Physics 570B, Spring 2011

Assignment 09

Due Tuesday, Mar. 29

Our midterm exam will be on Thursday, March 24. You may bring two pages of hand-written notes to this exam. You may not use calculators, cell phones or other aids during the exam.

Because of the midterm exam, this is a short assignment. It consists of a couple of problems reviewing the probability current, and some problems that are variants of problems that caused trouble in homework assignment 7. Specifically, many people did not understand the variational method well, or need some practice in calculation, and many people had trouble with thinking about eigenstates of angular momentum in Cartesian coordinates.

1. Show that the probability density $\rho = \Psi^*\Psi$ and the probability density current

$$\vec{j} = \frac{-i\hbar}{2m}\left(\Psi^*\left(\vec{\nabla}\Psi\right) - \left(\vec{\nabla}\Psi^*\right)\Psi\right) \tag{1}$$

satisfy the conservation equation

$$\frac{\partial\rho}{\partial t} = -\vec{\nabla}\cdot\vec{j} \tag{2}$$

for a nonrelativistic particle moving in a potential (in three dimensions).

2. Consider non-relativistic quantum mechanics in one dimension. A particle with momentum $\vec{p}$ (a plane wave) is incident from the left, and hits a discontinuity in the potential,

$$\begin{aligned} V(x) &= 0 \quad (x < 0) \\ V(x) &= V \quad (x > 0) \end{aligned} \tag{3}$$

where the energy of the incident particle is greater than V. Find the wave function, and show that probability is conserved in this problem.

3. The variational method problem in HW 7 didn't go so well, so we need a little more practice. This problem and the next problem each contain one obvious variational parameter. Please be sure to check dimensions — if you say something is an energy, be sure that it has dimensions of energy. And if you add two things together, be sure they have the same dimensions. This will catch a lot of errors.

I had to change this problem — the original potential led to an integral that you couldn't really do.
Consider a linear potential in three dimensions.

$$V(\vec{r}) = ar \tag{4}$$

Use a three dimensional harmonic oscillator wave function as a variational *ansatz*, and estimate the ground state energy. What is the variational parameter? Once you find the general expression, take $a = 1$ eV/Angstrom and give your answer in electron volts. Use $mc^2 = 511$ keV and $\hbar c = 1970$ eV-Angstrom.

4. Let's reverse the problem from HW 7:
   Use a hydrogen atom ground state wave function to estimate the ground state energy of a three dimensional harmonic oscillator. What do you think the variational parameter should be? Once you find the general expression, take $\hbar\omega = 10$ eV, and give your answer in electron volts.

5. In this problem, remember the in-class example of adding angular momenta, $1 \otimes 1 = 2 \oplus 1 \oplus 0$, and the discussion of this in terms of familiar objects in Cartesian coordinates. I know that you can do this problem using spherical harmonics, but that isn't the point — I am looking to see that you can understand this in a different way (and one that makes it trivial if you understand it)

   (a) What are the possible values for the total orbital angular momentum of two particles, each with orbital angular momentum one?

   (b) For two vectors $\vec{a}$ and $\vec{b}$, the cross product $\vec{a} \times \vec{b}$ is the (oriented) area of the parallelogram whose edges are $\vec{a}$ and $\vec{b}$. Show that this expression is completely antisymmetric under exchanges of the vectors.

   (c) $\vec{a}$ is a vector. What value of angular momentum does it correspond to? The expression above, $\vec{a} \times \vec{b}$, is a vector. What value of angular momentum does it correspond to?

   (d) The ground state configuration of carbon has two electrons in the 2P orbitals. What are the spin and orbital angular momenta of the ground state of carbon? What is the angular part of the wave function for these two electrons? Do this in Cartesian coordinates. Use $\vec{r}_1$ and $\vec{r}_2$ for the positions of the two electrons.