50) and (68), respectively
B	Overall broadband value of the atmospheric attenuation-coefficient for the basic atmosphere (dimensionless)
b_1, b_2, b_3	Coefficients defined in Eqs. (33), (50) and (68), respectively
C	Ratio of the diffuse radiation falling on a horizontal surface under a cloudless sky I_d to the direct normal irradiation at the Earth's surface on a clear day I_n (dimensionless)
C_n	Clearness number, the ratio of the direct normal irradiance calculated with local mean clear-day water vapour divided by the direct normal irradiance calculated with water vapour according to the basic atmosphere (dimensionless)
c_1, c_2, c_3, c'_3, c_4, c'_4	Coefficients defined in Eqs. (81)–(92)
$\dot{D}_a$	Aerosols scattered diffuse irradiance after the first pass through the atmosphere (W m^{-2})
$\dot{D}_m$	Broadband diffuse irradiance on the ground due to the multiple reflections between the Earth's surface and its atmosphere (W m^{-2})
$\dot{D}_r$	Rayleigh scattered diffuse irradiance after the first pass through the atmosphere (W m^{-2})
d_1, d_2, d_3, d_4, d_5, d_6	Coefficients defined in Eqs. (81)–(92)
E_o	Eccentricity correction factor of the Earth's orbit (dimensionless)
F_c	Fraction of forward scattering to total scattering (dimensionless)
F_{sg}	Angle factor between the surface and the Earth (dimensionless)
F_{ss}	Angle factor between the tilt surface and the sky (dimensionless)
G_b	Monthly average daily beam irradiance on a horizontal surface (MJ m^{-2})
G_c	Monthly average daily clear-sky global radiation incident on a horizontal surface (MJ m^{-2})
G_o	Monthly average daily extraterrestrial radiation on a horizontal surface (MJ m^{-2})
G_t	Monthly average daily global radiation on a horizontal surface (MJ m^{-2})
h	Elevation above sea level (km)
$\dot{I}_b$, I_b	Total beam irradiance on a horizontal surface (W m^{-2}); hourly beam irradiance on a horizontal surface (MJ m^{-2})
$\dot{I}_d$, I_d	Total diffuse irradiance on a horizontal surface (W m^{-2});

	hourly diffuse irradiance on a horizontal surface (MJ m^{-2})
$\dot{I}_n, I_n$	Direct normal irradiance (W m^{-2}); hourly normal irradiance (MJ m^{-2})
$\dot{I}_o, I_o$	Extraterrestrial radiation (W m^{-2}); hourly extraterrestrial radiation (MJ m^{-2})
$\dot{I}_r$	Reflected short-wave radiation (W m^{-2})
$\dot{I}_{sc}, I_{sc}$	Solar constant, 1367 W m^{-2}; solar constant in energy unit for 1 h (1367×3.6 kJ m^{-2})
$\dot{I}_t, I_t$	Global solar irradiance (W m^{-2}), hourly global solar irradiance (MJ m^{-2})
k_b	Atmospheric transmittance for beam radiation (dimensionless)
k_D	Atmospheric diffuse coefficient (dimensionless)
k_d	Diffuse fraction (dimensionless)
k_t	Clearness index (dimensionless)
l_{ao}	Aerosol optical-thickness (dimensionless)
l_{oz}	Vertical ozone layer thickness (cm)
l_{Ro}	Rayleigh optical-thickness (dimensionless)
m_a	Air mass at actual pressure (dimensionless)
m_r	Air mass at standard pressure of 1013.25 mbar (dimensionless)
N	Day number in the year (No.)
p	Local air pressure (mbar)
p_o	Standard pressure (1013.25 mbar)
q_{rat}	Downward radiation from the atmosphere due to long-wave radiation (W m^{-2})
RH	Monthly average daily relative humidity (%)
r_t	Ratio defined in Eq. (53) (dimensionless)
S	Monthly average daily sunshine-duration (h)
S_f	Monthly average daily fraction of possible sunshine, known as sunshine fraction (dimensionless)
S_o	Maximum possible monthly average daily sunshine-duration (h)
S_{om}	Modified average daily sunshine duration for solar zenith angle θ_z greater than 85° (h)
T	Monthly average daily-temperature (°C)
T_{at}	Atmosphere temperature at ground level (K)
T_{dew}	Dew-point temperature (°C)
T_L	Linke turbidity-factor defined in Eq. (112)
T_{sky}	Apparent sky-temperature (K)
U_1	Pressure-corrected relative optical-path length of precipitable water (cm)

U_3	Ozone relative optical path length under the normal temperature and surface pressure NTP (cm)
V_{is}	Horizontal visibility (km)
w	Precipitable water-vapour thickness reduced to the standard pressure of 1013.25 mbar and at the temperature T of 273K (cm)
w'	Precipitable water-vapour thickness under the actual conditions (cm)
Δ	Change of
Γ	Day angle (in radians)
α	Solar altitude (degrees)
β_1, β_2	Ångström turbidity-parameters (dimensionless)
δ	Declination, north positive (degrees)
δ_c	Declination of characteristics days, north positive (degrees)
ε_{at}	Atmosphere emittance for long-wave radiation (dimensionless)
ϕ	Latitude, north positive (degrees)
κ_g	Reflectance of the foreground (dimensionless)
λ	Wavelength (μm)
θ	Angle of incidence; the angle between normal to the surface and the Sun–Earth line (degrees)
θ_z	Zenith angle (degrees)
ρ_a	Albedo of the cloudless sky (dimensionless)
ρ_c	Albedo of cloud (dimensionless)
ρ_g	Albedo of ground (dimensionless)
σ	Stefan-Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4})
τ_r, τ_o, τ_g, τ_w, τ_a	Scattering transmittances for Rayleigh, ozone, gas, water and aerosols (dimensionless)
τ_{aa}	Transmittance of direct radiation due to aerosol absorptance (dimensionless)
τ_{as}	Transmittance of direct radiation due to aerosol scattering (dimensionless)
ω	Hour angle, solar noon being zero and the morning is positive (degrees)
ω_I	Hour angle at the middle of an hour (degrees)
ω_o	Single-scattering albedo (dimensionless)
ω_s	Sunset-hour angle for a horizontal surface (degrees)
ψ	Function
ζ	Tilt angle of a surface measured from the horizontal (degrees)
$< >$	Yearly average

- Parametric models
- Decomposition models

Parametric models require detailed information of atmospheric conditions. Meteorological parameters frequently used as predictors include the type, amount, and distribution of clouds or other observations, such as the fractional sunshine, atmospheric turbidity and precipitable water content [5,7–14]. A simpler method was adopted by the ASHRAE algorithm [5] and very widely used by the engineering and architectural communities. The Iqbal model [8] offers extra-accuracy over more conventional models as reviewed by Gueymard [13].

Development of correlation models that predict the beam or sky radiation using other solar radiation measurements is possible. Decomposition models usually use information only on global radiation to predict the beam and sky components. These relationships are usually expressed in terms of the irradiations which are the time integrals (usually over 1 h) of the radiant flux or irradiance. Decomposition models developed to estimate direct and diffuse irradiance from global irradiance data were found in the literature [15–23].

Solar radiation models commonly used for systems using solar energy are reviewed in this article. Comparisons of predicted values with these models and measured data in Hong Kong are made. The units and symbols follow those suggested by Beckman et al. [24].

2. Parametric models

2.1. Iqbal model C [8]

The direct normal irradiance $\dot{I}_{\mathrm{n}}$ (W m^{-2}) in model C described by Iqbal [8] is given by

$$\dot{I}_{\mathrm{n}} = 0.9751 E_{\mathrm{o}} \dot{I}_{\mathrm{sc}} \tau_{\mathrm{r}} \tau_{\mathrm{o}} \tau_{\mathrm{g}} \tau_{\mathrm{w}} \tau_{\mathrm{a}} \tag{1}$$

where the factor 0.9751 is included because the spectral interval considered is 0.3–3 μm; $\dot{I}_{\mathrm{sc}}$ is the solar constant which can be taken as 1367 W m^{-2}. E_{o} (dimensionless) is the eccentricity correction-factor of the Earth's orbit and is given by

$$E_{\mathrm{o}} = 1.00011 + 0.034221\cos\Gamma + 0.00128\sin\Gamma + 0.000719\cos2\Gamma + 0.000077\sin2\Gamma \tag{2}$$

where the day angle Γ (radians) is given by

$$\Gamma = 2\pi\left(\frac{N-1}{365}\right) \tag{3}$$

where N is the day number of the year, ranging from 1 on 1 January to 365 on 31 December.

τ_r, τ_o, τ_g, τ_w, τ_a (dimensionless) are the Rayleigh, ozone, gas, water and aerosols scattering-transmittances respectively. They are given by

$$\tau_r = e^{-0.0903 m_a^{0.84}\left(1+m_a-m_a^{1.01}\right)} \quad (4)$$

$$\tau_o = 1-\left[0.1611 U_3(1+139.48U_3)^{-0.3035} - 0.002715U_3\left(1+0.044U_3+0.0003U_3^2\right)^{-1}\right] \quad (5)$$

$$\tau_g = e^{-0.0127 m_a^{0.26}} \quad (6)$$

$$\tau_w = 1-2.4959U_1\left[(1+79.034U_1)^{0.6828}+6.385U_1\right]^{-1} \quad (7)$$

$$\tau_a = e^{-l_{ao}^{0.873}\left(1+l_{oa}-l_{ao}^{0.7808}\right)m_a^{0.9108}} \quad (8)$$

where m_a (dimensionless) is the air mass at actual pressure and m_r (dimensionless) is the air mass at standard pressure (1013.25 mbar). They are related by

$$m_a = m_r\left(\frac{p}{1013.25}\right) \quad (9)$$

where p (mbar) is the local air-pressure.

U_3 (cm) is the ozone's relative optical-path length under the normal temperature and surface pressure (NTP) and is given by

$$U_3 = l_{oz} m_r \quad (10)$$

where l_{oz} (cm) is the vertical ozone-layer thickness.

U_1 (cm) is the pressure-corrected relative optical-path length of precipitable water, as given by

$$U_1 = w m_r \quad (11)$$

where w (cm) is the precipitable water-vapour thickness reduced to the standard pressure (1013.25 mbar) and at the temperature T of 273 K: w is calculated with the precipitable water-vapour thickness under the actual condition w' (cm) by

$$w = w'\left(\frac{p}{1013.25}\right)^{\frac{3}{4}}\left(\frac{273}{T}\right)^{\frac{1}{2}} \quad (12)$$

l_{ao} (dimensionless) is the aerosol optical thickness given by

$$l_{ao} = 0.2758 l_{ao;\lambda|\lambda=0.38\ \mu m} + 0.35 l_{ao;\lambda|\lambda=0.5\ \mu m} \tag{13}$$

where λ (μm) is the wavelength.

The beam component $\dot{I}_b$ (W m^{-2}) is given by

$$\dot{I}_b = \cos\theta_z \dot{I}_n \tag{14}$$

where θ_z (degrees) is the zenith angle.

The horizontal diffuse irradiance $\dot{I}_d$ (W m^{-2}) at ground level is a combination of three individual components corresponding to the Rayleigh scattering after the first pass through the atmosphere, $\dot{D}_r$ (W m^{-2}); the aerosols scattering after the first pass through the atmosphere, $\dot{D}_a$ (W m^{-2}); and the multiple-reflection processes between the ground and sky $\dot{D}_m$, (W m^{-2}).

$$\dot{I}_d = \dot{D}_r + \dot{D}_a + \dot{D}_m$$

where

$$\dot{D}_r = \frac{0.79 \dot{I}_{sc} \sin\alpha \tau_o \tau_g \tau_w \tau_{aa} 0.5(1 - \tau_r)}{1 - m_a + m_a^{1.02}} \tag{16}$$

where α (degrees) is the solar altitude and is related to the zenith angle θ_z by the following equation

$$\cos\theta_z = \sin\alpha \tag{17}$$

τ_{aa} (dimensionless) is the transmittance of direct radiation due to aerosol absorptance and is given by

$$\tau_{aa} = 1 - (1 - \omega_o)\left(1 - m_a + m_a^{1.06}\right)(1 - \tau_a) \tag{18}$$

where ω_o (dimensionless) is the single-scattering albedo fraction of incident energy scattered to total attenuation by aerosols and is taken to be 0.9.

$$\dot{D}_a = \frac{0.79 \dot{I}_{sc} \sin\alpha \tau_o \tau_g \tau_w \tau_{aa} F_c (1 - \tau_{as})}{1 - m_a + m_a^{1.02}} \tag{19}$$

F_c (dimensionless) is the fraction of forward scattering to total scattering and is taken to be 0.84; τ_{as} (dimensionless) is the fraction of the incident energy transmitted after the scattering effects of the aerosols and is given by

$$\tau_{as} = \frac{\tau_a}{\tau_{aa}} \tag{20}$$

$$\dot{D}_{\rm m} = \frac{\left(\dot{I}_{\rm n}\sin\alpha + \dot{D}_{\rm r} + \dot{D}_{\rm a}\right)\rho_{\rm g}\rho_{\rm a}}{1 - \rho_{\rm g}\rho_{\rm a}} \tag{21}$$

where ρ_g (dimensionless) is the ground albedo; ρ_a (dimensionless) is the albedo of the cloudless sky and can be computed from

$$\rho_{\rm a} = 0.0685 + (1 - F_{\rm c})\,(1 - \tau_{\rm as}) \tag{22}$$

where $(1 - F_c)$ is the back-scatterance. Consequently, the second term on the right-hand side of this equation represents the albedo of cloudless skies due to the presence of aerosols, whereas the first term represents the albedo of clean air.

The global (direct plus diffuse) irradiance $\dot{I}_t$ (W m^{-2}) on a horizontal surface can be written as

$$\dot{I}_{\rm t} = \dot{I}_{\rm b} + \dot{I}_{\rm d} = \left(\dot{I}_{\rm n}\cos\theta_{\rm z} + \dot{D}_{\rm r} + \dot{D}_{\rm a}\right)\left(\frac{1}{1 - \rho_{\rm g}\rho_{\rm a}}\right) \tag{23}$$

2.2. *ASHRAE model [5]*

A simpler procedure for solar-radiation evaluation is adopted in ASHRAE [5] and widely used by the engineering and architectural communities. The direct normal irradiance $\dot{I}_n$ (W m^{-2}) is given by

$$\dot{I}_{\rm n} = (C_{\rm n})A{\rm e}^{-\frac{B}{\sec\theta_{\rm z}}} \tag{24}$$

where A (W m^{-2}) is the apparent extraterrestrial irradiance, as given in Table 1, and which takes into account the variations in the Sun–Earth distance. Therefore

Table 1
Values of A, B and C for the calculation of solar irradiance according to the ASHRAE handbook: applications [5]

Date	A (W m^{-2})	B (W m^{-2})	C (W m^{-2})
21 Jan	1230	0.142	0.058
21 Feb	1215	0.144	0.060
21 Mar	1186	0.156	0.071
21 Apr	1136	0.180	0.097
21 May	1104	0.196	0.121
21 Jun	1088	0.205	0.134
21 Jul	1085	0.207	0.136
21 Aug	1107	0.201	0.122
21 Sep	1152	0.177	0.092
21 Oct	1193	0.160	0.073
21 Nov	1221	0.149	0.063
21 Dec	1234	0.142	0.057

$$A = \dot{I}_{sc} E_o (\text{constant}) \tag{25}$$

The value of the multiplying constant is close to 0.9 [8]. The variable B (dimensionless) in Table 1 represents an overall broadband value of the atmospheric attenuation coefficient for the basic atmosphere of Threlkeld and Jordan [5]. C_n (dimensionless) is the clearness number and the map of C_n values for the USA is provided in the ASHRAE handbook: applications [5]. C_n is the ratio of the direct normal irradiance calculated with the local mean clear-day water-vapour to the direct normal irradiance calculated with water vapour according to the basic atmosphere.

Eq. (24) was developed for sea-level conditions. It can be adopted for other atmospheric pressures by

$$\dot{I}_n = (C_n) A e^{-\frac{B}{\cos\theta_z}\left(\frac{p}{p_o}\right)} = C_n A e^{-B\left(\frac{p}{p_o}\right)\sec\theta_z} \tag{26}$$

where p (mbar) is the actual local-air pressure and p_o is the standard pressure (1013.25 mbar).

In the above equation, the term $(p/p_o)\sec\theta_z$ approximates to the air mass, with the assumptions that the curvature of the Earth and the refraction of air are negligible.

The total solar irradiance $\dot{I}_t$ (W m^{-2}) of a terrestrial surface of any orientation and tilt with an incident angle θ (degrees) is the sum of the direct component $\dot{I}_n \cos\theta$ plus the diffuse component $\dot{I}_d$ coming from the sky, plus whatever amount of reflected short-wave radiation $\dot{I}_r$ (W m^{-2}) that may reach the surface from the Earth or from adjacent surfaces, i.e.,

$$\dot{I}_t = \dot{I}_n \cos\theta + \dot{I}_d + \dot{I}_r \tag{27}$$

The diffuse component is defined through a variable C (dimensionless) given in Table 1. The variable C is the ratio of the diffuse radiation falling on a horizontal surface under a cloudless sky $\dot{I}_d$ to the direct normal irradiation at the Earth's surface on a clear day $\dot{I}_n$. The diffuse radiation $\dot{I}_d$ is given by

$$\dot{I}_d = C \dot{I}_n F_{ss} \tag{28}$$

where for a horizontal surface,

$$C = \frac{\dot{I}_d}{\dot{I}_n} \tag{29}$$

F_{ss} (dimensionless), the angle factor between the surface and the sky, is given by

$$F_{ss} = \frac{1 + \cos\zeta}{2} \tag{30}$$

where ζ (degrees) is the tilt angle of a surface measured from the horizontal.

The reflected radiation $\dot{I}_r$ (W m^{-2}) from the foreground is given by

$$\dot{I}_r = \dot{I}_{t;\theta=0}\kappa_g F_{sg} \tag{31}$$

where κ_g (dimensionless) is the reflectance of the foreground; and $\dot{I}_{t;\theta=0}$ is the total horizontal irradiation ($\theta = 0°$). F_{sg}, the angle factor between the surface and the earth, is given by

$$F_{sg} = \frac{1 - \cos\zeta}{2} \tag{32}$$

3. Comparison of the Iqbal model C [8] and ASHRAE model [5]

The direct normal irradiance I_n was calculated with the Iqbal model C [8] for an ozone thickness of 0.35 cm (NTP) at the 21st day of each month for various values of the zenith angle θ_z for Hong Kong (22.3°N latitude, 114.3°E longitude). The beam, diffuse and global irradiances $\dot{I}_b$, $\dot{I}_d$ and $\dot{I}_t$ respectively, were computed using both models and are shown in Fig. 1(a)–(c). The predicted values for January and June are shown in Fig. 2(a)–(c) for comparison.

At $\theta_z = 0°$, a maximum difference for the beam irradiance of not more than 7% is obtained between the two models. The beam irradiance deviation of less than 8% was found for September–April, and less than 12% for May–August when $\theta_z < 60°$. The ASHRAE model would be inaccurate for the following reasons [2,5]. First, the solar constant used (1322 W m^{-2}) and the extraterrestrial spectral irradiance employed is outdated. Secondly, the attenuation coefficients are empirical functions and are based on site-specific data (Mount Wilson and Washington), and old instruments were used to take the data. Thirdly, when (ln $\dot{I}_n$) versus (p/p_o) sec θ_z was plotted from Eq. (26), the result was a straight line. The ordinate gives the apparent solar constant A, and the slope of the straight line is the overall broadband atmospheric attenuation-coefficient B. This linear relationship would be the approximation for broadband irradiance.

A large deviation (up to 41%) was found between the predicted values of diffuse irradiance with the two models as shown in the figure. For the ASHRAE model, the ground albedo is not included, so contributions by multiple reflections are ignored. The aerosol-generated diffuse irradiance is not considered in the ASHRAE model. This may introduce a large error as shown in Figs. 1(b) and 2(b). However, as diffuse radiation is only a small fraction of the direct radiation, the error is not serious in many applications. The deviation of the global irradiance $\dot{I}_t$ from the predicted $\dot{I}_t$ at θ_z at 0° for all months was only 5%. The predicted $\dot{I}_t$ shown in Figs. 1(c) and 2(c).

4. Correlation of average daily global radiation with hours of sunshine

Theoretical determinations of the direct, diffuse and directional intensities of diffuse irradiance would require data on the type and optical properties of clouds,

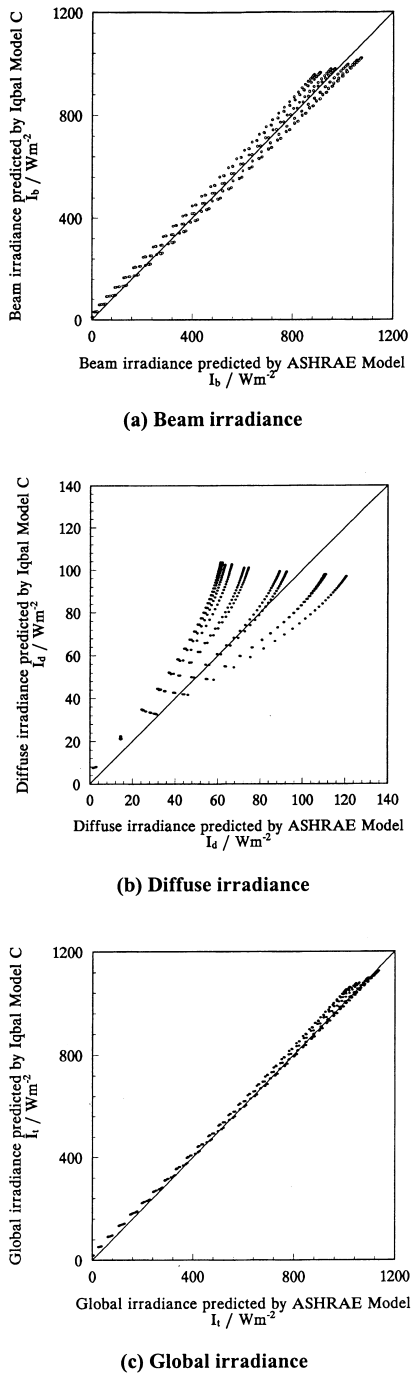
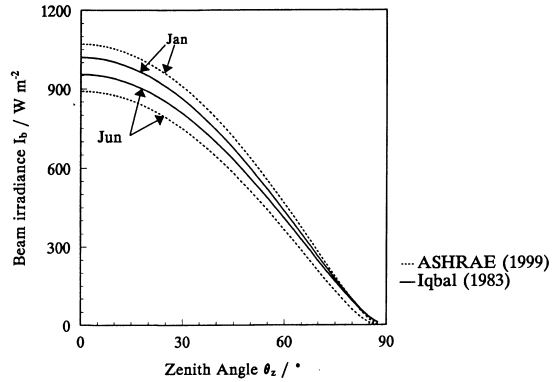
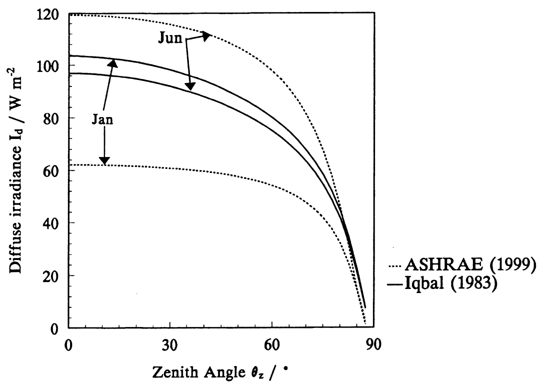


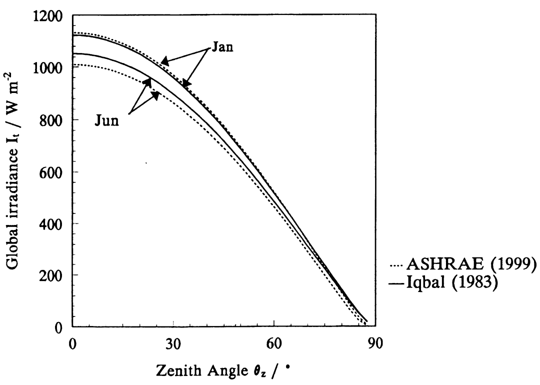
Fig. 1. Comparison of Iqbal model and ASHRAE model.



(a) Beam irradiance



(b) Diffuse irradiance



(c) Global irradiance

Fig. 2. Variation with zenith angle in January and June.

cloud amount, thickness, position and the number of layers. These data are rarely collected on a routine basis. However, sunshine hours and total cloud-cover data (i.e. the fraction of sky covered by clouds) are widely and easily available. Correlations, therefore, were developed to estimate insolation and sunshine hours.

The monthly average daily global radiation on a horizontal surface can be estimated through the number of bright sunshine hours. Ångström [25] developed the first model suggesting that the ratio of the average daily global radiation G_t (MJ m^{-2}) and cloudless radiation G_c (MJ m^{-2}) is related to the monthly mean daily fraction of possible sunshine (sunshine fraction) S_f (dimensionless) by

$$\frac{G_t}{G_c} = a_1 + (1 - a_1)S_f = a_1 + b_1 S_f \tag{33}$$

with the constant a_1 being obtained as 0.25 for Stockholm, Sweden.

The sunshine fraction S_f is obtained from:

$$S_f = \frac{S}{S_o} \tag{34}$$

where S (h) is the monthly average number of instrument-recorded bright daily sunshine hours, and S_o (h) is the average day length. In the Ångström model, $a_1 + b_1 = 1$, because on clear days, S_f is supposed to equal unity. However, because of problems inherent in the sunshine recorders, measurements of S_f would never equal unity. This leads to an underestimation of the insolation.

A modified form of the above equation, known as the Ångström–Prescott formula, was reported by Prescott in 1940 [26]:

$$\frac{G_t}{G_c} = a_1 + b_1 S_f \tag{35}$$

where $a_1 = 0.22$ and $b_1 = 0.54$.

In Eqs. (33) and (35), a_1 and b_1 correspond respectively to the relative diffuse radiation on an overcast day, whereas $(a_1 + b_1)$ corresponds to the relative cloudless-sky global irradiation. These equations assume that G_t corresponds to an idealised day over the month and can be obtained as the weighted sum of the global irradiations accumulated during two limiting days with extreme sky conditions, i.e. fully overcast and perfectly cloudless.

There may be problems in calculating G_c accurately. This equation was modified to be based on the extraterrestrial irradiation by Prescott in 1940 as reviewed by Gueymard et al. [26]:

$$\frac{G_t}{G_o} = a_1 + b_1 \frac{S}{S_o} \tag{36}$$

In equation (36), G_t and G_o (MJ m^{-2}) are the monthly mean daily global and extraterrestrial radiations on a horizontal surface, a_1 and b_1 are constants for different locations.

The hourly extraterrestrial radiation on a horizontal surface G_o (MJ m^{-2}) can be determined by [8]

$$G_o = \frac{24}{\pi} I_{sc} E_o \cos\phi \cos\delta \left[\sin\omega_s - \left(\frac{\pi}{180}\right)\omega_s \cos\omega_s \right] \tag{37}$$

where I_{sc} is the solar constant ($=1367\times3.6$ kJ m^{-2} h^{-1}), and ω_s (degrees) is the sunset-hour angle for a horizontal surface.

Rietveld [27] examined several published values of a_1 and b_1 and noted that a_1 is related linearly and b_1 hyperbolically to the appropriate yearly average value of S_f, denoted as $\left\langle \frac{S}{S_o} \right\rangle$, such that

$$a_1 = 0.10 + 0.24\left\langle \frac{S}{S_o} \right\rangle \tag{38}$$

$$b_1 = 0.38 + 0.08\left\langle \frac{S_o}{S} \right\rangle \tag{39}$$

If $\left\langle \frac{S}{S_o} \right\rangle = \frac{S}{S_o}$, the Rietveld model [27] can be simplified [26] to a constant-coefficient Ångström–Prescott equation by substituting Eqs. (38) and (39) into (36):

$$\frac{G_t}{G_o} = 0.18 + 0.62\frac{S}{S_o} \tag{40}$$

This equation was proposed for all places in the world and yields particularly superior results for cloudy conditions when $\frac{S}{S_o} < 0.4$.

Glover and McCulloch included the latitude ϕ (degrees) effect and presented the following correlation, as reviewed by Iqbal [8]:

$$\frac{G_t}{G_o} = 0.29\cos\phi + 0.52\frac{S}{S_o} \quad \text{for} \quad \phi<60^\circ \tag{41}$$

Gopinathan [28] suggested the regression coefficients a_1 and b_1 in terms of the latitude, elevation and percentage of possible sunshine for any location around the world. The correlations are:

$$a_1 = -0.309 + 0.539\cos\phi - 0.0693h + 0.29\frac{S}{S_o} \tag{42}$$

$$b_1 = 1.527 - 1.027\cos\phi + 0.0926h - 0.359\frac{S}{S_o} \tag{43}$$

where ϕ (degrees) is the latitude, and h (km) is the elevation of the location above sea level.

The coefficients a_1 and b_1 are site dependent [20,29–32]. A pertinent summary is shown in Table 2. The correlations of the average daily global radiation ratio and the sunshine duration ratio in Hong Kong (22.3°N latitude) are shown in Fig. 3 for comparison. The monthly average daily global radiation ratio registered in 1961–1990 in Hong Kong [33] is shown. It is very close to the predictions of Bahel et al. [29], Alnaser [30] and Louche et al. [20].

These coefficients would be affected by the optical properties of the cloud cover, ground reflectivity, and average air mass. Hay [34] studied these factors with data collected in western Canada and proposed a site-independent correlation

$$\frac{G_t}{G_o} = \frac{0.1572 + 0.556\left(\frac{S}{S_{om}}\right)}{1 - \rho_g\left[\rho_a\left(\frac{S}{S_{om}}\right) + \rho_c\left(1 - \frac{S}{S_{om}}\right)\right]} \tag{44}$$

where ρ_g (dimensionless) is the ground albedo, ρ_a is the cloudless-sky albedo taken as 0.25 and ρ_c is the cloud albedo taken as 0.6. S_{om} (h) is the modified day-length for a solar zenith angle θ_z greater than 85° and is given by

$$S_{om} = \frac{1}{7.5}\cos^{-1}\left(\frac{\cos 85 - \sin\phi \sin\delta_c}{\cos\phi\cos\delta_c}\right) \tag{45}$$

where δ_c (degrees) is the characteristic's declination, i.e. the declination at which the extraterrestrial irradiation is identical to its monthly average value.

Sen [32] employed fuzzy modelling to represent the relations between solar irradiation and sunshine duration by a set of fuzzy rules. A fuzzy logic algorithm has been devised for estimating the solar irradiation from sunshine duration measurements. The classical Ångström correlations are replaced by a set of fuzzy-rule bases

Table 2
Some examples of regression coefficients a_1 and b_1 used in different models

Reference	a_1	b_1	Data source
[27]	$0.1 + 0.24\left\langle\frac{\bar{S}}{\bar{S}_o}\right\rangle$	$0.38 + 0.08\left\langle\frac{\bar{S}}{\bar{S}_o}\right\rangle$	World-wide (42 places) ($6° < \phi < 69°$)
Simplified [27]	0.18	0.62	World-wide (42 places) ($6° < \phi < 69°$)
[8]	$0.29\cos\phi$	0.52	$\phi < 60°$
[29]	0.175	0.552	Dhahran, Saudi Arabia ($17° < \phi < 27°$)
[28]	$-0.309 + 0.539\cos\phi - 0.0693h + 0.29\frac{\bar{S}}{\bar{S}_o}$	$1.527 - 1.027\cos\phi + 0.0926h - 0.359\frac{\bar{S}}{\bar{S}_o}$	World-wide ($5° < \phi < 54°$)
[30]	0.2843	0.4509	Bahrain ($\phi = 27°$)
[20]	0.206	0.546	Southern France ($\phi = 42°$)
[31]	0.2006	0.5313	Northern India ($\phi = 27°$)

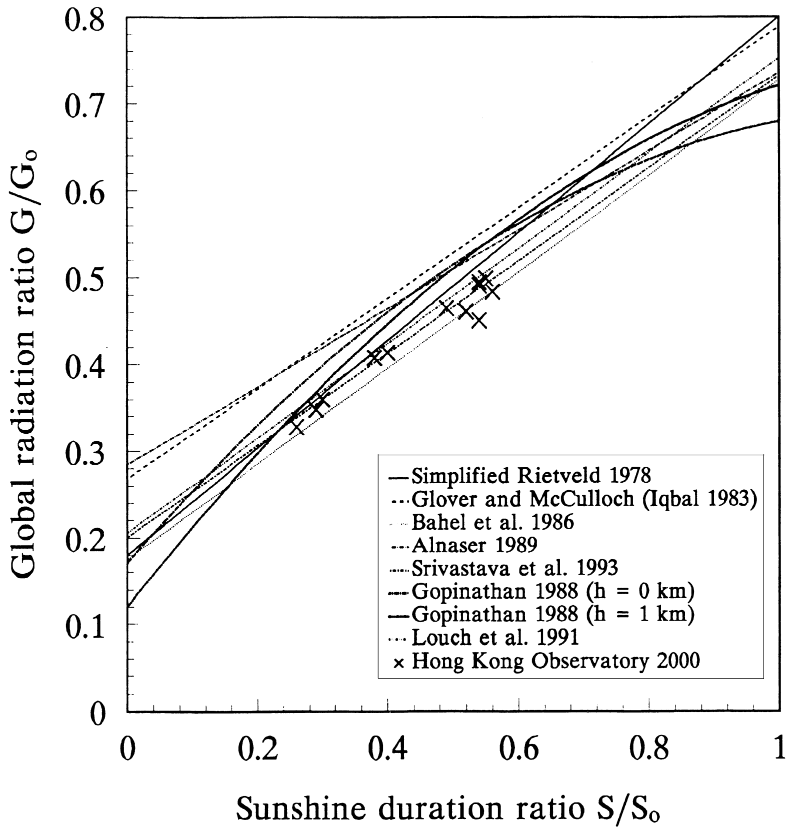


Fig. 3. Correlation of average daily global radiation with sunshine duration for Hong Kong ($\phi = 23°$).

and applied for three sites with monthly averages of daily irradiances in the western part of Turkey.

Mohandes et al. [35] studied the monthly mean daily values of solar radiation falling on horizontal surfaces using the radial basis functions technique. The predicted results by a multilayer perceptrons network were compared with the simplified Rietveld model [27]. The predicted results by the neural network were compared with those measured data for ten locations and good agreement was found.

5. Estimation of monthly-average hourly global radiation

As reviewed by Iqbal [8], Whiller investigated the ratios of hourly global radiation I_t (MJ m^{-2}) to daily global radiation G_t (MJ m^{-2}) in 1956 and assumed that

$$\frac{I_t}{G_t} = \frac{I_o}{G_o} \tag{46}$$

where I_o is the extraterrestrial hourly radiation (MJ m^{-2}), which can be determined by

$$I_o = I_{sc}E_o(\sin\delta\sin\phi + 0.9972\cos\delta\cos\phi\cos\omega_I) \quad (47)$$

where I_{sc} is the solar constant, E_o is the eccentricity correction factor, δ, ϕ and θ_I are the declination, latitude and hour angle respectively at the middle of an hour in degrees.

Whiller [8] correlated the hourly and daily beam radiations I_b (MJ m^{-2}) and G_b (MJ m^{-2}) respectively, as

$$\frac{I_b}{G_b} = \frac{I_o}{G_o} = \frac{\int^{1h} k_b(\omega)\dot{I}_{sc}E_o\cos\theta_z d\omega}{\int^{1day} k_b(\omega)\dot{I}_{sc}E_o\cos\theta_z d\omega} \quad (48)$$

where ω (degrees) is the hour angle, i.e., zero for solar noon and positive for the morning.

Assuming that the atmospheric transmittance for beam radiation $k_b(\omega)$ to be constant throughout the day, this equation can be rewritten as

$$\frac{I_b}{G_b} = \frac{I_o}{G_o} = \frac{\pi}{24}\frac{\frac{24}{\pi}\sin\left(\frac{\pi}{24}\right)\cos\omega_I - \cos\omega_s}{\sin\omega_s - \left(\frac{\pi}{180}\right)\omega_s\cos\omega_s} \quad (49)$$

where ω_s is the sunset-hour angle (in degrees) for a horizontal surface.

Liu and Jordan [15] extended the day-length range of Whiller's data and plotted the ratio $r_t = \frac{I_t}{G_t}$ against the sunset-hour angle ω_s. The mathematical expression was presented by Collares-Pereira and Rabl [36]

$$r_t = \frac{I_t}{G_t} = \frac{I_o}{G_o}(a_2 + b_2\cos\omega_I) \quad (50)$$

where

$$a_2 = 0.409 + 0.5016\sin(\omega_s - 60^\circ) \quad (51)$$

$$b_2 = 0.6609 - 0.4767\sin(\omega_s - 60^\circ) \quad (52)$$

With Eqs. (37) and (47) for I_o and G_o, Eq. (50) can be expressed as

$$r_t = \frac{\pi}{24}\left(\frac{\cos\omega_I - \cos\omega_s}{\sin\omega_s - \pi\frac{\omega_s}{180}\cos\omega_s}\right)(a_2 + b_2\cos\omega_I) \quad (53)$$

A graphical presentation is shown in Fig. 4 and the graph is applicable everywhere [e.g. 8,15,29].

6. Estimation of the hourly diffuse radiation on a horizontal surface using decomposition models

Values of global and diffuse radiations for individual hours are essential for research and engineering applications. Hourly global radiations on horizontal surfaces are available for many stations, but relatively few stations measure the hourly diffuse radiation. Decomposition models have, therefore, been developed to predict the diffuse radiation using the measured global data.

The models are based on the correlations between the clearness index k_t (dimensionless) and the diffuse fraction k_d (dimensionless), diffuse coefficient k_D (dimensionless) or the direct transmittance k_b (dimensionless) where

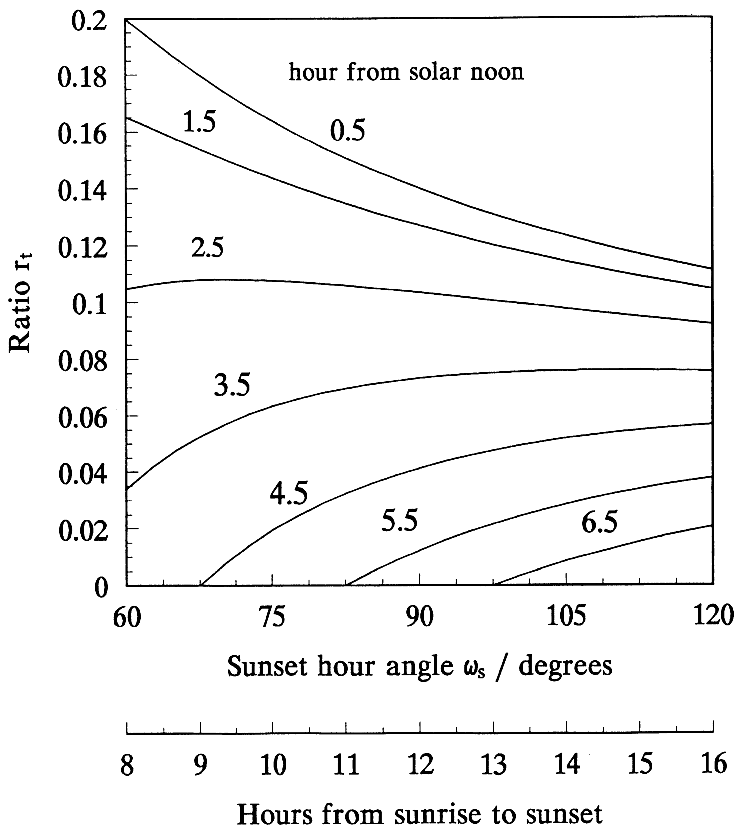


Fig. 4. Distribution of the monthly average hourly global radiation.

$$k_t = \frac{I_t}{I_o} \tag{54}$$

$$k_d = \frac{I_d}{I_t} \tag{55}$$

$$k_D = \frac{I_d}{I_o} \tag{56}$$

$$k_b = \frac{I_b}{I_o} \tag{57}$$

I_t, I_b, I_d and I_o being the global, direct, diffuse and extraterrestrial irradiances, respectively, on a horizontal surface (all in MJ m^{-2}).

6.1. Liu and Jordan model [15]

The relationships permitting the determination, for a horizontal surface, of the instantaneous intensity of diffuse radiation on clear days, the long-term average hourly and daily sums of diffuse radiation, and the daily sums of diffuse radiation for various categories of days of differing degrees of cloudiness, with data from 98 localities in the USA and Canada (19° to 55°N latitude), were studied by Liu and Jordan [15].

The transmission coefficient for total radiation on a horizontal surface is given by the intensity of total radiation (i.e. direct I_b plus diffuse I_d) incident upon a horizontal surface I_t divided by the intensity of solar radiation incident upon a horizontal surface outside the atmosphere of the Earth I_o. The correlation between the intensities of direct and total radiations on clear days is given by

$$k_D = 0.271 - 0.2939 k_b \tag{58}$$

Since

$$k_t = (I_b + I_D)/I_o = k_b + k_D \tag{59}$$

then

$$k_D = 0.384 - 0.416 k_t \tag{60}$$

6.2. Orgill and Hollands model [16]

Following the work by Liu and Jordan [15], the diffuse fraction was estimated by Orgill and Hollands [16] using the clearness index k_t as the only variable. The model was based on the global and diffuse irradiance values registered in Toronto (Canada, 42.8°N) in the years 1967–1971. The correlation between hourly diffuse fraction k_d and clearness index k_t is given by

$$k_d = 1 - 0.249k_t;\ k_t < 0.35 \tag{61}$$

$$k_d = 1.577 - 1.84k_t;\ 0.35 \leqslant k_t \leqslant 0.75 \tag{62}$$

$$k_d = 0.177;\ k_t > 0.75 \tag{63}$$

Once the hourly diffuse fraction is obtained from the $k_d - k_t$ correlations, the direct irradiance I_b is obtained by [23]

$$I_b = \frac{G_t(1 - k_d)}{\sin\alpha} \tag{64}$$

where α is the solar elevation angle.

6.3. Erbs et al. model [17]

Erbs et al. [17] studied the same kind of correlations with data from 5 stations in the USA with latitudes between 31 and 42°. The data were of short duration, ranging from 1 to 4 years. In each station, hourly values of normal direct irradiance and global irradiance on a horizontal surface were registered. Diffuse irradiance was obtained as the difference of these quantities. The diffuse fraction k_d is calculated by

$$k_d = 1 - 0.09k_t \ \text{ for } \ k_t \leqslant 0.22 \tag{65}$$

$$k_d = 0.9511 - 0.1604k_t + 4.388k_t^2 + 16.638k_t^3 + 12.336k_t^4 \ \text{ for } \ 0.22 < k_t \leqslant 0.8 \tag{66}$$

$$k_d = 0.165 \ \text{ for } \ k_t > 0.8 \tag{67}$$

6.4. Spencer model [21]

Spencer [21] studied the latitude dependence on the mean daily diffuse radiation with data from 5 stations in Australia (20–45°S latitude). The following correlation is proposed

$$k_d = a_3 - b_3k_t \ \text{ for } \ 0.35 \leqslant k_t \leqslant 0.75 \tag{68}$$

It is presumed that k_d has constant values beyond the above range of k_t. The coefficients a_3 and b_3 are latitude dependent.

$$a_3 = 0.94 + 0.0118|\phi| \tag{69}$$

$$b_3 = 1.185 + 0.0135|\phi| \quad (70)$$

where ϕ (degrees) is the latitude. There would be an increasing proportion of diffuse radiation at higher latitudes due to the increase in average air mass.

6.5. Reindl et al. [19]

Reindl et al. [19] estimated the diffuse fraction k_d using two different models developed with measurements of global and diffuse irradiance on a horizontal surface registered at 5 locations in the USA and Europe (28–60°N latitude). The first model (denoted as Reindl-1) estimates the diffuse fraction using the clearness index as input data and the diffuse fraction k_d is given by

$$k_d = 1.02 - 0.248k_t \text{ for } k_t \leqslant 0.3 \quad (71)$$

$$k_d = 1.45 - 1.67k_t \text{ for } 0.3 < k_t < 0.78 \quad (72)$$

$$k_d = 0.147 \text{ for } k_t \geqslant 0.78 \quad (73)$$

The second correlation (denoted as the Reindl-2 model) estimates the diffuse fraction in terms of the clearness index k_t and the solar elevation α. The diffuse fraction k_d is given by

$$k_d = 1.02 - 0.254k_t + 0.0123\sin\alpha \text{ for } k_t \leqslant 0.3 \quad (74)$$

$$k_d = 1.4 - 1.749k_t + 0.177\sin\alpha \text{ for } 0.3 < k_t < 0.78 \quad (75)$$

$$k_d = 0.486k_t - 0.182\sin\alpha \text{ for } k_t \geqslant 0.78 \quad (76)$$

6.6. Lam and Li [22]

Lam and Li [22] studied the correlation between global solar radiation and its direct and diffuse components for Hong Kong (22.3°N latitude) with the measured data in 1991–1994. A hybrid correlation model based on hourly measured data for the prediction of hourly direct and diffuse components from the global radiation for Hong Kong is given by

$$k_d = 0.977 \text{ for } k_t \leqslant 0.15 \quad (77)$$

$$k_d = 1.237 - 1.361k_t \text{ for } 0.15 < k_t \leqslant 0.7 \quad (78)$$

$$k_d = 0.273 \text{ for } k_t > 0.7 \quad (79)$$

6.7. Skartveit and Olseth model [18]

Skartveit and Olseth [18] showed that the diffuse fraction depends also on other parameters such as solar elevation, temperature and relative humidity. Similar arguments were found in the literature [19,37–39].

Skartveit and Olseth [18] estimated the direct irradiance I_b from the global irradiance G_t and from the solar elevation angle α for Bergen, Norway (60.4°N latitude) with the following equation

$$I_b = \frac{G_t(1-\psi)}{\sin\alpha} \tag{80}$$

where ψ is a function of k_t and the solar elevation α (degrees). The model was validated with data collected in Aas, Norway (59.7°N latitude), Vancouver, Canada (49.3°N latitude) and 10 other stations worldwide. Details of this function are given below:

If $k_t < c_1$,

$$\psi = 1 \tag{81}$$

where $c_1 = 0.2$.

If $c_1 \leqslant k_t \leqslant 1.09c_2$,

$$\psi = 1-(1-d_1)\left[d_2 c_3^{1/2} + (1-d_2)c_3^2\right] \tag{82}$$

where

$$c_2 = 0.87 - 0.56\mathrm{e}^{-0.06\alpha} \tag{83}$$

$$c_3 = 0.5\left\{1+\sin\left[\pi\left(\frac{c_4}{d_3}-0.5\right)\right]\right\} \tag{84}$$

$$c_4 = k_t - c_1 \tag{85}$$

$$d_1 = 0.15 + 0.43\mathrm{e}^{-0.06\alpha} \tag{86}$$

$$d_2 = 0.27 \tag{87}$$

$$d_3 = c_2 - c_1 \tag{88}$$

If $k_t > 1.09c_2$

$$\psi = 1 - 1.09c_2\frac{1-\xi}{k_t} \tag{89}$$

where

$$\xi = 1 - (1 - d_1)\left(d_2 c_3'\frac{1}{2} + (1 - d_2)c_3'^2\right) \quad (90)$$

$$c_3' = 0.5\left(1 + \sin\left[\pi\left(\frac{c_4'}{d_3} - 0.5\right)\right]\right) \quad (91)$$

$$c_4' = 1.09c_2 - c_1 \quad (92)$$

The constants have to be adjusted for conditions deviating from the validation domain.

6.8. Maxwell model [23]

A quasi-physical model for converting hourly global horizontal to direct normal insolation proposed by Maxwell in 1987, was reviewed by Batlles et al. (23). The model combines a clear physical model with experimental fits for other conditions. The direct irradiance I_b is given by:

$$I_b = I_o\left\{\psi - \left(d_4 + d_5 e^{m_a d_6}\right)\right\} \quad (93)$$

where I_o is the extraterrestrial irradiance, ψ is a function of the air mass m_a (dimensionless) and is given by

$$\psi = 0.866 - 0.122m_a + 0.0121m_a^2 - 0.000653m_a^3 + 0.000014m_a^4 \quad (94)$$

and d_4, d_5 and d_6 are functions of the clearness index k_t as given below:
If $k_t \leqslant 0.6$,

$$\begin{aligned} d_4 &= 0.512 - 1.56\ k_t + 2.286\ k_t^2 - 2.222\ k_t^3 \\ d_5 &= 0.37 + 0.962\ k_t \\ d_6 &= -\ 0.28 + 0.923k_t - 2.048\ k_t^2 \end{aligned} \quad (95)$$

If $k_t > 0.6$,

$$\begin{aligned} d_4 &= -5.743 + 21.77k_t - 27.49k_t^2 + 11.56k_t^3 \\ d_5 &= 41.4 - 118.5k_t + 66.05k_t^2 + 31.9k_t^3 \\ d_6 &= -47.01 + 184.2k_t - 222k_t^2 + 73.81k_t^3 \end{aligned} \quad (96)$$

6.9. Louche et al. model [20]

Louche et al. [20] used the clearness index k_t to estimate the transmittance of beam radiation k_b. The correlation is given by

$$k_b = -10.627k_t^5 + 15.307k_t^4 - 5.205k_t^3 + 0.994k_t^2 - 0.059k_t + 0.002 \quad (97)$$

The correlation includes global and direct irradiance data for Ajaccio (Corsica, France, 44.9°N latitude) between October 1983 and June 1985.

Other researchers [40,41] estimated the direct irradiance by means of $k_b - k_t$ correlations. They found that the solar elevation is an important variable in this type of correlation. The direct irradiance is then estimated with the following definition of direct transmittance [23]

$$I_b = k_b I_o \quad (98)$$

6.10. Vignola and McDaniels model [40]

Vignola and McDaniels [40] studied the daily, 10-day and monthly average beam-global correlations for 7 sites in Oregon and Idaho, USA (38–46°N latitude). The beam-global correlations vary with time of year in a manner similar to the seasonal variations exhibited by diffuse-global correlations. The direct transmittance k_b for daily correlation is given by

$$k_b = 0.022 - 0.28k_t + 0.828k_t^2 + 0.765k_t^3 \text{ for } k_t \geqslant 0.175 \quad (99)$$

$$k_b = 0.016k_t \text{ for } k_t < 0.175 \quad (100)$$

and, with seasonal terms,

$$k_d = 0.013 - 0.175k_t + 0.52k_t^2 + 1.03k_t^3 + \left(0.038k_t - 0.13k_t^2\right)\sin[2\pi(N - 20)/365] \quad (101)$$

for $k_t \geqslant 0.175$

$$k_b = 0.125k_t^2 \text{ for } k_t < 0.175 \quad (102)$$

where N is the year-day under consideration.

6.11. Al-Riahi et al. model [42]

Al-Riahi et al. [42] developed a clear-day model for the beam transmittance of the atmosphere based on 5-year daily global radiation data measured at the Solar Energy Research Center (Baghdad, Iraq, 33.2°N latitude). The model offers the prediction of clear-day hourly values of direct-normal and global solar radiations for any day of the year at a given location with no required meteorological inputs. The global irradiance I_t is given by

$$I_t = I_b(\sin\alpha + \psi) \quad (103)$$

where ψ is a fraction of the diffuse irradiance divided by the direct irradiance and would be a constant throughout a particular clear-day. The fraction ψ at a given day N (dimensionless) of a year is given by

$$\psi(N) = 0.0936 + 0.041\sin\left[\frac{(N-104.5)\pi}{167}\right] + 0.004773\sin\left[\frac{(N+24.4)\pi}{83.5}\right] \quad (104)$$

I_b is given by

$$I_b = I_o e^{-Bm_a} = I_o e^{-B\left(\frac{1}{\sin\alpha}\right)^{0.678}} \quad (105)$$

where m_a is the air mass, and B is the overall atmospheric coefficient of extinction which varies over the year. The exponent 0.678 compensates for the actual curvature of the atmosphere.

$B(N)$ is given by

$$B(N) = 0.3917397 - 5.596\times10^{-2}\sin\left(\frac{2\pi}{365}N\right) + 5.293\times10^{-3}\cos\left(\frac{2\pi}{365}N\right) + 1.3594\times10^{-2}\sin\left(\frac{4\pi}{365}N\right) + 4.0383\times10^{-3}\cos\left(\frac{4\pi}{365}N\right) \quad (106)$$

7. Comparison of decomposition models

The $k_a - k_t$ correlations of Liu and Jordan [15], Orgill and Hollands [16], Erbs et al. [17], Reindl-1 [19], Spencer [21] and Lam and Li [22] are plotted in Fig. 5. The correlations of Orgill and Hollands [16], Erbs et al. [17], Reindl-1 [19] and Lam and Li [22] are almost identical. The predicted diffuse fraction for Hong Kong by Lam and Li's correlations is higher than by other correlations for clear-sky conditions ($k_t > 0.7$). The Spencer [21] correlation plotted for ϕ yields consistently lower results.

The $k_d - k_t$ correlations of Reindl-2 [19] and Skartveit and Olseth [18] were plotted in Fig. 6 for solar elevations of 10, 40 and 90°. The results agree well with each other for $k_t \leqslant 0.3$ and agree reasonably well for $0.3 < k_t < 0.8$. A larger deviation was found for $k_t > 0.8$ and for a lower solar angle $\alpha = 10°$. The predicted diffuse fraction for Hong Kong [22] was plotted in the figure for comparison. The predicted results agree with those of the other two models.

The $k_b - k_t$ correlations proposed by Louche et al. [20], Vignola and McDaniels [40] and Maxwell [23] with air masses of 1, 2 and 4 are plotted in Fig. 7. The results of Louche et al. [20] and Vignola and McDaniels [40] are almost identical. The Maxwell modelled results agree well with the predictions of the other two models. The $k_b - k_t$ correlation for Hong Kong would be derived from the $k_d - k_t$ correlation

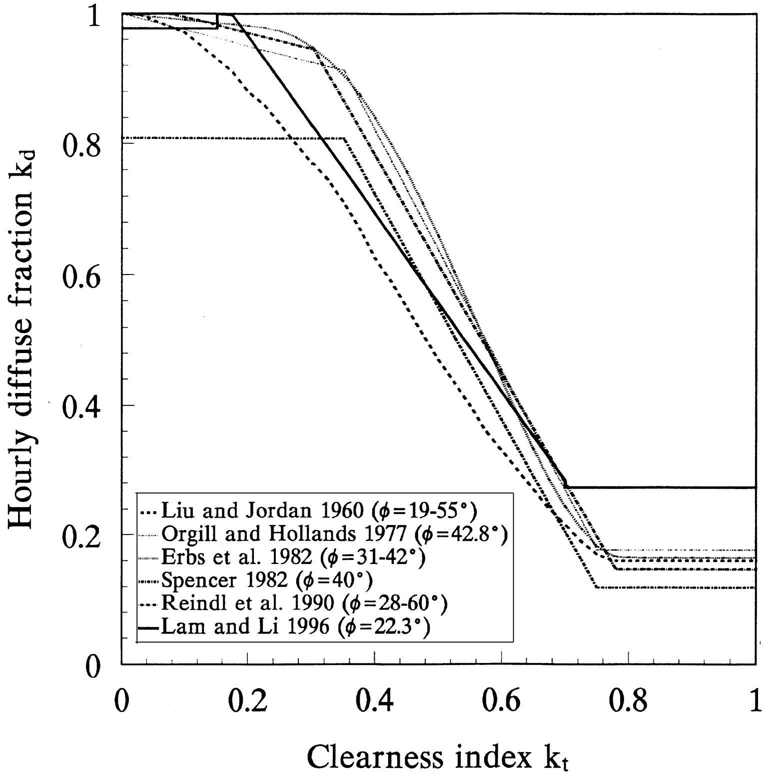


Fig. 5. Modelled hourly diffuse fraction k_d as a function of clearness index k_t.

[Eqs. (77) to (79)] proposed by Lam and Li [22]. With Eqs. (54), (55) and (57), k_b can be expressed as

$$k_b = (1 - k_d)k_t \tag{107}$$

The derived $k_b - k_t$ correlation for Hong Kong agrees well with k_b predicted by the others for $k_t < 0.7$ and falls below the predictions for $k_t > 0.7$ as shown in Fig. 7.

8. Comparison of decomposition models and parametric model

Batlles et al. [23] estimated the hourly values of direct irradiance using the decomposition models [16–20]. The results of these models were compared with those provided by an atmospheric transmittance parametric model [8] and with the measured direct irradiances for 6 stations (36.8–43°N latitude, 1.7–5.4°W longitude) reported by Batlles et al. [43].

The aerosol-scattering transmittance adopted by Batlles et al. [23] is calculated from the visibility.

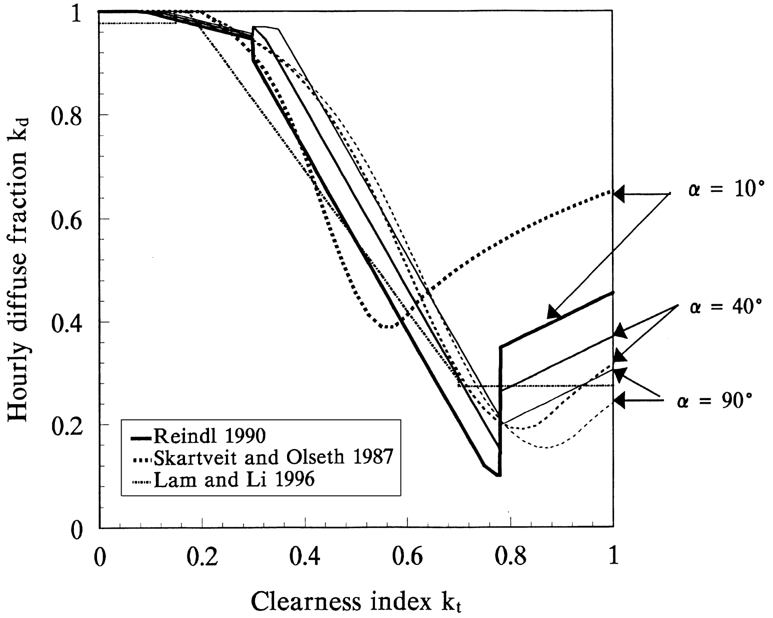


Fig. 6. Modelled hourly diffuse fraction k_d as a function of clearness index k_t for various values of the solar elevation α.

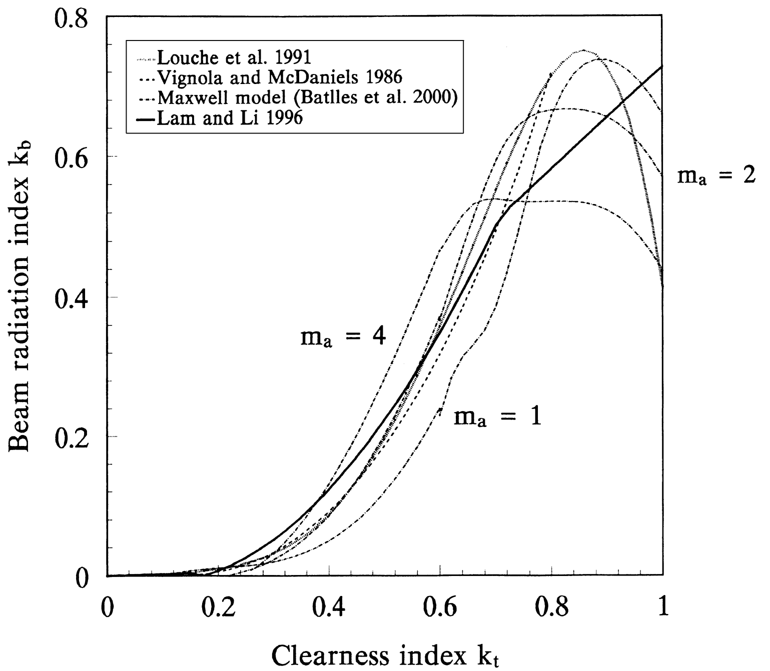


Fig. 7. Modelled beam radiation index k_b as a function of clearness index k_t.

$$\tau_a = \left(0.97 - 1.265 V_{is}^{-0.66}\right)^{m_a^{0.9}} \tag{108}$$

where visibility V_{is} (km) [2] is given by

$$V_{is} = 147.994 - 1740.523\left[\beta_1\psi - \left(\beta_1^2\psi^2 - 0.17\beta_1\psi + 0.011758\right)^{0.5}\right] \tag{109}$$

and

$$\psi = 0.55^{-\beta_2} \tag{110}$$

where β_1 and β_2 are the Ångström turbidity-parameters: β_2 is taken to be 1.3 and β_1 is calculated using the Linke turbidity-factor

$$T_L = \left(\frac{85 + \alpha}{39.5e^{-w'} + 47.4} + 0.1\right) + (16 + 0.22w')\beta_1 \tag{111}$$

where α is the solar elevation in degrees and w' is the precipitable water-content in cm. The Linke turbidity-factor T_L is defined as the number of Rayleigh atmospheres (an atmosphere clear of aerosols and without water vapour) required to produce a determined attenuation of direct radiation and is expressed by

$$T_L = (l_{Ro}m_a)^{-1}\ln\frac{I_o}{I_b} \tag{112}$$

where l_{Ro} (dimensionless) is the Rayleigh optical-thickness. In the study by Batlles et al. [23], hourly and monthly average hourly values of β_1 were used.

The decompositions are good for high values of the clearness index (i.e. cloudless skies and high solar elevations). In such conditions, the model proposed by Louche et al. [20] estimated the direct irradiance with a 10% root-mean-square (RMS) error and the estimated direct irradiance by the Iqbal model [8] using hourly values of β has a 4% RMS error. Both models have no significant mean bias error. Batlles et al. [23] concluded that the parametric model is the best when precise information of the turbidity coefficient is available. However, if there is no turbidity information available, decomposition models are a good choice.

The clearness index k_t and diffuse fraction k_d for Hong Kong (22.3°N latitude) were estimated with the parametric model C [8] as this model offers accurate prediction over a wide range of conditions [13]. The aerosols, Rayleigh, ozone, gas and water scattering transmittances τ_a, τ_r, τ_o, τ_g and τ_w respectively, were determined by Eqs. (108) and (4)–(7). The Ångström turbidity parameter β_2 is taken to be 1.3 and β_1 is taken to be 0, 0.1, 0.2 and 0.4 for the clean, clear, turbid and very turbid atmospheres respectively [8]. The corresponding visibility V_{is} in Eq. (109) is 337, 28, 11 and 5 km, respectively. The measured vertical ozone-layer thickness l_{oz} for the Northern Hemisphere [44] was used to determine the ozone relative optical-path length U_3 in Eq. (10). The air mass at standard pressure, m_r, is approximated by Kasten's formula [8]

$$m_r = \left[\cos\theta_z + 0.15(93.885 - \theta_z)^{-1.253}\right]^{-1} \tag{113}$$

The precipitable water-vapour thickness w' is approximated by Won's equation [8]

$$w' = 0.1\mathrm{e}^{2.2572+0.05454T_{\mathrm{dew}}} \tag{114}$$

where T_{dew} is the dew-point temperature (°C). The monthly average daily temperature T and monthly average daily dew-point temperature T_{dew} recorded between 1961 and 1990 in Hong Kong [33] were adopted in the calculations. The local air pressure p is taken to be 1013.25 mbar. The ground albedo ρ_g is taken to be 0.09, 0.35 and 0.74 for black concrete, uncoloured concrete and white glazed surfaces respectively (as Hong Kong is a developed city of many high-rise concrete buildings with curtain walls).

The extraterrestrial radiation $\dot{I}_o$ was determined by [8]

$$\dot{I}_o = \dot{I}_{sc}E_o\cos\theta_z \tag{115}$$

The clearness index k_t and the diffuse fraction k_d were determined with the modelled global solar irradiance $\dot{I}_t$ and total diffuse irradiance $\dot{I}_d$. The average values of the predicted diffuse fraction by the Iqbal model C [8] for clean, clear, turbid and very turbid atmospheres at ground albedo of 0.09, 0.35 and 0.74 are plotted in Fig. 8. The predicted k_d by Lam and Li's correlations [22] for Hong Kong is shown for comparison. Lam and Li's results agree with the modeled results for V_{is} of about 28 km for k_t between 0.4 and 0.7 and fall in the range of V_{is} between 11 and 28 km for k_t between 0.2 and 0.4.

9. Other studies of solar radiation models

Literature results [37,38] showed that the diffuse fraction depends also on other variables like atmospheric turbidity, surface albedo and atmospheric precipitable water.

Chendo and Maduekwe [39] studied the influence of four climatic parameters on the hourly diffuse fraction in Lagos, Nigeria. It was found that the diffuse fraction of the global solar-radiation depended on the clearness index k_t, the ambient temperature, the relative humidity and the solar altitude. The standard error of the Liu and Jordan-type [16,17,19] was reduced by 12.8% when the solar elevation, ambient temperature and relative humidity were included as predictor variables for the 2-year data set. Tests with existing correlations showed that the Orgill and Hollands model [16] performed better than the other two models [17,19].

Elagib et al. [45] proposed a model to compute the monthly average daily global solar-radiation G_t (MJ m^{-2}) for Bahrain (22°N latitude) from commonly measured

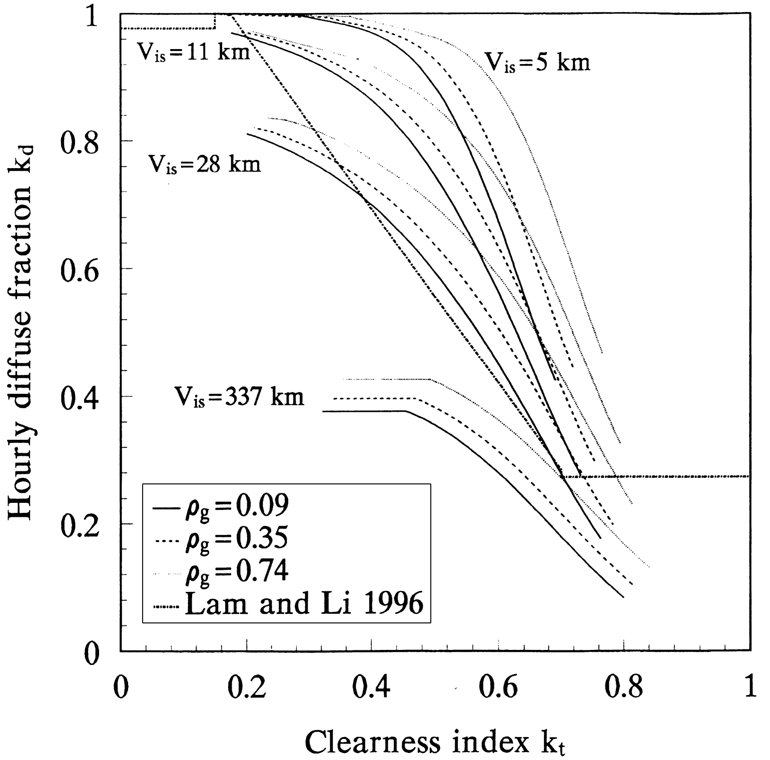


Fig. 8. Modelled hourly diffuse fraction k_d as a function of clearness index k_t for Hong Kong.

meteorological parameters. The parameters are monthly average daily relative-humidity RH (%), the monthly average daily temperature-range ΔT (°C), the monthly average daily temperature T (°C), the monthly average daily hours of sunshine S (h) and the monthly average daily extraterrestrial solar-radiation G_o (MJ m^{-2}).

For January–June, G_t can be computed by one of the following correlation equations:

$$G_t = 70.8385 - 0.7886RH \tag{116}$$

$$G_t = 55.8528 - 0.6440(RH - S) \tag{117}$$

$$G_t = 49.2637 - 0.6751(RH - \Delta T - S) \tag{118}$$

For July–December,

$$G_t = 4.5456e^{0.0453T} \tag{119}$$

$$G_t = -1.5358 - 0.6078G_o \quad (120)$$

$$G_t = 34.5577 - 0.4642(RH - T) \quad (121)$$

$$G_t = 18.9469 - 0.2663(RH - T - G_o) \quad (122)$$

The 3-variable models yielded the smallest errors yet the 1-variable models can give close estimations.

Tovar et al. [46] studied the probability density distributions of 1-min values of global irradiance, conditioned to the optical air-mass, considering them to be an approximation to the instantaneous distributions. It was found that the bimodality that characterizes these distributions increases with optical air-mass. The function based on Boltzmann's statistics can be used for the generation of synthetic radiation data. Expressing the distribution as a sum of two functions provides an appropriate modelling of the bimodality feature that can be associated with the existing two levels of irradiance corresponding to two extreme atmospheric situations — the cloudless and cloudy conditions.

In addition to the shortwave (0.3–2.0 μm) radiation from the Sun, the Earth receives longwave radiation (4–100 μm) from the atmosphere. The air conditions at ground level largely determine the magnitude of the incoming radiation [5] and the downward radiation from the atmosphere q_{rat} could be expressed as

$$q_{rat} = \varepsilon_{at}\sigma T_{at}^4 \quad (123)$$

where ε_{at} (dimensionless) is the atmosphere emittance, σ is the Stefan–Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4}) and T_{at} (K) is the ground-level temperature.

The apparent sky-temperature T_{sky} (K) is defined as the temperature at which the sky (as a blackbody) emits radiation at the rate actually emitted by the atmosphere at ground-level temperature with its actual emittance ε_{at}. Then,

$$\sigma T_{sky}^4 = \varepsilon_{at}\sigma T_{at}^4 \quad (124)$$

or

$$T_{sky}^4 = \varepsilon_{at} T_{at}^4 \quad (125)$$

The atmosphere emittance is a complex function of air temperature and moisture content [47]. A simple relationship [5], which ignores the vapour pressure of the atmosphere could be used to estimate the apparent sky-temperature

$$T_{sky} = 0.0552T_{at}^{1.5} \quad (126)$$

10. Conclusion

Solar radiation models are desirable for designing solar-energy systems and good evaluations of thermal environments in buildings. Two parametric models (Iqbal model C [8] and ASHRAE model [5]) were used to predict the beam, diffuse and global irradiances under clear-sky conditions for Hong Kong. At zero zenith angle, a maximum difference of beam irradiance of not more than 7% is obtained between the 2 models. The beam irradiance deviation of less than 8% was found for September–April, and less than 12% for May–August when the zenith angle is less than 60°. The deviation of the global irradiance predicted between the 2 models was only 5%. A large deviation, up to 41%, was found between the predicted values of the diffuse irradiance with the 2 models. The ASHRAE model would be less accurate for diffuse radiation predictions because the ground albedo and aerosol-generated diffuse irradiance are not included.

Sunshine hours and cloud-cover data are available easily for many stations: correlations for predicting the insolation with sunshine hours are reviewed. Seven models using the Ångström-Prescott equation to predict the average daily global radiation with hours of sunshine have been reviewed. Estimation of monthly average hourly global radiation was discussed. The 7 models were applied to predict the daily global radiation for Hong Kong and comparison with measured data was made. Predictions made by Bahel et al. [29], Alnaser [30] and Louche et al. [20] agree with the measured data, while the other models provide reasonable predictions.

Values of global and diffuse radiation for individual hours are essential for many research and engineering applications. Twelve decomposition models in the literature commonly used to predict the diffuse component using the measured hourly global data are reviewed. The models were used to predict the hourly diffuse fraction for Hong Kong and compared with the model based on Hong Kong measurements [22]. Good agreement was found among the model predictions, but lower diffuse fractions were predicted by some models [15–17,19,21] for clear-sky conditions. The beam-radiation index for Hong Kong was estimated by the Louche et al. [20], Vignola and McDaniels [40] and Maxwell [23] models and compared with the prediction of Lam and Li's model [22]. Again, good agreement was found except that a lower beam-radiation index was predicted by Lam and Li's model [22] under clear-sky conditions.

The clearness index k_t and diffuse fraction k_d for Hong Kong were also determined with the Iqbal model C [8] for 4 atmospheric conditions (clear, clean, turbid and very turbid atmospheres) for 3 cases of ground albedo (corresponding to black concrete, uncoloured concrete and white glazed surface). The results are then compared with those predicted by Lam and Li's model [22]. It was found that Lam and Li's predictions agree with the modeled results for V_{is} about 28 km for k_t between 0.4 and 0.7, and fall for V_{is} between 11 and 28 km for k_t between 0.2 and 0.4. The parametric model would provide accurate predictions of solar radiation as turbidity information is included and good evaluations of thermal environments in buildings are therefore possible. If precise atmospheric information, such as optical properties of clouds, cloud amount, thickness, position, the number of layers and turbidity, is not available, decomposition models based on measured hourly global radiation would be a good choice.

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