TMDs at Small-\(x\)

Feng Yuan (fyuan@lbl.gov)
Lawrence Berkeley National Laboratory
### TMD Summer School Program
**June 22nd - June 28th**

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6/28/17
Goal is to introduce some basic ideas about small-x physics

- Why is it interesting, how relevant
- Current theoretical approach

Connections to the TMDs

References for this lecture

- Dominguez, Marquet, Xiao, Yuan, 1101.0715
Gluon saturation inevitable at small-$x$
QCD evolution drives the gluon distribution rising at small-$x$

Figure 1.1: The processes related to the lowest order QCD splitting functions. Each splitting function $P_{p'p}(x/z)$ gives the probability that a parton of type $p$ converts into a parton of type $p'$, carrying fraction $x/z$ of the momentum of parton $p$

$$\mu \frac{d}{d\mu} f_{j/h}(x, \mu) = \sum_{k} \int_{x}^{1} \frac{dz}{z} P_{jk}(z, \alpha_s(\mu)) f_{k/h}(x/z, \mu)$$

$$P_{gg}(x) = \frac{x}{(1-x)_+} + \frac{1-x}{x} - x(1-x) + \delta(x-1)\beta_0.$$
BFKL evolution becomes relevant at small-$x$

- Balitsky-Fadin-Lipatov-Kuraev, 1977-78

\[
\frac{\partial N(x,r_T)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x,r_T)
\]

- Balitsky-Kovchegov: Non-linear term, 98

\[
\frac{\partial N(x,r_T)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x,r_T) - \alpha_s [N(x,r_T)]^2.
\]
**QCD** Phase structure of cold nuclei

- Hard processes probe the $k_t$-dependent gluon distributions directly.
- Saturation phenomena manifest in the observables.
Small-x approximation

- Take the leading contribution of high energy scattering (eikonal approx)
- Take the small-x limit whenever applicable, and neglect all higher order terms
  - There have been some recent developments to deal with sub-leading contributions, however, very subtle and complicated
Light-cone decomposition

\[ k^\pm = (k^0 \pm k^z) / \sqrt{2} \]

- Nucleon/nucleus moving in \(+z\) direction, the probe in \(-z\) direction

- Useful Fourier transform

\[
\int_{-\infty}^{+\infty} dx^- \Theta(-x^-) e^{-ik^+x^-} = \frac{i}{k^+ + i\epsilon} \quad \rightarrow \quad \Theta(-x^-) = \int_{-\infty}^{+\infty} dk^+ e^{ik^+x^-} \frac{i}{k^+ + i\epsilon}
\]

\[
\int \frac{d^2k_\perp}{(2\pi)^2} e^{ik_\perp \cdot x_\perp} \frac{k_\perp^\alpha}{k_\perp^2} = \frac{1}{2\pi} \frac{ix_\perp^\alpha}{x_\perp^2}
\]
Small-x factorization

- eikonal approximation in high energy scattering

\[ \int_{-\infty}^{+\infty} dx^- A^+(x^-, x_\perp) + \int_{-\infty}^{+\infty} dx_1^- dx_2^- \Theta(x_1^- - x_2^-) A^+(x_1^-, x_\perp) A^+(x_2^-, x_\perp) \]

\[ \int d^2 x_\perp e^{ik_{\perp \cdot x_\perp}} (U(x_\perp) - 1) \]

\[ U(x_\perp) = \mathcal{P} \exp \left( -ig \int_{-\infty}^{+\infty} dx^- A^+(x^-, x_\perp) \right) \]
Basic rules

\[(U(x_\perp) - 1)\]

\[(U^\dagger(x_\perp) - 1)\]

\[(W(x_\perp) - 1)_{ab} = \frac{1}{T_F} \text{Tr}[U(x_\perp)T^aU^\dagger(x_\perp)T^b - T^aT^b]\]
Example #1: $qA \rightarrow q + X$

\[ A \propto \bar{u}_i(k) \gamma^\mu u_j(k') p_\mu \int d^2 x_\perp e^{i k_{g\perp} \cdot x_\perp} (U(x_\perp) - 1))_{ij} \]

\[ \frac{d\sigma^{pA \rightarrow qX}_{\text{LO}}}{d^2 k_\perp dy} = \sum_f x q_f(x) \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-i k_\perp \cdot (x_\perp - y_\perp)} \frac{1}{N_c} \langle \text{Tr} U(x_\perp) U^\dagger(y_\perp) \rangle_Y \]
Dipole amplitude

- S-matrix describes quark-antiquark dipole scattering on nucleon/nucleus
  \[ S_Y^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c} \langle \text{Tr} U(x_\perp) U^\dagger(y_\perp) \rangle_Y \]

- Also referred as the un-integrated gluon distribution in heavy ion community

\[ F(k_\perp) = \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-i k_\perp \cdot (x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp) \]
Example #2: DIS

\[
\begin{align*}
\bar{u}(k_1) & \not{p} \frac{i}{\not{q} - k_2 - m} \gamma^\mu v(k_2) (U(x_\perp) - 1) \\
\bar{u}(k_1) & \gamma^\mu \frac{i}{k_1 - \not{q} - m} \not{p} v(k_2) (U^\dagger(x_\perp) - 1) \\
\bar{u}(k_1) & \not{p} \frac{i}{k_1 - k_{g1} - m} \gamma^\mu \frac{i}{k_{g2} - k_2 - m} \not{p} v(k_2) (U(x_\perp) - 1) (U^\dagger(x_\perp) - 1)
\end{align*}
\]
The amplitude (transverse photon) proportional to

\[
A \propto \int d^2x_{1\perp} d^2x_{2\perp} e^{ik_{1\perp} \cdot (x_{1\perp} - x_{2\perp})} e^{ik_{g\perp} \cdot x_{2\perp}}
\times \frac{k_{1\perp}^{\alpha} - k_{g1\perp}^{\alpha}}{(k_{1\perp} - k_{g1\perp})^2 + z(1-z)Q^2} (U(x_1)U^\dagger(x_2) - 1)
\]

\[
F_2(x, Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2 \alpha_{em}} \int_0^1 dz \int d^2x_\perp d^2y_\perp \left[ |\psi_T(z, r_\perp, Q)|^2 + |\psi_L(z, r_\perp, Q)|^2 \right]
\times [1 - S(r_\perp)], \quad \text{with} \quad r_\perp = x_\perp - y_\perp.
\]
Example #3: Drell-Yan

\[ \bar{u}(k_1) \frac{i}{k_1 + k_2 + i\epsilon} \gamma^\mu \phi u(p_2) (U(x_\perp) - 1) \]

\[ \bar{u}(k_1) \frac{i}{p_2 - k_2 + i\epsilon} \gamma^\mu u(p_2) (U(x_\perp) - 1) \]

\[ \frac{i}{p_2 + k_{g1} + i\epsilon} \frac{\gamma^\mu}{k_1 - k_{g2}} \rightarrow 0 \]
Amplitude squared proportional to

\[ |A|^2 \propto \frac{(k_{1\perp} + k_{2\perp})^2}{k_{2\perp}^2 (k_{2\perp} - z_2 k_{g\perp})^2} \int d^2x_{\perp} d^2y_{\perp} e^{ik_{g\perp} \cdot (x_{\perp} - y_{\perp})} \langle U(x_{\perp}) U^\dagger(y_{\perp}) \rangle \]

\[ = \frac{1}{k_{2\perp}^2 (k_{2\perp} - z_2 k_{g\perp})^2} k_{g\perp}^2 \mathcal{F}(k_{g\perp}) \]

Directly probe the dipole gluon distribution
Example #4: quark distribution at small-x

\[
xq^{(DY)}(x, k_\perp) = \frac{N_c}{4\pi^4} \int d^2 k_\perp F_x(k_\perp) \left( 1 - \frac{k_\perp \cdot (k_\perp - k_{g\perp})}{k_\perp^2 - (k_\perp - k_{g\perp})^2} \ln \frac{k_\perp^2}{(k_\perp - k_{g\perp})^2} \right)
\]

- It can be shown that the DIS quark is the same as the DY quark, although the diagrams are not
Example #4: one gluon radiation

- **BK evolution**

  - **Soft gluon limit**

\[
A \propto \int d^2x_1 \cdot d^2x_2 e^{ik_{g1\perp} \cdot (x_1 - x_2)} e^{ik_{g\perp} \cdot x_2} \left( \frac{k_{1\perp} - k_{g1\perp}}{(k_{1\perp} - k_{g1\perp})^2} - \frac{k_{1\perp} - k_{g\perp}}{(k_{1\perp} - k_{g\perp})^2} \right) \left( U(x_1) U^\dagger(x_2) T^a U(x_2) \right)
\]
BK: Real+Virtual

\[ M(x_\perp, z_\perp, y_\perp) = 4\pi g T^a \left[ \frac{\epsilon_\perp \cdot (x_\perp - z_\perp)}{(x_\perp - z_\perp)^2} - \frac{\epsilon_\perp \cdot (y_\perp - z_\perp)}{(y_\perp - z_\perp)^2} \right] \]

\[ \frac{\partial}{\partial Y} S_Y^{(2)}(x_\perp, y_\perp) = -\frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2b_\perp (x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2(y_\perp - b_\perp)^2} \left[ S_Y^{(2)}(x_\perp, y_\perp) - S_Y^{(4)}(x_\perp, b_\perp, y_\perp) \right] \]
TMD Gluons at small-x
Conventional gluon distribution

- Collins-Soper, 1981

\[ xG^{(1)}(x, k_{\perp}) = \int \frac{d\xi^- d^2\xi_{\perp}}{(2\pi)^3 P^+} \frac{e^{ixP^+\xi^- - ik_{\perp} \cdot \xi_{\perp}}}{P^+} \times \langle P|F^{+i}(\xi^-, \xi_{\perp})\mathcal{L}_\xi^\dagger \mathcal{L}_0 F^{+i}(0)|P\rangle \]

- Gauge link in the adjoint representation

\[ \mathcal{L}_\xi = \mathcal{P} \exp\left\{-ig \int_{\xi^-}^{\infty} d\zeta^- A^+ (\zeta, \xi_{\perp}) \right\} \]

\[ \mathcal{P} \exp\left\{-ig \int_{\xi_{\perp}}^{\infty} d\zeta_{\perp} A_{\perp} (\zeta^- = \infty, \zeta_{\perp}) \right\} \]
Physical interpretation

- Choosing light-cone gauge, with certain boundary condition (either one, but not the principal value).

\[ A_\perp(\zeta^- = \infty) = 0 \]

- Gauge link contributions can be dropped.

- Number density interpretation, and can be calculated from the wave functions of nucleus.
  - McLerran-Venugopalan
  - Kovchegov-Mueller
Classic YM theory: WW-gluon

- McLerran-Venugopalan

\[
xG^{(1)}(x, k_\perp) = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \int \frac{d^2 r_\perp}{(2\pi)^2} \frac{e^{-i k_\perp \cdot r_\perp}}{r_\perp^2} \left( 1 - e^{-\frac{r_\perp^2 Q_s^2}{4}} \right)
\]

- See also, Kovchegov-Mueller
- Weizsacker-Williams gluon distribution is the conventional one
DIS dijet probes $WW$ gluons

- Hard interaction includes the gluon attachments to both quark and antiquark
- The $q_\perp$ dependence is the gluon distribution w/o gauge link contribution at this order
Fundamental representation

\[ xG^{(1)} (x, k_{\perp}) = 2 \int \frac{d\xi - d\xi_{\perp}}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_{\perp}\cdot\xi_{\perp}} \times \langle P | \text{Tr} [F^{+i}(\xi^-, \xi_{\perp}) U^{[+]\dagger} F^{+i}(0) U^{[+]}] | P \rangle \]

\[ U^{[+]}_{\xi} = U^n [0, +\infty; 0] U^n [+\infty, \xi^-; \xi_{\perp}] \]

- Apply the following identity

\[ \partial_i U(v) = i g_S \int_{-\infty}^{\infty} dv^+ U[-\infty, v^+; v] (\partial_i A^-(v^+, v)) U[v^+, \infty; v] \]

\[ - \langle \text{Tr} [\partial_i U(v)] U^\dagger (v') [\partial_j U(v')] U^\dagger (v) \rangle_{x_g} = \]

\[ g_S^2 \int_{-\infty}^{\infty} dv^+ dv'^+ \langle \text{Tr} [F^i-(\vec{v}) U^{[+]\dagger} F^j-(\vec{v}') U^{[+]}] \rangle_{x_g} \]
Dipole calculation

\[ A \propto \int d^2x_{1\perp} d^2x_{2\perp} e^{ik_{g1\perp} \cdot (x_{1\perp} - x_{2\perp})} e^{ik_{g\perp} \cdot x_{2\perp}} \times \frac{k_{1\perp}^{\alpha} - k_{g1\perp}^{\alpha}}{(k_{1\perp} - k_{g1\perp})^2 + z(1 - z)Q^2} (U(x_1)U^\dagger(x_2) - 1) \]
Expansion in the correlation limit, $q_+ \ll P_+$

- There is cancellation between two-point and four-point functions
- Final result

\[
\frac{d\sigma}{dP\cdot S} = \alpha_em^2 e_q^2 \alpha_s \delta (x_{\gamma*} - 1) z (1 - z) (z^2 + (1 - z)^2) \frac{P_+^4 + \epsilon_f^4}{(P_+^2 + \epsilon_f^2)^4} 
\times (16\pi^3) \int \frac{d^3v d^3v'}{(2\pi)^6} e^{-iq_\perp \cdot (v-v')} 2 \left\langle \text{Tr} F^i+(v) U^{[+]\dagger} F^i+(v') U^{[+]\dagger} \right\rangle_{xy}
\]

Agrees with the TMD result
Photon-jet correlation probes the dipole gluon distribution

There is no color structure corresponding to this. We have to express the gluon Distribution in the Fundamental representation.
Dipole gluon distribution

\[ xG^{(2)}(x, k_\perp) = 2 \int \frac{d\xi^-d\xi_\perp}{(2\pi)^3P^+} e^{ixP^+\xi^- - ik_\perp \cdot \xi_\perp} \]

\[ \langle P | \text{Tr} \left[ F^{+i}(\xi^-, \xi_\perp) U^{-}[\cdot] F^{+i}(0) U^{+[\cdot]} \right] | P \rangle \]

This is the dipole gluon distribution, also called unintegrated gluon distribution

\[ xG^{(2)}(x, q_\perp) \simeq \frac{q_\perp^2 N_c}{2\pi^2 \alpha_s} S_\perp \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp \cdot r_\perp} S_{xg}^{(2)}(0, r_\perp) \]
Intuitive explanations

- Final state interactions in DIS can be eliminated by choosing the light-cone gauge → number density interpretation
- Photon-jet correlation have both initial/final state interactions, can not be eliminated by choosing LC gauge → there is no number density interpretation → dipole gluon distribution
Dijet-correlation at RHIC

- Initial state and/or final state interactions

- Standard (naïve) Factorization breaks!

Boer-Vogelsang 03

Becchetta-Bomhof-Mulders-Pijlman, 04-06
Collins-Qiu 08; Vogelsang-Yuan 08
Rogers-Mulders 10; Xiao-Yuan, 10
Modified factorization

- Dilute system on a dense target, in the large $N_c$ limit,
Hard partonic cross section

\[ H_{qq\rightarrow qq}^{(1)} = \frac{\hat{u}^2 (\hat{s}^2 + \hat{u}^2)}{-2\hat{s}\hat{u}\hat{t}^2}, \quad H_{qq\rightarrow qq}^{(2)} = \frac{\hat{s}^2 (\hat{s}^2 + \hat{u}^2)}{-2\hat{s}\hat{u}\hat{t}^2} \]

\[ H_{gg\rightarrow q\bar{q}}^{(1)} = \frac{1}{4N_c} \frac{2 (\hat{t}^2 + \hat{u}^2)^2}{\hat{s}^2\hat{u}\hat{t}}, \quad H_{gg\rightarrow q\bar{q}}^{(2)} = \frac{1}{4N_c} \frac{4 (\hat{t}^2 + \hat{u}^2)}{\hat{s}^2} \]

\[ H_{gg\rightarrow gg}^{(1)} = \frac{2 (\hat{t}^2 + \hat{u}^2) (\hat{s}^2 - \hat{t}\hat{u})^2}{\hat{u}^2\hat{t}^2\hat{s}^2}, \quad H_{gg\rightarrow gg}^{(2)} = \frac{4 (\hat{s}^2 - \hat{t}\hat{u})^2}{\hat{u}\hat{t}\hat{s}^2} \]

\[ H_{gg\rightarrow gg}^{(3)} = \frac{2 (\hat{s}^2 - \hat{t}\hat{u})^2}{\hat{u}^2\hat{t}^2}, \]
Kt-dependent gluon distributions

\[ \mathcal{F}_{qq}^{(1)} = xG^{(2)}(x, q_\perp), \quad \mathcal{F}_{qq}^{(2)} = \int xG^{(1)}(q_1) \otimes F(q_2), \]

\[ \mathcal{F}_{gg}^{(1)} = \int xG^{(2)}(q_1) \otimes F(q_2), \quad \mathcal{F}_{gg}^{(2)} = \int \frac{q_1 \cdot q_2}{q_1^2} xG^{(2)}(q_1) \otimes F(q_2) \]

\[ \mathcal{F}_{gg}^{(3)} = \int xG^{(1)}(q_1) \otimes F(q_2) \otimes F(q_3), \]
- Naïve $kt$-factorization breaks down, violation effect is 100%
- Integrate out $q_t$, recover the inclusive dijet cross section
- At large $q_t$: $P_t \gg q_t \gg Q_s, \Lambda_{QCD}$, collinear factorization result for the correlation,
  - Qiu-Vogelsang-Yuan 2007
Violation effects

\[ xG^{(1)} \]

\[ xG^{(2)} \]

\[ xG^{(1)}xF \]

\[ qg \rightarrow qg \]
Sudakov (CSS) Resummation

- Sudakov double logs can be re-summed in the small-x saturation formalism
- Radiated gluon momentum
  \[ k_g = \alpha g p_1 + \beta g p_2 + k_{g\perp} \]
- Soft gluon, \( \alpha \sim \beta \ll 1 \)
- Collinear gluon, \( \alpha \sim 1, \beta \ll 1 \)
- Small-x collinear gluon, \( 1 - \beta \ll 1, \alpha \to 0 \)
- Rapidity divergence

Mueller, Xiao, Yuan, PRL110,082301 (2013)
Xiao, Yuan, Zhou, NPB 2017
Big Applause for the Organizers, and in particular, Andreas and Sean!

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Back-ups
Feynman rules

- Propagators

- Gluon

\[
\frac{\Gamma_{\text{gluon}}}{p^2 + i0} = \frac{-g_{\mu\nu} + (1 - \xi) \frac{p_\mu p_\nu}{p^2 + i0}}{p^2 + i0}
\]

- Quark

\[
\frac{\Gamma_{\text{quark}}}{p^2 - m_f^2 + i0} = \frac{i(\phi + m_f) p_\sigma}{p^2 - m_f^2 + i0}
\]

- Ghost

\[
\frac{\Gamma_{\text{ghost}}}{p^2 + i0} = \frac{i}{p^2 + i0}
\]
Feynman rules

- Interaction vertices

- Quark-gluon

- Ghost-gluon
Feynman rules

- Non-abelian part

Three-gluon

\[-g \mu^\epsilon f_{\alpha \beta \gamma} \left[ (p - q)^\nu g^{\lambda\mu} + (q - r)^\lambda g^{\mu\nu} + (r - p)^\mu g^{\nu\lambda} \right]\]

Four-gluon

\[-ig^2 \mu^2 \epsilon f_{\epsilon \alpha \beta} f_{\epsilon \gamma \delta} (g^{\kappa \mu} g^{\lambda \nu} - g^{\kappa \nu} g^{\lambda \mu})
- ig^2 \mu^2 \epsilon f_{\epsilon \alpha \gamma} f_{\epsilon \beta \delta} (g^{\kappa \lambda} g^{\mu \nu} - g^{\kappa \nu} g^{\mu \lambda})
- ig^2 \mu^2 \epsilon f_{\epsilon \alpha \delta} f_{\epsilon \beta \gamma} (g^{\kappa \lambda} g^{\mu \nu} - g^{\kappa \nu} g^{\mu \lambda})\]
Small-x factorization

- eikonal approximation in high energy scattering

\[
\int_{-\infty}^{+\infty} dx^- A^+(x^-, x_\perp) + \int_{-\infty}^{+\infty} dx_1^- dx_2^- \Theta(x_1^- - x_2^-) A^+(x_1^-, x_\perp) A^+(x_2^-, x_\perp)
\]

\[
\int d^2 x_\perp e^{i k_{\perp} \cdot x_\perp} (U(x_\perp) - 1)
\]

\[
U(x_\perp) = \mathcal{P} \exp \left( -ig \int_{-\infty}^{+\infty} dx^- A^+(x^-, x_\perp) \right)
\]

Mueller, 1994
Splitting function

- Dipole amplitude

\[ S^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c} \langle |U^\dagger(x_\perp)U(y_\perp)| \rangle \]

- At one-loop order

\[
\frac{\partial}{\partial Y} S_Y^{(2)}(x_\perp, y_\perp) = -\frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2 b_\perp (x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2(y_\perp - b_\perp)^2} \left[ S_Y^{(2)}(x_\perp, y_\perp) - S_Y^{(4)}(x_\perp, b_\perp, y_\perp) \right]
\]

\[ Y \sim \text{Log}(1/x) \]

BK-JIMWLK