

Physics 570B, Spring 2011
Assignment 10
Due Tuesday, Apr. 5

1. At time $t = 0$ two lights are flashed, one at $x = 0$ and the other at $x = L$. Find the space and time coordinates, x' and t' of these flashes in a coordinate system moving with velocity v relative to the coordinate system in which the flashes are simultaneous. Do this problem using the four-vector (and 4×4 matrix) formalism used in the lectures. Since the point of this and the next problem are to practice with this formalism, this is a requirement.
2. Now consider the same two positions, 0 and L in the unprimed frame. This time there is a marker at each position. Find the distance between these two markers as seen in the primed frame. Do this by finding the position of the markers at a given time t' in the primed frame. Explain why this distance is different than the distance found in the previous problem. Again, you must use the four-vector (and 4×4 matrix) formalism.
3. If you perform a Lorentz boost to a reference frame moving with velocity v in the \hat{x} direction relative to the original frame, then a boost to a frame moving with velocity v in the \hat{y} direction relative to this new frame, then a boost to a frame moving with relative velocity $-v$ in the \hat{x} direction relative to this last frame, then a boost to a frame moving with relative velocity $-v$ in the \hat{y} direction, do you get back to the original reference frame? Verify your answer.
The exact calculation is a mess, so you may expand in powers of the velocity and keep terms up to and including order v^2 .
Discuss the physical interpretation of your result.
4. Find the **ground state** energy for a spinless charged particle in a Coulomb potential including the lowest relativistic corrections. Do this by following *Baym* pages 524-529. However, since I'm only asking for the ground state, you need only consider $l = 0$ and $\nu = 0$ (where ν shows up in Eq. 22-128). However, since we haven't done chapter 10 on Regge poles, you will have to proceed a little differently. After you reach Eq. 22-124, make the conventional substitution $\rho = \sqrt{\frac{8m'|E'|}{\hbar^2}} r$. Remember that E' is negative for a bound state, so $|E'| = -E'$. Then extract the terms that are important at large ρ and at small ρ . Here you would normally assume $U(r) = R(r)/r = \rho^s P(\rho)e^{-\rho/2}$ where s is a power that I will leave for you to find and $P(\rho)$ is a polynomial. Because I only want the ground state, you may, by a stroke of genius, assume that P terminates immediately when plugged into a recursion relation — in other words, $P(\rho) = 1$. This should get you to Eq. 22-128 (the i in this equation disappears because this equation doesn't contain the absolute value on E' .) Then proceed following *Baym*, making expansions in powers of $Z\alpha$ as appropriate.
This is a fairly long problem, so it will count as three problems.

5. Now that you have verified *Baym* Eq. 22-142 when $n = 1$ and $l = 0$, I'll let you use it for all l and n . To this order in α , how are the energies of the $n = 1$ and 2 states modified from the nonrelativistic answer? Why are the 2S and 2P states degenerate in the nonrelativistic case but nondegenerate in the relativistic case? (This last question is mostly about the nonrelativistic case)