Astr/Phys 589 – Theoretical Astrophysics – Fall 2005
Homework #4
Due Date: Wednesday November 16, 11am

1. Synchrotron-Self Compton model of Relativistic Jets:

   (a) Consider a photon of energy $E = h\nu << 511 keV$, that is being scattered off a very relativistic electron with $\gamma >> 1$. Show that, on average, the energy gain of the photon is proportional to γ^2.
   Hint: Boost the photon energy to the electron rest frame, consider no energy exchange between the photon and the electron at that frame, and then boost the photon energy back out to the observer's frame.

   (b) Discuss and plot qualitatively the radiation spectrum that emerges from a distribution of relativistic electrons with density n_e and an average Lorentz factor γ_e, in the presence of a random magnetic field with magnitude B. Consider both the emission of synchrotron photons by the relativistic electrons, as well as the Compton scattering of these photons by the same electrons.

   This is the leading model for the high-energy emission from relativistic jets in systems such as Blazars.

2. The Flux of Solar Neutrinos
   Estimate the number flux on earth of the neutrinos that are produced in the ${}_5B^8$ decay in the sun. Use the fact that the luminosity of the sun is $\simeq 4 \times 10^{33}$ erg/s, that in every pp-chain the amount of energy releases is ~ 26.7 MeV, and that only 1 in 10^4 pp-chains occur through the ppIII cycle. *Hint:* How many pp-chains per second occur in the sun?

3. The 3α reaction
   The energy generation rate in the 3α reaction is given by

$$\epsilon_{3\alpha} \simeq 3.9 \times 10^{11} \frac{\rho^2 Y^3}{T_8^3} f \exp\left(\frac{-42.94}{T_8}\right) \text{erg/g/s} , \qquad (1)$$

   where ρ is the plasma density, $T_8 = T/10^8$ K is the temperature, Y is the helium mass fraction, and f is the electron screening factor.

   (a) Plot the quantity $\epsilon_{3\alpha}/(\rho^2 Y^3 f)$ as a function of temperature

   (b) Show that for small temperature variations around a temperature T_0 the energy generation rate can be approximated by

$$\epsilon_{3\alpha} = \epsilon_{3\alpha}(T_0) \left(\frac{T}{T_0}\right)^n , \qquad (2)$$

   with

$$n = \frac{42.94}{T_8} - 3 . \qquad (3)$$

(c) Show that for $T_8 \simeq 1$, the energy generation rate is approximately

$$\epsilon_{3\alpha} \simeq 4.4 \times 10^{-8} \rho^2 Y^3 f T_8^{40} \mathrm{erg/g/s} \; . \tag{4}$$

and plot this approximate expression on top of the curve for the complete expression that you plotted for part (a).