Finite Lambertian source analysis of concentrators: application to solar reflectors

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The maximum power cast on a collector from a source of finite angular extension by a concentrator of fixed position occurs when the collector sees the concentrator as a Lambertian source. Concentrators not fully meeting this requirement are evaluated using a parameter called the shape quality factor. Lambertian concentrators can be obtained with mirrors but not with lenses of finite $n$. In many cases they are not ideal, and some rays are cast outside the collector leading to an intercept factor below unity. Among those mirrors with the highest intercept factor, shape inaccuracies reduce this parameter and the shape quality factor to the extent which is analyzed. Rules for effective cost design are given leading to the conclusion that Lambertian concentrators must be used if the collector cost is high, while ideal concentrators should be used if the concentrator cost is high.

1. Introduction

The solar source for one-axis tracking concentrators can be considered as an extended source due to tracking errors (and to a lesser extent to the sun’s finite diameter). If an on–off mechanism is used for tracking, any time the sun is outside an interval $\Delta \phi$, the tracking mechanism passes to the on state. If $\Delta \phi$ is small, this interval $\Delta \phi$ is traversed at constant relative velocity by the sun, and the source can be represented by the infinite $\theta$-averaged angular restricted Lambertian source considered in Appendix A.

A concentrator works under the principle of transforming the rays coming from the source with relatively small angular spread into rays coming from the concentrator into the collector with higher angular spread. The higher is this angular spread, the higher the concentration will be. In this way the concentrator can be considered as a secondary source, and two factors now influence the power cast into the collector. First, the position of the concentrator that must surround as much as possible the collector to produce the highest angular spread. Second, this secondary source, as shown in this paper, must be Lambertian or much approach as near as possible to that condition. It is clear that if both conditions are fulfilled, the collector becomes isotropically illuminated, which is the condition for maximum power in the collector.2

Classical compound parabolic concentrators (CPC) and related concentrators are able to achieve both conditions, but if $\Delta \phi$ is small they become very deep. More practical concentrators are those which present the concentrating surface normals close to the sun’s direction. Of course, these concentrators do not surround the collector. If higher power is to be cast into the collector, a second stage must be designed which considers the first stage as a secondary finite Lambertian source.3

In Sec. II we analyze the figures of merit of a given concentrator with respect to its behavior in casting the incident power into the collector (intercept factor $I$) and with respect to its ability to illuminate the collector as a Lambertian source (shape quality factor $Q$).

Restricting our interest to reflecting concentrators, a previous analysis4 has analyzed the shape and relative position of a reflecting concentrator when all the rays impinging on the mirror are cast into the collector ($I = 1$). Mirrors fulfilling this condition are called ideal. Inaccuracies in the concentrator’s shape lead some rays to be cast outside the collector. The intercept factor of these concentrators ($I < 1$) is considered in Sec. III.

A result of the present work is that ideal concentrators casting all the rays into the collector allow for an optimal use of the mirror surface but prevent the mirror to be seen as Lambertian by the collector. On the other hand Lambertian concentrators cast maximum power into the collector with an efficient use of its surface. A cost effective optimum is obtained in Sec. IV which depends on cost considerations of both collector and concentrator.

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II. Theoretical Bases

Let us consider a cylindrical Lambertian source $S_L$ of infinite length with a $\theta$ independent brightness $B(\phi)$ (see the system of coordinates in Fig. 8). The source outline is represented in Fig. 1 (labeled mirror). The power cast by this source per unit of length on an elementary collector strip of infinite length and width $d l'$ is (see Appendix A)

$$dW_1 = dl' \frac{BA}{2} \int d\phi' \cos \phi',$$  

where $B_A$ is the $\theta$-averaged brightness described in Appendix A, and $\phi'$ is the angle of the projection of the rays on the plane of the paper with respect to the normal to the collector surface. By using the ray direction variable $p'$ defined as

$$p' = \sin \phi',$$  

we can write the power cast by the Lambertian source on a unit of length of the collector $O O'$ as

$$W_L = \frac{\pi}{2} B_A \int_{D_e} dl'dp' = \frac{\pi}{2} B_L E_L,$$  

where the domain of integration in the $l',p'$ space include the $p'$ directions of the projected rays issuing from point $l'$ and reaching the Lambertian source at every $l'$ position; $E_L$ is the étendue (or Lagrange invariant) at the collector.

Let us consider now a cylindrical concentrator formed by the same collector $O O'$ and a mirror $A A'$ used instead of the Lambertian source (Fig. 1). The light source $S_l$ is now a source placed at infinity, with a constant $\theta$-averaged brightness $B_s$ within the angle interval $(\phi_0, \phi_0 + \Delta\phi)$ and zero elsewhere.

As represented in Fig. 1, the angles $\phi$ are now considered with respect to the normal to the entry aperture $A A'$. The power cast by this source per unit length of entry aperture is (see Appendix A)

$$W_c = \frac{\pi}{2} B_s \int_{0}^{L_e} dy \int_{D_c} dp' = \frac{\pi}{2} B_s L_e = \frac{\pi}{2} B_s E_c.$$  

where $p = \sin \phi$, and $p_0$ and $p_0 + \Delta p$ are the values of $p$ corresponding to the source extreme rays $\phi_0$ and $\phi_0 + \Delta \phi$. The integrals in Eq. (4) are the étendue at the entry aperture for the source under study.

In the rest of this paper we shall use the word ray to refer to the class of rays having a common projection in the plane of the paper (plane normal to the cylinder's generatrices). Such a ray is defined by its coordinates $(y,p)$ at the entry aperture. The same ray strikes the mirror at a point defined by the arc length $l$ from the origin $A$ and the sinus $p_m$ of the angle $\phi_m$ with the normal to the mirror at $l$. We shall call these coordinates $(l,p_m)$. At the mirror the ray is reflected and sent toward the collector. If it actually reaches the collector it can be defined again by the coordinates $(l',p')$ (which is the length of arc at the collector from origin $O$) and $p'$ defined in Eq. (2). Ray-tracing establishes a correspondence among the spaces $(y,p)$, $(l,p_m)$, and $(l',p')$. It can be shown that

$$dx dp = dl dp_m = dl' dp'.$$  

Since all the rays impinging on the entry aperture reach the mirror we can also write the étendue $E_c$ of Eq. (4) on the mirror as

$$E_c = \int_{D_c} dl dp_m.$$  

This integral cannot be separated into the product of two integrals as in Eq. (4), since for each point $l$ the normal of the mirror is different, and so they are the coordinates $p_m$ of the existing rays. $D_e$ represents the domain of existing rays at any position on the mirror.

The power reaching the collector per unit length after reflection on the lossless mirror is

$$W_c = \frac{\pi}{2} B_s \int_{D_c} dl dp_m t(l,p_m) = \frac{\pi}{2} B_s E_c,$$  

where $t(l,p_m)$ is a function defined as one when the ray $(l,p_m)$ reaches the collector and as zero otherwise.

Again we can write the integral of Eq. (7) using the new function $t_{-1}^{-1}(l',p')$. This function is defined as one for each ray that, issuing from the collector, reaches the source after reflection on the mirror and zero otherwise. If $(l',p')$ is the point corresponding to $(l,p_m)$ it is clear that $t(l,p_m)$ and $t_{-1}^{-1}(l',p')$ are one simultaneously. Therefore using Eq. (5) we can write

$$W_c = \frac{\pi}{2} B_s \int_{D_c} dl dp_m t_{-1}^{-1}(l',p') = \frac{\pi}{2} B_s E_c,$$  

where $D_c$ now represents all the rays that, issuing from the collector, reach the mirror whether they reach the source after reflection there or not. $D_e$ is the same domain of integration used in Eq. (3).

We can define now the optical intercept factor as the fraction of power collected by the lossless mirror that is cast on the collector: this fraction is

$$I = \frac{W_c}{W_c} = \frac{E_c}{E_e} \leq 1.$$  

![Fig. 1. Mirror and collector outlines.](image-url)
The inequality can be easily deduced from Eqs. (4), (6), and (7). The equality occurs when \( t(l,p_m) = 1 \) for all rays. Concentrators with \( I = 1 \) are called ideal. Also we see that the maximum power that can be cast on the collector for a given mirror outline occurs when \( t^{-1}(l',p') = 1 \) for all \((l',p')\). In that case \( W_c \) becomes \( W_{L} \), i.e., the energy cast by a Lambertian source of the same outline [see Eq. (3)]. A shape quality factor \( Q \) can now be defined as

\[
Q = \frac{W_c}{W_L} = \frac{E_c}{E_L} \leq 1. \tag{10}
\]

It is easy to show that under isotropic illumination (\( \Delta \phi = \pi \)) any mirror becomes a Lambertian source of normal luminance \( B_e \). Therefore we can define again the shape quality factor \( Q \) as the ratio of power cast by the mirror on the collector, when illuminated by the source of restricted angular extension to that cast when illuminated by an isotropic source.

A function \( t_s(y,p) \) can be defined on the entry aperture so that \( t_s(y,p) = t(l,p_m) \). With this function the étendue at the collector can be written also as [see Eq. (5)]

\[
E_c = \int_0^{\phi_s+\Delta \phi} dp \int_0^{l_s} dy t_s(y,p), \tag{11}
\]

and a directional intercept factor can be defined, according to Jones, as [see Eqs. (4) and (11)]

\[
\alpha(p) = \frac{\partial E_c}{\partial p} = \frac{1}{L_e} \int_0^{l_s} dy t_s(y,p). \tag{12}
\]

By considering the definition of \( I \) [Eq. (9)] it can be seen that it is the average value of \( \alpha(p) \):

\[
I = \frac{1}{\Delta \phi} \int_0^{\phi_s+\Delta \phi} \alpha(p) dp. \tag{13}
\]

We want to note that the straight-line entry aperture has been selected to use ray direction coordinates that are the same regardless of the mirror shape. This is not possible if we use the coordinates \( p_m \) there. A local Lambertian illumination degree \( \Lambda(l') \) can be defined at the collector as [see Eqs. (3) and (8)]

\[
\Lambda(l') = \frac{\partial E_c}{\partial l'} = \frac{1}{p_1(l') - p_2(l')} \int_{l'}^{p_2(l')} t^{-1}(l',p') dp', \tag{14}
\]

where \( p_1(l') \) and \( p_2(l') \) are the extreme values of \( p \) when the mirror is seen from the point \( l' \) at the collector. It is easy to show, by considering the definition in Eq. (10), that \( Q \) is the weighted average of \( \Lambda(l') \):

\[
Q = \frac{\int_0^{1} \Lambda(l') [p_1(l') - p_2(l')] dl'}{\int_0^{1} [p_1(l') - p_2(l')] dl'}. \tag{15}
\]

Finally a local mirror shape quality factor \( M(l) \) can be defined as

\[
M(l) = \frac{\partial E_c}{\partial l} = \frac{1}{p_{m1}(l) - p_{m2}(l)} \int_{p_{m1}}^{p_{m2}} t(l,p_m) dp_m, \tag{16}
\]

where \( p_{m1}(l) \) and \( p_{m2}(l) \) are the extreme values of \( p_m \) reaching the collector from \( l \).

Again \( Q \) is the weighted average of \( M(l) \):

\[
Q = \frac{\int_0^{L} M(l)[p_{m1}(l) - p_{m2}(l)] dl}{\int_0^{L} [p_{m1}(l) - p_{m2}(l)] dl}. \tag{17}
\]

Equations (14) and (17) allow for the local analysis of collector and mirror. Regions with low \( \Lambda(l') \) or \( M(l) \) can be removed with increase of \( Q \).

If \( I \) is known, \( Q \) can be easily calculated. In fact, using Eqs. (10), (11), and (13)

\[
\frac{L_e}{E_L} \int_0^{p_{m1}+\Delta p} \alpha(p) dp = \frac{L_e}{E_L} \Delta p I. \tag{18}
\]

In this equation \( E_L \) can be calculated regardless of the specific mirror shape using the equation

\[
E_L = \frac{\Delta A}{\Delta A_\infty} + \frac{\Delta A}{\Delta A_\infty} - \frac{\Delta A}{\Delta A_\infty}, \tag{19}
\]

which only depends on the position of the mirror ends with respect to the collector.

As can be easily shown a mirror under full isotropic illumination becomes a Lambertian source so that \( Q = 1 \). This can be used to check our calculation of \( \alpha(p) \) since

\[
L_e \int_0^{1} \alpha(p) dp = E_L. \tag{20}
\]

If the concentrator is a Fresnel lens instead of a mirror, Eq. (20) is not valid since a lens is not a Lambertian source even under isotropic illumination (unless it has an infinite refractive index).

The concentrator's geometrical gain \( G_0 \) is merely the ratio of concentrator entry aperture \( L_e \) to collector width \( L_o \) \( (L_e = \overline{O' \prime A}) \). The concentrator's optical gain \( G_0 \) is the ratio of the energy cast on the collector by the lossless mirror to the energy on the collector if placed outside the mirror with the same orientation as its entry aperture. According to Eqs. (4), (8), and (9)

\[
G_0 = \frac{E_e}{L_e \alpha_p} = IG_e. \tag{21}
\]

This equation is also valid for two-stage concentrators provided that the second stage is an ideal concentrator which considers the first stage a finite Lambertian source.

III. Directional Intercept Factor of Mirrors with Shape Inaccuracies

Ideal high concentration reflecting concentrators have been analyzed elsewhere. As represented in Fig. 2 they are arcs of a parabola of vertical axis, vertex at \( A \), and focus at \( O' \) \( (AO' \infty) \) unity and arbitrary length. Once the parabola ending in, say, \( A' \) has been built, the other collector end \( O' \) must be on the circle passing through \( OAA' \) of center \( C \) so that angle \( OCO' \) is \( 2 \Delta \phi \). Obviously this concentrator can be arbitrarily scaled up or down. As well as this trivial degree of freedom the concentrator has another degree of freedom which is the absissa \( D \) of \( A' \). This absissa is the inverse of the concentrator's \( f \) No.; in other words \( D \) is the mirror luminosity.

This concentrator is ideal in the sense that all the rays in the interval \( \Delta \phi \) are cast into the collector. At the
same time this concentrator achieves the maximum possible geometrical gain for a shallow mirror compatible with ideality. This collector can also be the entry aperture of a second-stage concentrator designed for a finite Lambertian source. 

Technological inaccuracies modify the ideal shape considered before. We have restricted our analysis to the family of conic curves tangent to the ideal parabola at points $A$ and $A'$. In Appendix B we give the mathematical details of these curves. A manufacturing error is defined as (Fig. 2)

$$\varepsilon = \frac{EP}{MP} \approx \frac{HP}{NP}.$$  \hspace{1cm} (22)

For these profiles we have calculated the directional intercept factor $\alpha(\psi)$ (see Fig. 1 for definition of angles). For that [see Eq. (12)]

$$\alpha(\psi) = \frac{\partial E_c/\partial \psi}{\partial E_c/\partial \phi},$$

using the definition of $E_c$ on the mirror surface [Eq. (7)] and the change of variables,

$$\rho_m = \text{sen} \phi_m = \text{sen}(\psi + \gamma),$$  \hspace{1cm} (23)

$$dl = dx/cos\gamma,$$  \hspace{1cm} (24)

so that

$$\frac{\partial E_c}{\partial \psi} = \int_0^D dx \frac{\cos(\psi + \gamma)}{\cos \gamma} t(x,\psi) dx,$$  \hspace{1cm} (25)

using the definition of $E_c$ on the entry aperture [Eq. (4)] and the change of variables

$$p = \text{sen} \phi = \text{sen}(\psi + \beta),$$  \hspace{1cm} (26)

$$dy = dx/cos\beta,$$  \hspace{1cm} (27)

we obtain

\begin{align}
\alpha(\psi) &= \frac{\partial E_c}{\partial \psi} = D \cos(\psi + \beta) \\
&= \frac{\cos \beta}{\cos \gamma} t(x,\psi) \frac{dx}{D}.
\end{align}

(29)

A numerical integration of Eq. (29) has been done for several cases. Typical shapes of $\alpha(\psi)$ are presented in Fig. 3. It is observed that a fraction of rays outside the acceptance angle $\psi_i$ are collected by the concentrator $[\alpha(\psi) > 0]$. At the same time if some degree of manufacturing error is tolerated, some rays within the acceptance angle are rejected $[\alpha(\psi) < 1]$. Both effects are more important in low acceptance mirrors. We present in Fig. 4 the interval of $\psi$ corresponding to a value of $\alpha(\psi)$ higher than a certain minimum for several values of the luminosity $D$ and of the manufacturing error.

Equation (20) shows the relationship between $E_L$ calculated from the geometrical positions of the mirror extremes $A$ and $A'$ (with respect to $OO'$) and the integral of $\alpha(p)$ extended to all the angles no matter what the specific mirror shape. This relationship has been verified in all cases.

IV. Intercept Factor and Shape Quality Factor: Ideal Concentrators and Lambertian Concentrators

In Fig. 5 we represent $\alpha(p)$ for a certain concentrator with manufacturing errors. According to Eq. (13) the dotted area in this figure determines the reduction of $I$. This curve is similar to those in Fig. 3 except that the
Fig. 4. Percent of \( \Delta \psi \) corresponding to a value of \( \alpha(\psi) \) higher than a certain minimum for several values of error \( \epsilon \) and luminosity \( D \).

Fig. 5. Acceptance function \( \alpha(\psi) \) for a concentrator with an error for \( \Delta \psi = 5^\circ \) corresponding to \( \Delta \psi_0 = 0.0872 \).
variable in the abscissa has been changed. From the shape of $\alpha(\psi)$ it is clear that the $\varepsilon$-free concentrators are ideal.

Once $\alpha(\psi)$ is known with the procedure in the preceding section the intercept factor $I$ can be calculated using Eq. (13). This equation can be written in terms of $\psi$ using the change of variables of Eq. (26) as

$$I = \frac{\int_0^{\phi} \alpha(\psi) \cos(\psi + \beta) d\psi}{\sin(\phi + \beta) - \sin \beta}.$$  

(30)

For reflecting concentrators (not for lenses) Eqs. (18) and (20) can be combined to give the quality factor $Q$:

$$Q = \frac{\int_{-\phi}^{\phi} \alpha(p) dp}{1 - \int_{-\phi}^{\phi} \alpha(p) \cos(\phi) dp}.$$  

(31)

The dashed area in Fig. 5 is responsible for the $G$ reduction. It is clear now that all the concentrators in Fig. 3 are not Lambertian for the tracking step $\Delta \psi_i$. For that reason the optical gain of the ideal concentrators [which, since $I = 1$ equates their geometrical gain according to Eq. (21)] is below its maximum achievable value.

The non-Lambertian characteristics of the ideal mirrors in this work can also be explained using the concept of local Lambertian illumination degree $\Lambda(l')$ of Eq. (14) and the concept of mirror shape quality factor $M(l)$ of Eq. (16). In Fig. 6 we represent the $\varepsilon$-free concentrator for tracking step $\Delta \psi_i$. Since the mirror point $M$ is an interior point of the circle $C_1$ (which subtends $O\alpha'$ with an angle $\Delta \psi_i$), the beam of angle $\Delta \psi_i$ (dashed) incident and reflected by the point $M$ is represented in the figure. Not all the points of the collectors are illuminated by the mirror, for example, point $N$. Therefore some values of $t(l_m, p_m) = 0$ and $M(l_N) \leq 1$.

At the same time, the collector point $N$ is not illuminated by all the mirror points leading to some values of $t^{-1}(l_N, p') = 0$. In particular this happens with the mirror point $M$ which is seen dark by the collector point $N$. Therefore $\Lambda(l_N) < 1$. The same will occur with the average value $Q$ of $M(l)$ and $\Lambda(l')$ leading to non-Lambertian characteristics.

If the tracking error is now increased (see Fig. 5), the shape quality factor $Q$ increases, so that they both become eventually Lambertian ($Q = 1$). At the same time the intercept factor decreases becoming less ideal. The opposite is true when the tracking error is reduced. In Fig. 7 we present a set of curves of $Q$ and $I$ vs $\Delta \psi$. We observe that Lambertian concentrators are always obtained for high $\Delta \psi$, even if manufacturing errors exist. This is a consequence of the fact that any mirror illuminated by a hemispheric source becomes Lambertian. Ideality is not always achieved for low $\Delta \psi$ since for high manufacturing errors $\alpha(\psi) < 1$ for all $\psi$.

Taking into account the shape of $\alpha(\psi)$ in Fig. 3 (or the results presented in Fig. 7), the least increase of $\Delta \psi$ to achieve a Lambertian concentrator is obtained with $\varepsilon$-free outlines. Since for a given position of $O\alpha'AA'$ the $E_{\ell}$ is constant, according to Eq. (18), the $\varepsilon$-free outline is the concentrator with the lowest $\alpha(\psi)$ value to become Lambertian ($Q = 1$) and therefore the one with the highest $I$ among those with given position $O\alpha'AA'$.

This Lambertian concentrator is represented in Fig. 6. Now all the mirror points must be outside the circle $C_i$ subtending the new tracking error $\Delta \psi_i$ to insure illumination of the whole collector. Obviously some rays do not now reach the collector leading to $I < 1$, as predicted from the observation of $\alpha(\psi)$.

Fig. 6. Error-free concentrator for tracking step $\Delta \psi_i$ and the Lambertian concentrator. The new tracking step is $\Delta \psi_e$.

Fig. 7. Intercept factor, quality factor, and parameter of cost $|\log [C_{\text{cell}}/(C_{\text{con}} \cdot G_{\text{OM}})]|$ vs angle $\Delta \psi$. 4198
In consequence it is possible to achieve Lambertian concentrators leading, by using two-stage configurations, to the maximum optical gain (the thermodynamical limit) as the Winston mirror does. These concentrators are not ideal, and therefore [see Eq. (21)] their entry aperture is bigger than that of the corresponding Winston mirrors. However, because of the deep outline of the Winston mirrors for low $\Delta \psi_i$ the two-stage mirror uses less reflecting area. We must recall that only deals with the reduction of entry aperture, while $Q$ is related to the power cast on the collector.

V. Cost Effective Concentrator Design for Photovoltaic Systems

In photovoltaic applications the objective of concentration is to reduce the system’s cost. In general, the tracking error $\Delta \psi$ is a datum for the problem. A reduction of this error can lead to a huge cost increase. The cost of the concentrating system can be written

$$C(\$/W) = \left[ \frac{C_{\text{conc}}(\$/m^2) + C_{\text{cell}}(\$/m^2)/G_{OM}}{\eta_{\text{cell}} \cdot \eta_{\text{ref}} \cdot I} \right],$$

where $\eta_{\text{cell}}$ and $\eta_{\text{ref}}$ are constant related to the cell efficiency and the reflection efficiency of the optical surfaces, while $I$ is the intercept factor, i.e., the efficiency taking into account the energy effectively cast into the collector.

If the concentrator is two-stage with the second stage adapted to the first, considered as a Lambertian source, Eq. (32) can be written, with the help of Eqs. (21) and (10), as

$$C_{\text{ref}} \eta_{\text{ref}} = \frac{C_{\text{conc}}}{I} + C_{\text{cell}} \frac{1}{G_{OM} \cdot Q},$$

where $G_{OM}$ is the maximum achievable optical gain (the thermodynamical limit), which is only a function of $\Delta \psi$.

For an $I$-free concentrator the curves $I(\Delta \psi)$ and $Q(\Delta \psi)$ are approximately the same no matter the specific value of $\psi_i$ (with an error of $<5\%$ for $\Delta \psi_c < 20\%$) for a given value of $D$. In consequence the minimum condition of Eq. (33) is

$$F(\Delta \psi) = \frac{d I/I}{d(\Delta \psi)} = \frac{1/\Delta \psi}{d(\Delta \psi)} = \frac{C_{\text{cell}}}{C_{\text{conc}} G_{OM}}.$$

The curve on the right-hand side of this equation has been represented vs $\Delta \psi/\Delta \psi_i$ in Fig. 7 together with the curves of $I$ and $Q$. Depending on the value of $C_{\text{cell}}/C_{\text{conc}} G_{OM}$ whose logarithm is represented in ordinates, the corresponding value of $\Delta \psi/\Delta \psi_i$ is obtained.

For example, if $G_{cell}/C_{conc}G_{OM}$ is very low (cell cost low), the cost effective design corresponds to $\Delta \psi$ close to $\Delta \psi_i$, and the ideal concentrator becomes the cost effective one. This type of concentrator is that which better uses the concentrator surface. On the other hand, if $C_{\text{cell}}/C_{\text{conc}} G_{OM}$ is very high (cell cost high), the most effective design is that with $\Delta \psi = \Delta \psi_i$, approaching the Lambertian condition. Obviously this design is the one which makes better use of the collector. In general by selecting $\Delta \psi$ from both extreme values the most cost effective concentrator is obtained.

VI. Conclusions

We have demonstrated that a concentrator seen by the collector as a Lambertian source is that which casts maximum energy on the collector. The shape quality factor $Q$ is a figure-of-merit to measure the extent to which a concentrator approaches this condition. The additional condition to cast maximum energy in the collector required to achieve a surrounding structure around the collector has been treated adequately elsewhere and is beyond the scope of the present work.

The effectiveness of using the mirror surface is measured through the intercept factor $I$. This parameter as well as $Q$ can be obtained from the directional intercept parameter $\psi$. Parameters giving the local properties of the concentrator and collector surface are also given.

The effect of manufacturing errors in ideal concentrators with highest gain designed for one-axis high concentration systems was analyzed. We conclude that all outlines become Lambertian when the tracking error is high enough leading to mirrors that cast the same power on the collector as a CPC mirror while using less mirror surface.

A method for cost analysis is presented leading to the conclusion that mirrors with $Q$ close to 1 (Lambertian) must be used for high cost collectors, while mirrors with $I$ close to 1 (ideal) should be used for collectors when cost is low compared with concentrator cost.

Appendix A: Two-Dimension Brightness of Linear Concentrator

Let us consider a Lambertian source of brightness $B(\theta)$ which has an arbitrary dependence on the polar coordinate $\theta$ in Fig. 8 but which is constant with the polar coordinate $\phi$. The power cast by this structure on a collector element is
\[ dW = \frac{dydz}{(\Omega A)^2 \cos(\theta A)} \left[ B(\theta) \cos(\theta A) \right] \cos(\Omega X A), \]

(A1)

where \( \Omega A \) is the normal to the source at \( A \), and \( \phi \) and \( \theta \) are the polar coordinates of point \( A \). The first set of brackets in the integrand is the source area intercepted by an element of solid angle. The second set deals with the power cast on the collector by the Lambertian source in the proper direction. The far right cosine deals with the angle of incidence on the collector. By simplifying and putting \( \cos(\Omega X A) \) in terms of \( \theta \) and \( \phi \), we can write

\[ dW = \frac{dydz}{(\Omega A)^2} \frac{\pi}{2} B_A \int_{\phi_1}^{\phi_2} \cos\phi d\phi \int_{\theta_1}^{\theta_2} B(\theta) \cos^2 \theta d\theta, \]

(A2)

where \( \phi_1 \) and \( \phi_2 \) are the extreme values of \( \phi \) at which the source is seen by the collector, and

\[ B_A = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} B_A(\theta) \cos^2 \theta d\theta. \]

(A3)

According to Eq. (A2) a finite or infinite 3-D source of \( \phi \)-independent brightness \( B(\theta) \) as described in this Appendix can be treated as a 2-D source of brightness \((\pi/2) B_A\).

By placing the source at infinity an infinite \( \theta \)-averaged Lambertian source is obtained with limited angular extension \( A\theta \) corresponding to the extreme values \( \phi_1 \) and \( \phi_2 \) of \( \phi \).

Appendix B: Outlines of the Mirrors Analyzed

The general equation of the family of curves under consideration in rectangular coordinates is

\[ SY^2 + TX^2 + UXY + VY = 0, \]

(B1)

where

\[ S = 1 - \lambda, \]

\[ T = -\lambda Y_A^2/X_A, \]

\[ U = m_A + 2\lambda Y_A/X_A, \]

\[ V = m_A X_A - Y_A, \]

\( \lambda \) is a parameter for selecting a given curve from the family, and \( X_A, Y_A, \) and \( m_A \) are the coordinates and slope of point \( A \), which are the following functions of the luminosity \( D \):

\[ X_A = D; \]

\[ Y_A = D^2/4; \]

\[ m_A = \tan(\tan^{-1} [X_A/(1 - Y_A)/2]). \]

(B3)

(B4)

(B5)

For \( \lambda = 1 \) the optimal parabola is obtained. For values of \( \lambda \) close to 1, ellipses or hyperbolas are obtained depending on whether \( \lambda > 1 \) or \( \lambda < 1 \) corresponding to curves above or below the parabola, respectively.

The manufacturing error corresponding to a given curve of the family, defined in Eq. (22), is related to the family parameter \( \lambda \) by

\[ e = (\lambda^{1/2} - 1)/(\lambda^{1/2} + 1). \]

(B9)

The manufacturing error can be defined also as \( \varepsilon = EP/D \) or \( \varepsilon = HP/D \). In this case

\[ \varepsilon = \frac{\varepsilon/16}{(1 + D^2/16)^{1/2}}. \]

(B10)

References