Astro/Phys 589 – Topics in Theoretical Astrophysics
Problem Set 3
due Monday, October 17, 2005 at 11am

1. The Coma cluster is a galaxy cluster with a radius of $R = 3$ Mpc. X-ray observations reveal a large amount of hot gas, with a thermal spectrum indicating a temperature of $T = 8.8 \times 10^7$ K. Assume that the gas consists of 75% hydrogen and 25% helium by mass. Wherever necessary, use a Gaunt factor of 1.2.

(a) Find out whether or not the gas can be considered fully ionized.
(b) Verify that most of the emission is radiated in the X-ray regime and calculate the average free electron density in the gas, if the X-ray luminosity is $L_X = 5 \times 10^{37}$ W. Assume here the gas is optically thin.
(c) Consider bremsstrahlung absorption and Thomson scattering and find at which frequency the gas becomes optically thick.
(d) What is the cooling time for this gas?

2. Consider a pure hydrogen nebula surrounding a hot star of radius $R$. At some distance $r$ from the star, the ionization equilibrium equation

$$n_{H^0} \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu d\nu = n_p n_e \alpha(T)$$

becomes

$$\frac{n_{H^0} R^2}{r^2} \int_{\nu_0}^{\infty} \frac{\pi F_\nu(R)}{h\nu} a_\nu e^{-\tau_\nu} d\nu = n_p n_e \alpha(T),$$

where the optical depth is

$$\tau_\nu(r) = \int_0^r n_{H^0}(r') a_\nu(r') dr',$$

and the ionization cross section is approximately given by

$$a_\nu = 6.3 \times 10^{-18} \left(\frac{\nu_0}{\nu}\right)^3 \text{ cm}^2,$$

where $\nu_0$ corresponds to the ionization threshold for hydrogen at 912 Å. We will assume that the star emits like a blackbody. Define the ionization fraction $x$ such that $n_{H^+} = x n_H$, $n_{H^0} = (1-x)n_H$.

(a) Integrate numerically equation (2) to plot the ionization fraction as a function of the distance from the star in parsecs. Repeat the exercise for these two cases:

i) $T_{\text{eff}} = 45,000$ K, $R/R_\odot = 11$

ii) $T_{\text{eff}} = 40,000$ K, $R/R_\odot = 20$
Assume that $T_e = 10,000$ K in the nebular gas so that $\alpha = 2.59 \times 10^{-13}$ cm$^3$s$^{-1}$, and $n_H = 10$ cm$^{-3}$ throughout the nebula. You may assume that $a_\nu = 6.3 \times 10^{-18}$ cm$^2$ in the calculation of the optical depth.

(b) Calculate the number of ionizing photons for the two black bodies using

$$Q(H^0) = \int_{v_0}^{\infty} \frac{L_\nu}{h\nu} dv.$$  \hspace{1cm} (5)

(c) Calculate the Strömgren radius of both stars using $Q(H^0)$ from part (b).