

Phys 321, Theoretical Mechanics I, Spring 2006

Homework #4

**Due Date: Friday April 14, 10am**

1. Show explicitly that the areal velocity is constant for a particle moving under the influence of an attractive force given by  $F(r) = -kr$ .
2. Two stars of mass  $M_1$  and  $M_2 < M_1$  are orbiting each other in a circular orbit. The heavy star ( $M_1$ ) experiences a supernova explosion, losing most of its mass in a spherically symmetric outflow (i.e., without losing any angular momentum) and leaving a small neutron star of mass  $M_{\text{NS}}$ . Show that if the mass lost is larger than half of the total mass of the system, then the binary is disrupted.
3. Use Kepler's 1st and 2nd law to show that the gravitational force must be central and that the radial dependence must be  $1/r^2$  (i.e., repeat what Newton must have done to obtain his gravitational law).
4. **For honor students only:** Consider a particle of mass  $m$  in a circular orbit in the potential

$$V(r) = -\frac{GM}{r} \exp(-r/\rho), \quad (1)$$

which is predicted by a class of Quantum Gravity theories with  $\rho$  a parameter related to the mass of the graviton.

- (a) Show that such a circular orbit exists only if the angular momentum of the orbit is smaller than a critical value  $l_{\text{crit}}$ .
- (b) Evaluate the critical angular momentum  $l_{\text{crit}}$ .