Physics 570B, Spring 2011

Assignment 11

Due Thursday, Apr. 14

1. Consider a superposition of two plane waves:

$$\Psi = \left(Ae^{ikx} + Be^{iky}\right) e^{-i\omega t} \tag{1}$$

   Find the probability density current $\vec{j}$. Is probability conserved?

2. Show that you can't find 2x2 matrices $\vec{\alpha}$ and β that satisfy the anticommutation rules needed for the Dirac equation.

3. Show that for the single particle Dirac equation the positive definite quantity

$$\int d^3x\, \psi^\dagger \psi \tag{2}$$

   is conserved, where the corresponding current is $c\psi^\dagger \vec{\alpha} \psi$, where ψ is the four-component Dirac spinor. (At this level, it looks just like the nonrelativistic Schroedinger equation formula.)

4. Show that for the Dirac equation, just as for the Schroedinger equation,

$$\frac{d}{dt}\langle\psi\left|A\right|\psi\rangle = \langle\psi\left|\frac{\partial A}{\partial t}\right|\psi\rangle + i\langle\psi\left|[H, A]\right|\psi\rangle \quad , \tag{3}$$

   where A is an operator and ψ is a Dirac spinor. The Dirac Hamiltonian is $\vec{\alpha} \cdot \vec{p} + m\beta$. Find the operator corresponding to the velocity, $\frac{d}{dt}\langle\vec{x}\rangle$. What are the eigenvalues of this operator?