

**Astro/Phys 589 – Topics in Theoretical Astrophysics  
Final Exam**

*due Friday, December 9, 2005 at 4pm*

Please bring the completed exams to FO's or DP's office, or leave them in our mailboxes by 4pm.

The exam consists of two parts. Part I has a number of short, primarily conceptual questions from all 4 subjects we covered this semester. Part II has three longer problems.

Please explain all your calculations and answers. You may need to make additional assumptions in your solutions that were not stated explicitly in the problems. If you do, please state these assumptions in your answers.

## Part I

1. What is the hydrogen Gunn-Peterson effect and has it been detected? Is it easier or harder to detect than the He Gunn-Peterson effect? Explain why.

2. Complete the following table for the dynamical timescales for open clusters, globular clusters, and elliptical galaxies.

Please consult references to find the most recent values of the mass and radius for each of the systems. Choose typical values for  $M$  and  $R$  in each case.

System	$\tau_{\text{mix}}$	$\tau_{\text{rec}}$	$\tau_{\text{evap}}$
Open clusters			
Globular clusters			
Elliptical galaxies			

3. In fluid dynamics, radiative transfer, and N-body dynamics, we often study ensembles of particles, photons, and stars, respectively, using the Navier-Stokes equations, the radiative transfer equation, and the Fokker-Plank equation. Describe the assumptions underlying the three approaches.

4. What are the parameters that determine the abundances of  $^2\text{D}$ ,  $^4\text{He}$ , and  $^7\text{Li}$  produced during the era of nucleosynthesis in the early universe? Argue that the values of these parameters that best describe the current observational data, when viewed in conjunction with the high- $Z$  supernova and the WMAP results, rule out the possibility that dark matter in the universe can be in the form of compact objects (i.e., white dwarfs, neutron stars, and black hole of stellar origin).

## Part II

**5.** For a sequence of white-dwarf models, the mass of the white dwarf initially increases with central density, has a maximum at a central density  $\rho_0$ , and then decreases at even higher densities. Show that all the models with central density higher than  $\rho_0$  are unstable. *Hint:* Use the virial theorem or the solution to the polytropic equations.

**6.** A spherical shell of supernova ejecta moving at  $v \approx 4 \times 10^4$  km/s interacts with the dense circumstellar matter, mostly consisting of hydrogen, that was previously ejected from the progenitor star. The number density of the circumstellar material is  $100 \text{ cm}^{-3}$  before the interaction with the shock front.

(i) Find the density and temperature of the post-shock material.

(ii) Given your answer to part (i), estimate whether you expect to see emission lines from trace high-Z elements in the ejecta such as O, Si, and Ni.

(iii) Assuming a tangled magnetic field and an appropriate particle distribution function, calculate the spectrum and luminosity of synchrotron emission from this region. You may consult the literature for the relevant quantities.

**7.** Infrared observations of stellar orbits in the vicinity of the supermassive black hole in the center of the Milky Way show that a non-luminous object with mass  $2 \times 10^6 M_\odot$  is confined to a volume of at most  $10^{-6} \text{ pc}^3$ . Argue that the possibility of a cluster of compact objects with this total mass, confined in the same volume, is not a plausible alternative to the black-hole hypothesis (assume that each compact object has a mass larger than  $\sim 0.1 M_\odot$ ).