MONTHLY-AVERAGED DAILY SHADING FACTOR
FOR A COLLECTOR FIELD

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Abstract—An analytical expression is derived for the monthly-averaged daily shading factor $f$ for a field of flat plate collectors. The elements of the collector field are oriented towards the equator and tilted with a constant value of tilt angle. Uniform spacing is maintained between various rows of collectors in the field. The expression of $f$ is found to depend on the site latitude angle, month, length and width of each collector, spacing between any successive rows of collectors, number of collectors in a row, and the value of the tilt angle for collectors. Graphical presentation for the value of $f$ at wide range of its independent variables is given. The graphs and the analytical expression of $f$ assist in improving the accuracy of predicting the monthly-averaged daily solar radiation received by a collector field.

1. INTRODUCTION

Long-term performance of a single flat plate collector depends on the monthly-averaged daily radiation $\bar{H}$ received by the collector surface. The value of $\bar{H}$ is usually estimated using the following commonly accepted isotropic model.

$$\bar{H} = \left(1 - \frac{\bar{H}_d}{\bar{H}}\right) \bar{R}_b + \frac{1}{2} \bar{H}_d \left(1 + \cos \beta \right) + \frac{1}{2} \rho_p (1 - \cos \beta) \quad (1)$$

where the meanings of the various symbols are given in the nomenclature list. In a previous work[1] we showed that the above equation for $\bar{H}_r$ must be corrected before being applied to a collector field. The correction is necessary to account for shading of beam irradiance by other collectors in the preceding rows and also to account for shielding of the collector from ground and sky by collectors in the preceding rows. It was shown[1] that each term in the right hand side of eqn (1) is multiplied by one or more correction factors. To account for shading of beam irradiance the first term in eqn (1) must be multiplied by $(1 - f)$, i.e., to read

$$\left(1 - \frac{\bar{H}_d}{\bar{H}}\right) \bar{R}_b (1 - f)$$

where $f$ is the monthly-averaged daily shading factor for the $N$ elements row. The expression of $f$ is given as follows[1]

$$f = \frac{1}{\bar{H}_d \bar{R}_b} \frac{1}{M} \sum_{j=1}^{M} \int_{\theta_{in}}^{\theta_{out}} \frac{\cos \theta}{\cos \theta_1} G, \, d\theta. \quad (2)$$

In the above expression the instantaneous shading factor $f$ is defined as follows

$$f = \frac{\text{shaded area of all elements in the row}}{\text{total area of all elements in the row}}. \quad (3)$$

Calculation of the daily beam radiation received by a tilted, partially or fully shaded surface have been studied previously by several investigators. Applebaum and Bany[2] studied the geometry of shading of vertical and inclined poles. They then derived an analytical expression for the instantaneous shading factor $f$ (see eqn 3) for south facing flat plate collector fields. Utzinger and Klein[3] used the same expression given by eqn (2) to determine $f$ for vertical flat plate receivers shaded by overhangs. After deriving the expression for $f$, they used it in eqn (2) to determine $f$ by carrying out a numerical integration. Finally, they established charts to provide $f$ at various parametric conditions. Jones and Bukhart[4] considered a large flat plate collector field with rows of infinite width. With this simplification, they derived an analytical expression for the daily beam and daily total radiation on collector surfaces. Sharp[5] also studied a simple overhang on a surface at any tilt and azimuth and derived an expression for the daily- or monthly-averaged daily beam radiation received by the surface. Since his results are only for a special geometry they could not be applied therefore for collector fields. Recently, Elsayed and Al-Turki[6] studied in detail the calculation of the shading factor $f$ (eqn 3) for equator facing collector fields of finite or infinite row width. A simple analytical expression is derived for $f$ using the separation of variable technique. The results could be used to determine the instantaneous shading factor $f$ for any collector in a row or for a whole row as caused by one or more collectors in a preceding row.

The objective of the present paper is to derive a mathematical expression for $f$ for collector fields with elements oriented towards the equator and tilted with any arbitrary tilt angle.
2. INSTANTANEOUS SHADING FACTOR

Consider a collector field of several rows of \( N \) elements per row and with collectors aligned behind each other and oriented toward the equator (to south in the northern hemisphere), as shown in Fig. 1. The spacing between rows is maintained at \( \Delta y \) and each element has width \( W \) and length \( L \). In this geometry we have showed in a previous work [6] that the shading factor for any row (other than the first) is given as follows:

\[
f = \left(1 - \tan |G_1| \right) \left(1 - \tan \frac{G_2 \cdot NW}{L} \right),
\]

\[
at -1 < \tan G_1 < 1 \\
and 0 < \tan G_2 < \frac{L}{NW} \]

\[
f = 0; \quad \text{at } |\tan G_1| \geq 1 \\
and/or \tan G_2 \geq \frac{L}{NW} \quad (4)
\]

where the angles \( G_1 \) and \( G_2 \) are defined as follows [6]

\[
\tan G_1 = \frac{\Delta y/(NW) \cdot \sin \beta \sin \theta \sin \psi}{\cos \theta \cos \beta + \sin \theta \cos \psi \sin \beta}, \quad (5a)
\]

\[
\tan G_2 = \frac{\Delta y/(NW) \cdot \cos \theta}{\cos \theta \cos \beta + \sin \theta \cos \psi \sin \beta}, \quad (5b)
\]

By simple algebraic manipulation the angles \( G_1 \) and \( G_2 \) could be expressed in the following forms

\[
\tan G_1 = \frac{\Delta y}{NW} \sin \beta \sin \theta \sin \psi \frac{\sin \omega}{\lambda + \cos(\phi - \beta)\cos \omega}, \quad (6a)
\]

\[
\tan G_2 = \frac{\Delta y}{NW} \cos \phi \frac{\cos \omega - \cos \omega_1}{\lambda + \cos(\phi - \beta)\cos \omega}, \quad (6b)
\]

where \( \omega_1 \) and \( \lambda \) are expressed as follows

\[
\omega_1 = \cos^{-1}(\tan \beta \tan \phi), \quad (7a)
\]

\[
\lambda = \tan \phi \left[ \tan \phi \cos(\phi - \beta) - \frac{\sin \beta}{\cos \phi} \right]. \quad (7b)
\]

By definition, the angle \( G_1 \) is an equivalent solar azimuth angle and the angle \( G_2 \) is an equivalent solar altitude angle [6]. The angle \( G_2 \) is symmetric around solar noon while the angle \( G_1 \) is symmetric in magnitude only around solar noon. The change in the sign of \( G_1 \) is only due to the change in the sign of the solar hour angle \( \omega \) before and after solar noon. The expression of \( f \) in eqn (4) could then be rewritten as follows

\[
f = \left[1 - \frac{\Delta y}{NW} \sin \beta \frac{\sin \omega}{\lambda + \cos(\phi - \beta)\cos \omega} \right] \times \left[1 - \frac{\Delta y}{L} \cos \omega - \cos \omega_1 \frac{\cos \omega}{\lambda + \cos(\phi - \beta)\cos \omega} \right],
\]

\[
0 \leq \omega_1 \leq |\omega| \leq \omega_2 \leq \omega_T \quad (8a)
\]

\[
f = 0, \quad |\omega| \leq \omega_1 \text{ or } |\omega| \geq \omega_2 \quad (8b)
\]

where \( \omega_1 \) and \( \omega_2 \) are the boundaries of \( \omega \) for the daily shading period after solar noon (shading period is symmetric around solar noon). Values of \( \omega_1 \) and \( \omega_2 \) are determined as given in Section 4. The sunset solar hour angle \( \omega_T \) for the collector surface is determined from the following eqn [7]

\[
\omega_T = \max \{\omega_1, \cos^{-1}[\tan \delta \tan(\phi - \beta)]\}. \quad (9)
\]

3. THE MONTHLY-AVERAGED DAILY SHADING FACTOR

In the present section the expression of \( f \) given by eqns (8a) and (8b) is used in eqn (2) and integration

![Fig. 1. Collector field geometry.](image-url)
is carried out to find $f$. Following the same procedure used by Liu and Jordan[8] and Klein[9] in the determination of $R_b$, the extraterrestrial irradiance on a horizontal plane $G_0$ is used as the weighing irradiance instead of actual value of $G_r$ in eqn (2). The expression for $G_n$ is given as follows[10].

$$G_n = G_0 \left[ 1 + 0.033 \cos \left( \frac{360n}{365} \right) \right] \cos \theta$$  \hspace{1cm} (10)

where $G_n$ is the solar constant and $n$ is the Julian day. Equation (2) then takes the following form

$$f = \frac{\bar{G}_n}{\bar{H}_o \bar{R}_b} \sum_{n=0}^{M} \left[ 1 + 0.033 \cos \left( \frac{360n}{365} \right) \right] \cos \theta$$  \hspace{1cm} (11a)

Using the average day $\bar{n}$ of the month as suggested by Klein[9], eqn (11a) becomes

$$f = \frac{\bar{G}_n}{\bar{H}_o \bar{R}_b} \left[ 1 + 0.033 \cos \left( \frac{360\bar{n}}{365} \right) \right] \int_{0}^{\theta} \cos \theta \, dt.$$ \hspace{1cm} (11b)

Using the following expressions for $\bar{H}_o$ and $\bar{R}_b$[9],

$$\bar{H}_o = \frac{24 \times 3600}{\pi} G_n \left[ 1 + 0.033 \cos \left( \frac{360\bar{n}}{365} \right) \right] \cos \phi \cos \delta \sin \omega + \frac{\pi \omega_i}{180} \sin \phi \sin \delta,$$ \hspace{1cm} (12)

$$\bar{R}_b = \frac{\cos(\phi - \beta) \cos \delta \sin \omega + \frac{\pi \omega_i}{180} \omega_i \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \sin \omega + \frac{\pi \omega_i}{180} \sin \phi \sin \delta}$$ \hspace{1cm} (13)

the expression for $f$ then becomes

$$f = \frac{J}{\cos(\phi - \beta) \cos \delta \sin \omega + \frac{\pi \omega_i}{180} \omega_i \sin(\phi - \beta) \sin \delta}$$ \hspace{1cm} (14)

where

$$J = \frac{1}{2} \int_{\omega_i}^{\omega_f} \cos \theta f \, d\omega = \int_{0}^{\omega_f} \cos \theta f \, d\omega$$ \hspace{1cm} (15)

since both $f$ and $\theta$ are symmetric around solar noon. Substituting the expression for $\cos \theta f$[10] and the expression of $f$ using eqn (8) yields

$$J = \int_{\omega_i}^{\omega_f} \left\{ \sin(\phi - \beta) \sin \omega + \frac{\pi \omega_i}{180} \omega_i \sin(\phi - \beta) \sin \omega \right\} \frac{\sin \omega}{\lambda + \cos(\phi - \beta) \cos \omega} \, d\omega$$

$$\times \left\{ 1 - \frac{\Delta Y}{l_0} \sin \beta \cdot \frac{\sin \omega}{\lambda + \cos(\phi - \beta) \cos \omega} \right\} \frac{\sin \omega}{\lambda + \cos(\phi - \beta) \cos \omega} \, d\omega.$$ \hspace{1cm} (16)

Carrying out the above integration, the expression for $f$ given by eqn (14) becomes

$$f = \frac{1}{\cos(\phi - \beta) \cos \delta \sin \omega + \frac{\pi \omega_i}{180} \omega_i \sin(\phi - \beta) \sin \delta}$$

$$\times \left\{ C_1 \frac{\pi}{180} (\omega_2 - \omega_1) + C_2 (\sin \omega_2 - \sin \omega_1) - C_3 (\cos \omega_2 - \cos \omega_1) + C_4 [I(\omega_2) - I(\omega_1)] - C_5 \sin \left( \frac{\eta + \cos \omega_2}{\eta + \cos \omega_1} \right) \right\}$$ \hspace{1cm} (17)

Fig. 2. Presentation of the functions $F_1(\omega)$ and $F_2(\omega)$. 

Since both $f$ and $\theta$ are symmetric around solar noon.
Fig. 3. Presentation of the functions $F_3(\omega)$ and $F_4(\omega)$.

where

$$\eta = \frac{\tan \delta}{\cos \phi} \left( \sin \phi - \frac{\sin \beta}{\cos(\phi - \beta)} \right)$$  \hspace{1cm} (18a)

$$C_1 = \frac{\sin \delta}{\cos(\phi - \beta)} \left\{ \sin(\phi - \beta) \left[ \cos(\phi - \beta) - \cos \phi \frac{\Delta y}{L} \right] - \sin \beta \frac{\Delta y}{L} \right\}$$  \hspace{1cm} (18b)

$$C_2 = \cos \delta \left[ \cos(\phi - \beta) - \cos \phi \frac{\Delta y}{L} \right]$$  \hspace{1cm} (18c)

$$C_3 = \sin \beta \cos \delta \frac{\Delta y}{NW} \left[ \frac{\Delta y}{L} - \cos \phi \cos(\phi - \beta) - 1 \right]$$  \hspace{1cm} (18d)

$$C_4 = \cos \phi \sin \delta \frac{\Delta y}{L} \left[ \tan(\phi - \beta) - \tan \phi + \frac{\sin \beta}{\cos \phi \cos(\phi - \beta)} \right]$$  \hspace{1cm} (18e)

$$C_5 = \frac{\Delta y}{NW} \sin \beta \sin \delta \left\{ \left[ \tan(\phi - \beta) \cos \phi \right. \right.$$

$$+ \frac{2 \sin \beta}{\cos(\phi - \beta)} \sin \phi \left. \frac{\Delta y}{L} - \sin(\phi - \beta) \right\} - \frac{\sin \beta}{\cos \phi} \tan \phi \cos(\phi - \beta) \right\}$$  \hspace{1cm} (18f)

Fig. 4. Variation of $\vec{f}$ with $\Delta y/(NW)$ at $\phi = 0^\circ$, $\beta = 15^\circ$, and $L/\Delta y = 1$ and 1.5.
Monthly-averaged daily shading factor for a collector field

4. DAILY SHADING PERIOD

In the previous section the expression for \( \tilde{I} \) is found to depend on the boundaries \( \omega_1 \) and \( \omega_2 \) of the afternoon part of the shading period on the average day of the month. We shall now determine the expressions for \( \omega_1 \) and \( \omega_2 \).

4.1 Condition for \( G_1 \)

The angle \( G_1 \) is defined as an equivalent solar azimuth angle measured from south to west [6]. Thus, \( G_1 \) is symmetric in magnitude around solar noon but has negative sign before solar noon and positive sign in the afternoon. For shading to exist at any instance during a day, the angle \( G_1 \) must satisfy the following inequality indicated by eqn (4)

\[
-1 \leq \tan G_1 \leq 1.
\]

This condition is reduced to the following inequality, when only considering the afternoon period of the day

\[
0 \leq \sin \beta \frac{\Delta y}{NW} \sin \omega \leq \lambda + \cos(\phi - \beta) \cos \omega
\]

or

\[
0 \leq F_1(\omega) \leq F_2(\omega)
\]

where eqn (6A) is used to replace \( \tan G_1 \) in the in-
The coefficients $a$, $b$, and $c$ are given by the following expressions:

$$a = \sin^2(\phi - \beta) \left( \frac{\Delta y}{NW} \right)^2 + \cos^2(\phi - \beta) \quad (26a)$$

$$b = 2\lambda \cos(\phi - \beta) \quad (26b)$$

$$c = \lambda^2 - \sin^2(\phi) \left( \frac{\Delta y}{NW} \right)^2 \quad (26c)$$

### 4.2 Condition for $G_2$

As given in [6], the angle $G_2$ is an equivalent solar altitude angle. The angle $G_2$ is symmetric around solar noon. Using eqns (4) and (6b), the condition for $G_2$ to cause shading at a given instance in a day is that

$$\sin \beta \frac{\Delta y}{NW} \sin \omega = \lambda + \cos(\phi - \beta) \cos \omega. \quad (24)$$
Monthly-averaged daily shading factor for a collector field

Monthly-averaged daily shading factor for a collector field

\( \phi = 30^\circ \text{N}, \beta = 15^\circ, \text{ and } L/\Delta y = 1 \text{ and } 1.5. \)

**Fig. 9.** Variation of \( f \) with \( \Delta y/(NW) \) at \( \phi = 30^\circ \text{N}, \beta = 15^\circ, \text{ and } L/\Delta y = 1 \text{ and } 1.5. \)

\[
0 \leq \frac{\Delta y}{L} \cos \phi (\cos \omega - \cos \omega_2) \leq (\lambda + \cos(\phi - \beta) \cos \omega) \tag{27}
\]

or

\[
0 \leq F_3(\omega) \leq F_4(\omega) \tag{28}
\]

where:

\[
F_3(\omega) = \frac{\Delta y}{L} \cos \phi (\cos \omega - \cos \omega_2) \tag{29}
\]

\[
F_4(\omega) = \lambda + \cos(\phi - \beta) \cos \omega. \tag{30}
\]

By plotting the general trends for the functions \( F_3(\omega) \) and \( F_4(\omega) \) as shown in Fig. 3, the lower and upper limits \( \omega_1^* \) and \( \omega_2^* \) of the range of \( \omega \) to satisfy the inequality in eqn (28) could be determined in accordance with the relative values of \( F_3(0) \) and \( F_4(0) \), and \( F_3(\omega_1) \) and \( F_4(\omega_2) \) as follows:

a) \( F_3(0) > F_4(0) \) and \( F_3(\omega_1) > F_4(\omega_2) \); no possible shading, let

\[
\omega_1^* = \omega_2^* = 0. \tag{31a}
\]

b) \( F_3(0) > F_4(0) \) and \( F_3(\omega_1) < F_4(\omega_2) \):

\[
\omega_1^* = \cos^{-1} P, \quad \omega_2^* = \omega_2. \tag{32b}
\]

c) \( F_3(0) < F_4(0) \) and \( F_3(\omega_1) > F_4(\omega_2) \):

\[
\omega_1^* = 0, \quad \omega_2^* = \cos^{-1} P. \tag{32c}
\]

In the above equations the parameter \( P \) is the solution of the equation \( F_3 = F_4 \), and is given as follows

\[
P = \frac{(\Delta y/L) \cos \phi \cos \omega_2 + \lambda}{(\Delta y/L) \cos \phi - \cos(\phi - \beta)}. \tag{33}
\]

4.3 Shading range for \( \omega \)

As given by eqn (4), shading takes place only when both the inequalities given by eqns (20) and (28) for \( G_1 \) and \( G_2 \), respectively are satisfied. In this case, the shading period is symmetric around solar noon. In the post solar noon period, shading extends from \( \omega_1 \) to \( \omega_2 \) where \( \omega_1 \) and \( \omega_2 \) are given as follows

\[
\omega_1 = \max \{ \omega_1^*, \omega_2^* \} \tag{34}
\]

\[
\omega_2 = \max \{ \omega_1, \min(\omega_2^*, \omega_2^*) \}. \tag{35}
\]

The determination of the lower and upper limits \( \omega_1 \) and \( \omega_2 \) of the shading period (after solar noon) during

**Fig. 10.** Variation of \( f \) with \( \Delta y/(NW) \) at \( \phi = 30^\circ \text{N}, \beta = 30^\circ, \text{ and } L/\Delta y = 1 \text{ and } 1.5. \)
5. SIMPLIFIED EXPRESSION FOR THE CASE $\beta = \phi$

In many practical cases the collectors are tilted with tilt angle $\beta = \phi$. In this case, the expression for $f$ is determined following the same procedure outlined previously for the general case of $\beta = \phi$. The expression of $f$ in this case takes the following form

$$f = \frac{1}{\sin \omega_1} \left[ \left( 1 - \frac{\Delta y}{L} \cos \phi \right) \left( \sin \omega_2 - \sin \omega_1 \right) ight. + \frac{\Delta y}{L} \frac{\cos \phi \cos \omega_1 \pi}{180} \left( \omega_2 - \omega_1 \right) + \frac{\Delta y}{NW} \sin \phi \left( 1 - \frac{\Delta y}{L} \cos \phi \right) \left( \cos \omega_2 - \cos \omega_1 \right) 

+ \frac{\Delta y}{NW} \frac{\Delta y}{L} \sin \phi \cos \phi \cos \omega_1 \ln \left( \frac{\cos \omega_2}{\cos \omega_1} \right) \right] \quad (36)$$

where $\omega_1$ and $\omega_2$ are then given as follows:

a) $\omega_1 < 90$:

$$\frac{L}{\Delta y} > \cos \phi (1 - \cos \omega_1): \quad \omega_1 = 0,$$

$$\omega_2 = \min(\omega_{x1}, \omega_{x2}) \quad (37)$$

$$\frac{L}{\Delta y} < \cos \phi (1 - \cos \omega_1):$$

$\Omega_1 > \Omega_2$: No shading, let $\omega_1 = \omega_2 = 0$ (38a)

$\Omega_1 < \Omega_2$: $\omega_1 = \Omega_1$, $\omega_2 = \min(\omega_{x1}, \omega_{x2})$ (38b)

b) $\omega_1 > 90$:

$$\frac{L}{\Delta y} > \cos \phi (1 - \cos \omega_1): \quad \omega_1 = 0,$$

$$\omega_2 = \min(\omega_{y1}, \Omega_1, \Omega_2) \quad (39b)$$

$\Omega_1 > \Omega_2$: No shading, let $\omega_1 = \omega_2 = 0$ (39a)

$\Omega_1 < \Omega_2$: $\omega_1 = \Omega_1$, $\omega_2 = \min(\omega_{y1}, \Omega_1, \Omega_2)$ (39b)
The angles $\Omega_1$ and $\Omega_2$ are defined as follows

$$\Omega_1 = \cos^{-1} \left( \frac{\cos \phi \cos \omega_1}{\cos \phi - \Delta y/L} \right) \quad (40a)$$

$$\Omega_2 = \tan^{-1} \left( \frac{1}{\sin \phi \Delta y/NW} \right). \quad (40b)$$

The above expressions for $\Omega_1$, $\omega_1$, and $\omega_2$ are also verified by putting $\beta = \phi$ in the expressions for $\Omega_1$, $\omega_1$, and $\omega_2$ in Sections 3 and 4.

6. VERIFICATION OF THE RESULTS

A computer program is prepared to numerically evaluate the integral $J$ given by eqn (15) and $f$ from eqn (14). In this program the expression of $f$ given by eqn (4) is used, and the lower limit $\omega_1$ and the upper limit $\omega_2$ to make $f = 0$ are determined numerically in accordance with the conditions for $G_1$ and $G_2$ in eqn (4). The results for $\omega_1$ and $\omega_2$ coincide with those obtained by eqns (34) and (35). The value of $f$ obtained from the program is also the same as that obtained from the analytical expression by eqn (17).

7. GRAPHICAL PRESENTATION OF $f$

Values of $f$ calculated using eqn (17) with $\omega_1$ and $\omega_2$ are determined using eqns (34) and (35) for the

Fig. 13. Variation of $f$ with $\Delta y/(NW)$ at $\phi = 45^\circ$N, $\beta = 30^\circ$, and $L/\Delta y = 1$ and 1.5.

Fig. 14. Variation of $f$ with $\Delta y/(NW)$ at $\phi = 45^\circ$N, $\beta = 45^\circ$, and $L/\Delta y = 1$ and 1.5.

Fig. 15. Variation of $f$ with $\gamma/(NW)$ at $\phi = 45^\circ$N, $\beta = 60^\circ$, and $L/\Delta y = 1$ and 1.5.
Table 1. Summary of calculation results for a four month period

<table>
<thead>
<tr>
<th>Month</th>
<th>( N = 3 )</th>
<th>( N = 6 )</th>
<th>( \text{Error} )</th>
<th>( \text{Error} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H_{b,t} ) (MJ/m²)</td>
<td>( \bar{f} )</td>
<td>( H_{\bar{b},t} ) (MJ/m²)</td>
<td>( \bar{f} )</td>
</tr>
<tr>
<td>March</td>
<td>10.62</td>
<td>0.124</td>
<td>9.3</td>
<td>14.1</td>
</tr>
<tr>
<td>June</td>
<td>7.13</td>
<td>0.000</td>
<td>7.13</td>
<td>0.0</td>
</tr>
<tr>
<td>September</td>
<td>10.39</td>
<td>0.085</td>
<td>9.51</td>
<td>9.3</td>
</tr>
<tr>
<td>December</td>
<td>7.67</td>
<td>0.323</td>
<td>5.19</td>
<td>47.8</td>
</tr>
</tbody>
</table>

average day of each month as recommended by Klein[9], with the following parametric conditions

\[
\begin{align*}
\phi & \quad \beta \\
0^\circ & \quad 15^\circ \quad 1, 1.5 \quad 0.04 \rightarrow 4 \\
15^\circ \text{N} & \quad 15^\circ, 30^\circ \quad 1, 1.5 \quad 0.04 \rightarrow 4 \\
30^\circ \text{N} & \quad 15^\circ, 30^\circ, 45^\circ \quad 1, 1.5 \quad 0.04 \rightarrow 4 \\
45^\circ \text{N} & \quad 30^\circ, 45^\circ, 60^\circ \quad 1, 1.5 \quad 0.04 \rightarrow 4
\end{align*}
\]

The results are plotted in Figs. 4–15. As indicated by the figures, the monthly-averaged daily shading factor increases by the increase of latitude angle, tilt angle, and/or the ratio \( L/\Delta y \). Also, the value of \( f \) increases as the ratio \( \Delta y/(NW) \) is decreased. Although values of \( L/\Delta y = 1 \) and 1.5 are only given in the charts, other values could be obtained by interpolation or extrapolation of the plotted results.

8. AN ILLUSTRATIVE EXAMPLE

To demonstrate the use of the correction factor \( f \) to improve the accuracy of predicting the monthly-averaged daily radiation received by collector surfaces, consider several rows of FPC located at Madison, WI. Each collector has a width \( W = 0.5 \text{ m} \), length \( L = 1.5 \text{ m} \), and spacing between rows \( \Delta y = 1.5 \text{ m} \). Using the same data and the results of example 2.16.1[10], the monthly-averaged daily beam radiation received per unit collector area is given as follows

\[
\bar{H}_{b,t} = \left( 1 - \frac{\bar{H}_d}{\bar{H}_t} \right) \bar{H}_{b,t}
\]

where the effect of shading is excluded. Including the effect of shading of a preceding row then gives

\[
\bar{H}_{b,t} = \left( 1 - \frac{\bar{H}_d}{\bar{H}_t} \right) \bar{H}_{b,t}(1 - \bar{f}).
\]

The percentage error in predicting \( \bar{H}_{b,t} \) is

\[
\Delta \bar{H}_{b,t} = 100 \left( \bar{H}_{b,t} - \bar{H}_{b,t} \right) / \bar{H}_{b,t}\]

Table 1 summarizes the results of calculations for rows with three and six elements for four months within the year where \( \bar{f} \) is calculated using eqn (17) (it could be also determined by interpolation from Figs. 12 and 15). During the heating season (December) the error is about 47.8% for three elements rows and 93% for six elements rows. Increasing the spacing \( \Delta y \) between rows would decrease the effect of shading but with the penalty of increasing land area per unit collector area. An optimization procedure must then be followed by the designer searching for the optimum spacing between rows.

9. CONCLUSIONS

An analytical expression is derived to estimate the monthly-averaged daily shading factor for any row of collectors in a collector field. This expression is given by eqn (17) and could be applied for fields oriented toward the equator with collectors aligned behind each other and of \( N \) collectors in a row. To use the expression of \( f \) it is necessary to determine the solar hour angles at the start and end of the shading period, \( \omega_1 \) and \( \omega_2 \), respectively, on the afternoon part of the average day of each month. The expressions for \( \omega_1 \) and \( \omega_2 \) are given respectively by eqns (34) and (35). At the special case when the collectors are tilted with an angle equal to the latitude angle, i.e., \( \beta = \phi \), the expressions for \( f \), \( \omega_1 \), and \( \omega_2 \) are simplified to those given in Section 5.

NOMENCLATURE

\( C_6 \) coefficients defined by eqns (18B) to (18G)
\( F_1 \) function defined by eqn (21)
\( F_2 \) function defined by eqn (22)
\( F_3 \) function defined by eqn (29)
\( F_4 \) function defined by eqn (30)
\( \bar{f} \) instantaneous shading factor, see eqn (3)
\( \bar{f} \) monthly-averaged daily shading factor
\( G_{0e} \) extraterrestrial solar irradiance on a horizontal surface
\( G_k \) solar constant
\( G_1 \) equivalent solar azimuth angle defined by eqn (5A)
\( G_2 \) equivalent solar altitude angle defined by eqn (5B)
\( \bar{H} \) monthly-averaged daily total radiation on a horizontal surface
\( \bar{H}_b \) monthly-averaged daily beam radiation on a horizontal surface
\( \bar{H}_d \) monthly-averaged daily diffuse radiation on a horizontal surface
\( \bar{H}_{0e} \) monthly-averaged daily extraterrestrial solar radiation on a horizontal surface
\( \bar{I} \) function defined by eqn (19)
\( J \) integral defined by eqn (15)
\( W \) length of a single collector
\( M \) number of days in a month
\( N \) number of collectors in a row
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