Physics 570B, Spring 2011
Assignment 13
Due Tuesday, May. 3

Our final exam is Tuesday, May 10 at 8:00 AM. You may bring two pages of handwritten notes to this exam.

This assignment is somewhat different than your usual assignments in that it requires you to put together quite a number of different ideas to finally get an answer. On the other hand, it is a straightforward extension of the problem that I will work in detail in lecture. I hope that you find this valuable in learning the difficult subject of calculating decay rates.

1. Generalize the model of a mechanical oscillator coupled to a stretched string that we discussed in class to an oscillator coupled to a stretched membrane. To be specific, consider a mechanical harmonic oscillator with mass M and spring constant κ and a stretched membrane with mass per unit area ρ and tension τ. (Remember that for a membrane a "tension" is a force per length.) The membrane is square, covering the region $-L/2 < x < L/2$ and $-L/2 < y < L/2$. The mechanical oscillator is coupled to the center of the membrane with a perturbing Hamiltonian $V = \lambda q z(0,0)$ where q is the coordinate of the oscillator and $z(x,y)$ is the displacement of the membrane at position (x,y). At time $t = 0$ the membrane is in its ground state and the mechanical oscillator is in state N. Calculate the decay rate for the excitation of the oscillator. **Explain your work in detail.**

   (a) Write the Hamiltonian for this problem

   (b) What are the unperturbed eigenstates?

   (c) What is the density of states for excitations of the membrane?

   (d) Write the Hamiltonian for the membrane using raising and lowering operators

   (e) Use Fermi's golden rule to compute the decay rate

   (f) How would the decay rate change if the size of the membrane were increased? For example, if L was replaced by $2L$.

   This counts as four problems.