Physics 570B, Spring 2011
Assignment 12
Due Tuesday, Apr. 26

1. Consider an attractive delta function potential on one dimension: $V(x) = -\lambda\delta(x)$. Find the energy of the ground state for a particle in this potential. In doing this, rederive the conditions on the wave function at $x = 0$ rather than just quoting them from your notes or a book.
   Hint: Look at $\int_{-\epsilon}^{\epsilon} \frac{\partial^2\Psi}{\partial x^2}$.

2. Consider the periodic (repulsive) delta function potential in one dimension:

$$V(x) = \sum_{n=-\infty}^{\infty} \lambda\delta(x - na) \tag{1}$$

   Use degenerate states perturbation theory to find the band gap at $k = \pm\pi/a$. What condition must λ satisfy for perturbation theory to be a good approximation? Make a rough sketch of the energy as a function of k, where k is the "wave number" in Bloch's theorem.
   Hints: Check dimensions. It may be useful to assume that the whole system is inside a very large box.

3. Consider a periodic step potential in one dimension:

$$\begin{aligned} V(x) &= 0 \ : \ \frac{-a}{4} < x < \frac{a}{4} \\ V(x) &= V \ : \ \frac{a}{4} < x < \frac{3a}{4} \\ V(x+a) &= V(x) \end{aligned} \tag{2}$$
$$\tag{3}$$

   First, make a graph of the potential.
   Use perturbation theory to find the first two band gaps. Make a sketch of the energy as a function of crystal momentum including the first two gaps. What condition is necessary for perturbation theory to be accurate here?

4. What is the density of states (number of states per unit energy) for a stretched string of length L? The string has mass per unit length ρ and tension τ, and vibrates in only one direction.

5. What is the density of states in two dimensions for spinless particles? For spin 1/2 particles? for photons? Assume that the particles are confined in an $L \times L$ box.