

Physics 570B, Spring 2011
Assignment 01
Due Tuesday, Jan. 25

These problems are not really quantum mechanics problems, they are reviews of some classical mechanics concepts that are background to some of the things we will talk about in the first few lectures.

- Show that the principle of least action, or $\frac{\delta S}{\delta q(t)} = 0$, leads to Lagrange's equation for a nonrelativistic particle moving in a potential $V(q)$. Here δ is the variational derivative. In doing this, draw a picture or two to illustrate a variation of the path $q(t)$ and how you handle the derivative in the action.
- Show that the **time dependent** Schroedinger equation is invariant under a gauge transformation, including the change of phase of the wave function:

$$\begin{aligned}\Psi(\vec{x}, t) &\rightarrow \Psi(\vec{x}, t)e^{ief(\vec{x}, t)/(\hbar c)} \\ \vec{A}(\vec{x}, t) &\rightarrow \vec{A} + \vec{\nabla}f(\vec{x}, t) \\ \phi(\vec{x}, t) &\rightarrow \phi - \frac{1}{c} \frac{\partial f}{\partial t}\end{aligned}\tag{1}$$

- Show that the classical Hamiltonian for a particle in a time independent magnetic field, $H = \frac{1}{2m} \left(\vec{p} - \frac{e}{c} \vec{A}(\vec{x}) \right)^2$ gives the expected equation of motion, $m \frac{d^2}{dt^2} \vec{x} = \frac{v}{c} \times \vec{B}$ using Hamilton's equations, $\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i}$.

Hints: Remember that p_i is a canonical momentum, and is not the same as mv_i . (v_i is $\frac{dx_i}{dt}$). Also remember that when you differentiate $\vec{A}(\vec{x})$ you are differentiating it at the position of the particle, which is changing with time.