

Physics 570B, Spring 2011
Assignment 03
Due Tuesday, Feb. 8

1. A clever problem from *Modern Quantum Mechanics*, by J.J. Sakurai:
An electron moves in the presence of a uniform magnetic field in the \hat{z} direction, $\vec{B} = b\hat{z}$.
 - (a) Evaluate $[\Pi_x, \Pi_y]$, where $\Pi_x \equiv p_x - \frac{eA_x}{c}$ is the x component of the “mechanical momentum”, etc. (p_x is the canonical momentum, and it is the canonical momentum that satisfies $[x, p_x] = i\hbar$, and is represented by $-i\hbar \frac{\partial}{\partial x}$ in the coordinate representation.)
 - (b) By comparing the Hamiltonian and the commutation relation obtained above with those of the one-dimensional harmonic oscillator, show that we can immediately write the energy eigenvalues as

$$E_{k,n} = \frac{\hbar^2 k^2}{2m} + \left(\frac{|eB|\hbar}{mc} \right) \left(n + \frac{1}{2} \right) \quad (1)$$

where $\hbar k$ is the continuous eigenvalue of the p_z operator and n is a non-negative integer. *You don't have to be too rigorous here – just show that the two problems have the same form after a simple rescaling of the operators. After all, we have done this problem in class, and even counted the solutions for each energy.*

2. *Baym* problem 3.11 (page 80).
3. *Baym* problem 3.12 (page 81). This is an electrostatic analogue of the magnetic Aharonov-Bohm effect discussed in class, since the particles are never in a region with an electric field.
4. In this and the next few problems, I will write spin wave functions for two electrons. As usual, in each term the arrow on the left represents the first electron (or the one going to observer A), and the arrow on the right the second electron.
Is this state entangled? Explain.

$$\Psi_{spin} = \frac{1}{\sqrt{2}} (\uparrow\uparrow + \uparrow\downarrow) \quad (2)$$

5. Is this state entangled? Explain.

$$\Psi_{spin} = \frac{1}{\sqrt{2}} (\uparrow\uparrow + \downarrow\downarrow) \quad (3)$$

6. In the EPR experiment discussed in class, two electrons are emitted from a source, one going to observer A and the other to observer B. The spin wave function of the two electrons is

$$\frac{1}{\sqrt{2}}(\uparrow_z\downarrow_z - \downarrow_z\uparrow_z) \quad (4)$$

where the subscript indicates that we are using the basis of eigenstates of S_z , and it is understood that the first arrow corresponds to the electron going to observer A, and the second to the electron going to observer B.

Suppose observer A measures S_z and gets $\pm\frac{1}{2}\hbar$. What are the possible results, and the probability of each result, if observer B measures S_z ? What if observer B measures S_x ?

Now suppose observer A measures S_x and gets $\pm\frac{1}{2}\hbar$. What are the possible results, and the probability of each result, if observer B measures S_x ? What if observer B measures S_z ?

7. Repeat the previous problem, except change the sign in the spin wave function:

$$\frac{1}{\sqrt{2}}(\uparrow_z\downarrow_z + \downarrow_z\uparrow_z) \quad (5)$$

8. A beam of electrons is prepared with half of the electrons in the state \uparrow_z and half of the electrons in the state \uparrow_x . Find the density matrix.

9. *This counts as two problems*

Read the New York Times article linked from the course web site. Then, for context, read the first page of the Nature article by Simmons *et al.*. Don't worry about the details here, just figure out what state they are talking about. The answer is that their entangled state is just the entangled state of two spins that we used in our measurement example. Then read the first 1.5 pages of the Physical Review Letters paper by Peres (Phys. Rev. Lett. **77**, 1413 (1996)), also linked from the course web site. (You can stop at the point "As a second example"). Now explicitly write out the components of the density matrix $\rho_{m\mu,n\nu}$ and the partially transposed density matrix σ (Eqs. 7 and 4, respectively). Compute the eigenvalues of these two matrices, and verify the claim in the paper about when they are positive.

OK, I admit that most of this problem consists of sorting out the notation, but I thought it would be fun to give a problem from current literature. As a hint, note that when a matrix contains a row with only one nonzero element, that instantly tells you one eigenvector and eigenvalue. After reducing the problem in this way, you only have to think about two by two matrices. Write the matrices as a linear combination of the identity matrix, for which any vector is an eigenvector, and one of the Pauli matrices, whose eigenvectors and eigenvalues you know.