

0.1 Homework 7

Reminder: the assignment next week (homework 8, due October 19) will be the Midterm Project. That means all work should be strictly individual, as it would be for a take-home exam.

This week (homework 7) is normal.

Due: Monday, October 12, 2007 at 10:00 pm

This week you will be modeling the motion of objects in two dimensions. You will use the second order Runge-Kutta method to study the orbit of a planet about the Sun.

1. Write a program to solve for the orbit of the earth around the Sun. Use r in AU and time in years. See the class notes for hints on how to compute GM_o in these units. (M_o is the mass of the sun.) *You should not have to look up G or M_o .* Begin by showing that you get a circular orbit with the expected period. A time-step of 1 day is a reasonable first choice. Verify that energy and angular momentum are nearly conserved; report the level of non-conservation. Find the period of your orbit (perhaps by starting along the positive x axis, and finding the time where it again crosses the positive x axis). Show that the error in the period has the expected dependence on step size.

Include calls to `philplot` to animate the motion of your planet.

Note that since we are now solving for motion in two dimensions, the acceleration requires two values to return. There are several ways to do this, but a nice one is to pass arrays, e.g. with a prototype

```
void acceleration(double time, int ndim, double pos[], \
                 double vel[], double acc[]);
```

where `ndim` will be the length of the arrays. Type ‘void’ simply means that the function doesn’t have a return value; you exit the function with simply `return;`, not something like `return 0;`.

2. Show that circular orbits with radii 2 AU and 0.5 AU have the expected periods.
3. Now modify your initial conditions to model an elliptical orbit. Use the orbital equations given in the lecture notes to determine the initial conditions for a planet with a semi-major axis of 1 AU and eccentricity of 0.5. It is easy to set the initial conditions if you start at perihelion (the closest approach to the Sun); at this point, the velocity is purely tangential and can be computed from the equations in the notes. Again, check conservation of energy and angular

momentum. Make a plot showing the kinetic energy, the potential energy and the total energy as a function of time (all on the same plot.)

Use philplot to animate the motion of this elliptical orbit.

Consider the error in the energy as a function of the time step. What time step do you need to get the answer to a part in 10^4 ? What time step do you need if the orbit has $e = 0.9$? It is a curious fact that the leading order errors in the energy and momentum in the second order Runge-Kutta method average to zero over an orbit, so you should look at the difference between the maximum and minimum energy and angular momentum seen during an orbit. (You can plot these versus time, and read them off a graph.)

4. Make a plot of your orbit in velocity space, and verify that it has the expected form. Again, note that in doing this it is important that the s and y ranges of your graph be the same, otherwise it will look distorted.
5. Now alter the central force law to be

$$F = -\frac{GMm}{r^2} + \frac{\alpha GMm}{r^3} \quad (1)$$

where $\alpha = 0.1$ AU. Use the initial positions and velocities appropriate for $e = 0.5$ and $a = 1$ AU (for the normal $\alpha = 0$ case). Make a plot of the position, covering at least 10 orbits.