The Concave Grating Spectrometer
Rowland Mounting

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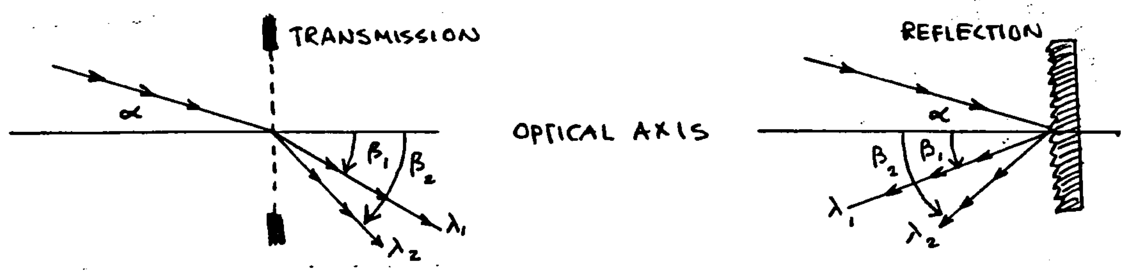
INTRODUCTION

The ray diagrams below show how a plane transmission and plane reflection grating diffract light.



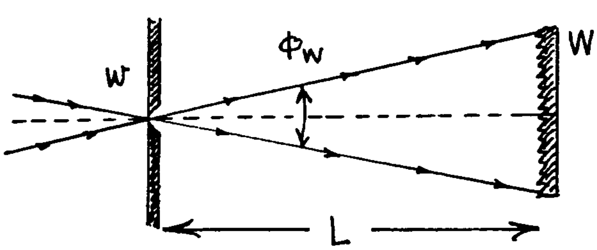
In most spectrographs the angle α of the incident ray is fixed and the spectrum is spread out according to β. In some monochrometers and spectrometers, α can be varied. The detector is scanned through β and get the spectrum from "red to green to blue". An important question is: What is the best value for α? In this Rowland Spectrometer experiment you will examine the various parameters to consider when building a grating spectrograph.

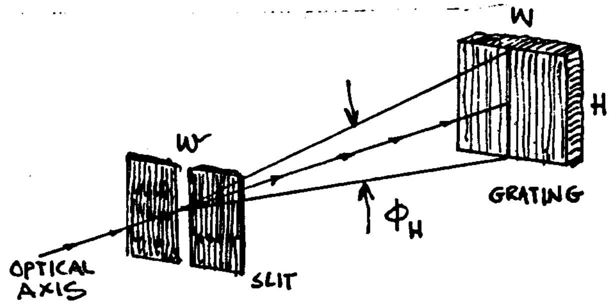
Before we begin lets examine the Rowland spectrograph. The instrument consists of a slit (A), a concave grating (B) and a back focal plane with eyepiece (C).

THE SLIT

Examine the slit, What is its minimum and maximum opening. (the dial goes from 0 to 100). What do these numbers mean? Do both slit jaws or just one jaw move during the slit width adjustment? How accurate can you determine the slit width? Is the slit dial calibrated - i.e. what do the numbers mean? How does the slit image (spectral line) change when you change the slit? Pay careful attention to the slit because its proper adjustment is the key to getting good results.

The entrance slit is the "window" to a spectrograph, spectrometer, monochrometer. All light that passes through the slit should hit the grating. Doubling the slit width will double the amount of light on the grating. The slit-grating combination defines an optical axis which passes through the center of the slit to the center of the grating. The optical axis extends from grating to slit to infinity - or to the wall or to the light source - which ever is closer.



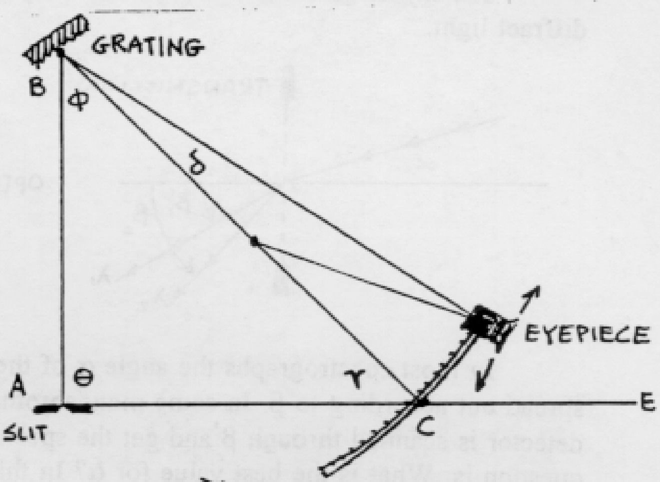


A light source (spectroscopic source) is used to illuminate the slit and therefore the grating. The main goal is to put the light source at some point on the optical axis so that light will go through the slit, illuminate and fill the grating.

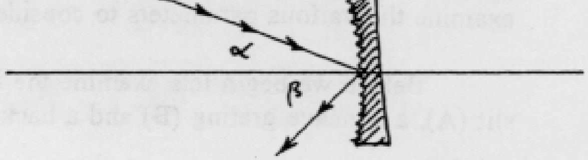
Although it is not important right now, we note some important geometrical relations between angles, sizes and distances: The slit has width w, height h and area $A_s = w_s h_s$. The grating has width W, height H and area $A_g = W_g H_g$. The angles formed at the center of the slit by the grating is $\phi_H = H/L$ and $\phi_W = W/L$. The solid angle formed at the center of the slit is $d\Omega = (H/L)(W/L) = HW/L^2 = A/L^2$. Note the angles ϕ_H and ϕ_W extend out through the slit.

THE MOVEABLE ARMS

Note that point C on arm BC can be moved along AE. Perform this motion and determine which arm stays fixed (in length) as ϕ is changed. For fixed ϕ, slide the eyepiece along the curved track. Do the eyepiece and grating stay fixed when this is done? What angles change? What is the relationship between the angles θ, ϕ and γ?



Pick ϕ fixed and rotate δ along the back focus. How are the angles θ, ϕ, δ and γ related to the grating angles α and β in the equation $m\lambda = d(\sin\beta - \sin\alpha)$? What is the relationship between AB, BC and CA?



Note that BC is constant as the eyepiece is slid along the track. Also δ can be kept constant as ϕ (or γ) is changed. Locate the pull tab under arm BC. Pull it down and turn it 90° to fasten it in the pull-down position. Now arm BD can move in an arc at a fixed distance BD from the grating.

What do the numbers on the linear scale (cm) mean?
What is the smallest/largest angle that can be achieved for ϕ, γ?
What is the smallest/largest length of AC?
What is the smallest/largest value for δ?

How does the instrument work - i.e. what does it do? What is it good for? Why is it important to keep BC constant? For many years gratings were ruled on flat surfaces and lenses (or mirrors) were required to bring the diffracted rays to a focus. In 1882 Henry Rowland invented the concave grating which performed three functions: 1. it gathered light, 2. diffracted it through angle β and 3. focused it onto a back focus (slit or photographic plate). Thus chromatic aberration and the absorption caused by the lenses were eliminated.

Rowland discovered that the concave grating has a unique relationship in the relative position of the source slit, the grating and its center of curvature, and the focusing point of the spectrum. These are always located on a circle drawn tangent to the midpoint of the grating with a diameter equal to the radius of curvature of the grating. This circle is called the Rowland circle.

THE ROWLAND CIRCLE

Refer to the figure on the next page:

AE is the width of the concave grating
KM is the radius of curvature of the concave grating
KM is the diameter of the Rowland circle
CM is the radius of the Rowland circle
MCK is the normal to the grating at point M

~~Recall that for a plane grating (transmission or reflection)~~ the following relationship holds

$$m\lambda = d(n_\beta \sin\beta - n_\alpha \sin\alpha) = dn(\sin\beta - \sin\alpha) = d(\sin\beta - \sin\alpha)$$

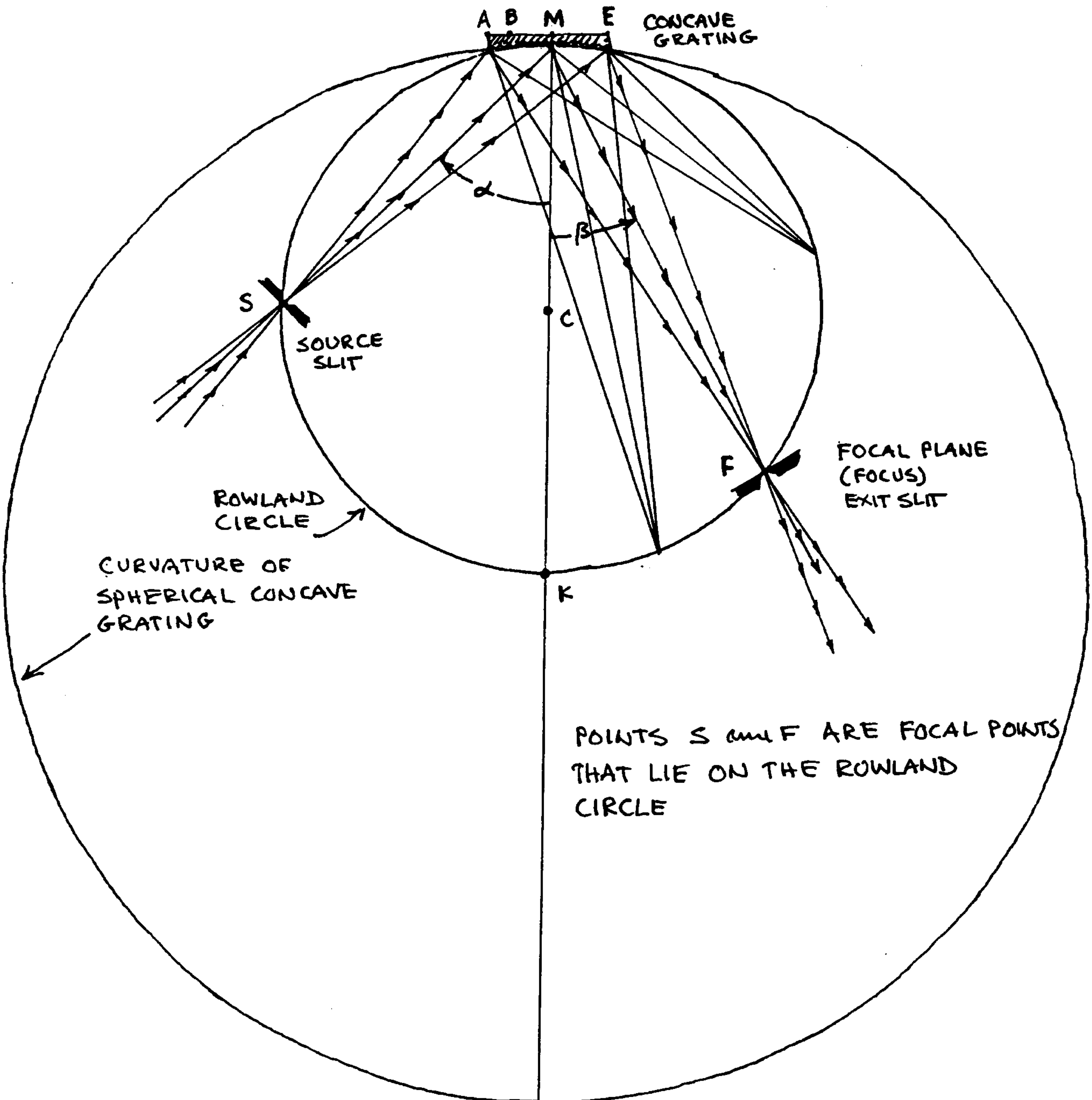
where:

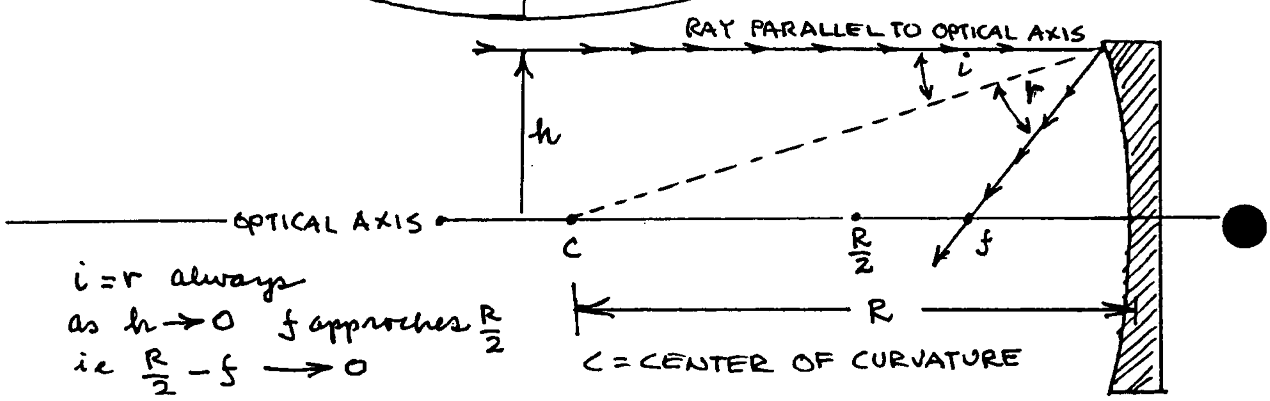
α is the angle of incidence of light from the source slit
β is the angle of diffraction of light to the exit slit (eye piece)
λ is the wavelength of the incident light
d is the grating groove spacing in mm (1/d is the number of grooves/mm)
m (an integer) is the diffraction order number
n_β and n_α are the refractive indicees on the β and α sides respectively
$n = n_\alpha = n_\beta$ (n = 1.000293 ~ 1.000 for air. n = 1 exactly for a vacuum)

How would you derive this relationship? Compare it to the derivation of the Rowland grating.
What assumptions are made with respect to the Rowland grating derivation?
We want to show that this same relationship holds for a concave grating.
Refer to Figures on the next two pages.

Draw SAF and SBF where AB = d is the distance between two successive grating spacings.
These rays are almost parallel. (What is the angle at their apex?)
For constructive interference the optical path difference SAF - SBF must equal $m\lambda$
Construct "perpendiculars" Aa and Bb. Aa ⊥ SB and Bb ⊥ FA. Note that Aa and Bb are arcs that are approximately straight lines since AS » Aa and AF » Bb
So we can write Ab - Ba = SAF - SBF = $m\lambda$
From similar triangles this becomes: $d\sin\beta - d\sin\alpha = m\lambda = d(\sin\beta - \sin\alpha)$

Now consider the next set of rays from S to any other two consecutive reflecting spaces: say B and C. These rays have incident and refracting angles β' and α' respectively. For constructive interference they must satisfy the equation: $d(\sin\beta' - \sin\alpha') = m\lambda$

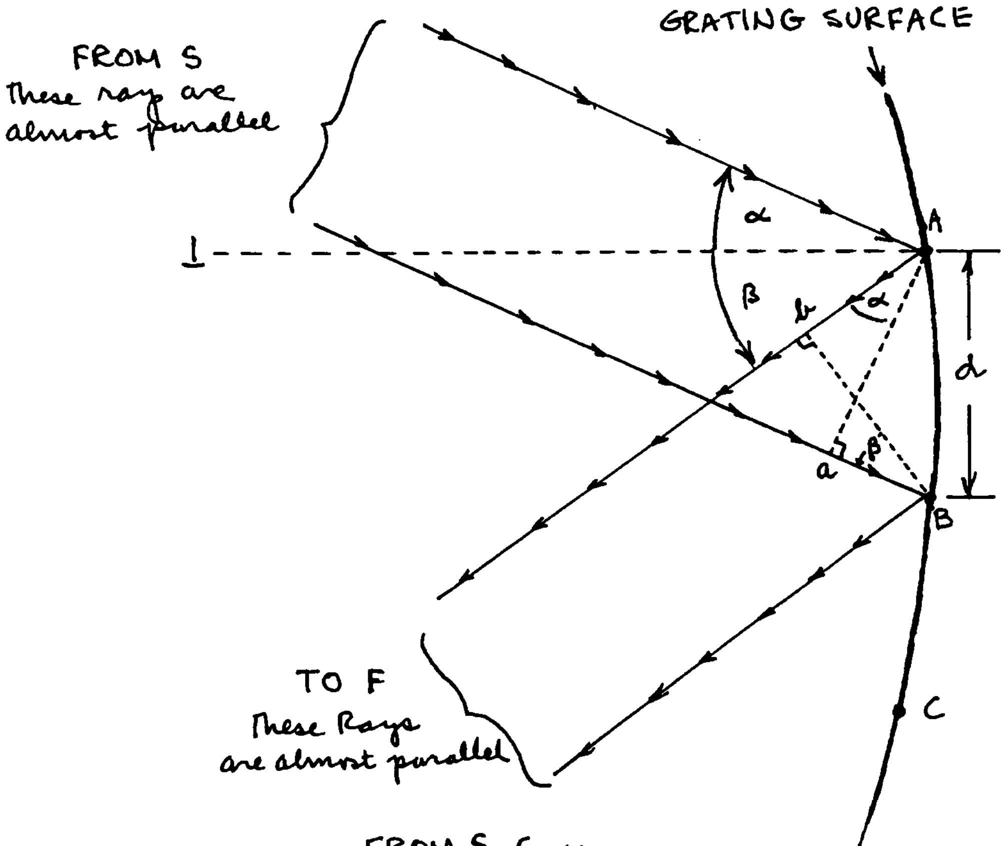


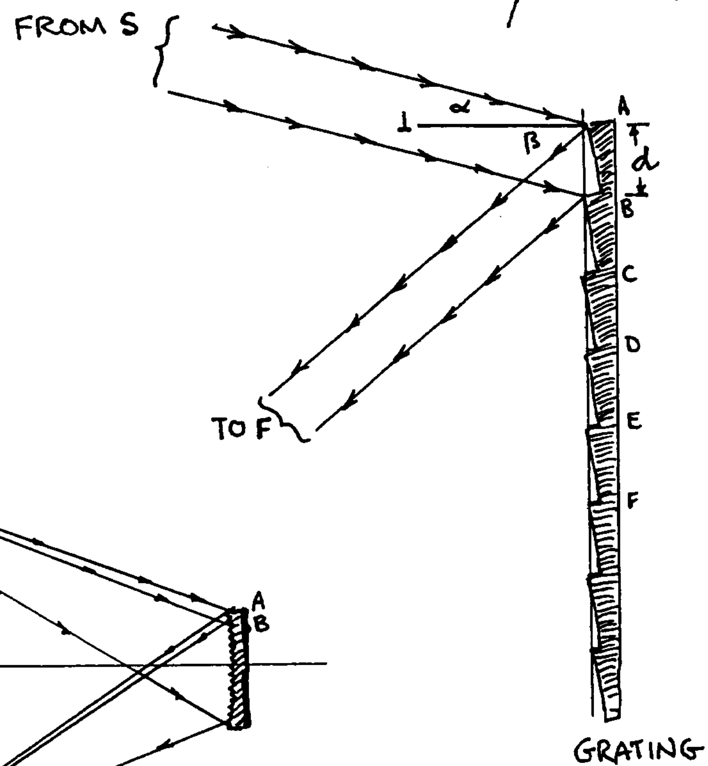


$i = r$ always

as $h \to 0$ f approaches $\frac{R}{2}$

ie $\frac{R}{2} - f \longrightarrow 0$


GRATING SURFACE
FROM S
These rays are almost parallel
⊥
α
β
b
a
d
A
B
C
TO F
These Rays are almost parallel


FROM S
⊥
α
β
A
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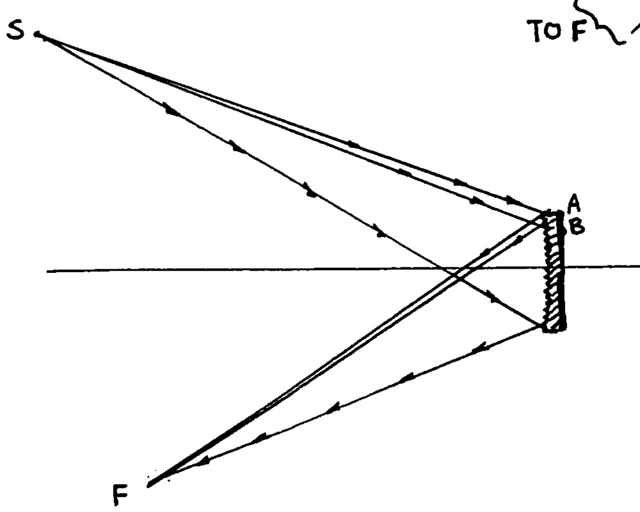
F
TO F
GRATING


S
A
B
F

Now Assumptions (Faith) Verify that these assumptions are valid in this case.

1. The grating surface width is small compared to the radius of curvature therefore all elements of the grating are very nearly on the circle KSMF.
2. The angles SAK = β and SDK = β' are approximately subtended by the same arc KS. Therefore these angles are equal ($\beta = \beta'$) and ($\alpha' = \alpha'$)
3. All light from all grating spacings constructively interferes at F when $d(\sin\beta - \sin\alpha) = m\lambda$ for the concave grating. This equation is identical to the plane grating equation.

THE GRATING

Locate the grating. Discuss its condition, function and other properties easily seen by the eye. (no accurate measurements are needed here) Not all angles, lengths, dimensions are accessible, however their allowed values are governed by important and interesting relationships. What experiments can we do with this dedicated but versatile instrument?

1. We can measure wavelengths very accurately using $m\lambda = d(\sin\beta - \sin\alpha)$ if we know the grating constant d and angles α and β
2. We can determine grating constants if we know the wavelengths and angles.
3. We can prove the Rowland circle geometry is valid for a concave grating.
4. Since all concave grating spectrometers employ the Rowland circle we can determine which values for lengths and angles will optimize the desired λ-range for work in the ir, visible and uv.
5. We can study spectral line widths as a function of slit width.

PROCEDURE

Open the slit to a width of about 0.5 mm and rotate the slit to a vertical position. Place a mercury light source in front of the slit to illuminate the slit and grating.Focus the telescope on the cross hair. Locate the green a spectral line Hg (λ5461Å) on the cross hair of the eyepiece. Turn the screws at the top of the grating holder to produce the sharpest image of the line.

NOTE

The spectral lines you see are nothing more than images of the entrance slit in the colors of the spectrum!!! Therefore the spectrometer must be in focus

It may be necessary to make some final adjustments to the slit width and slit angle. However, when adjusted, they should remain constant throughout the experiments. The instrument is now ready to take data.

THE EXPERIMENTS

1. With the reflecting angle β equal to zero, obtain the positions of the visible lines in the spectrum of mercury (or helium). Plot the wavelengths λ as ordinates against the scale positions as abscissas. Draw the curve for each order. Compute the slopes and relate the slope values to the order of the spectrum.

2. Use your data of part 1 above to plot a full page calibration curve, wavelength as a function of scale reading for the first order of the mercury (or helium) spectrum. Cover the wavelength range from λ4000 to 7000Å and scale readings from 10 to 18.

3. With the angle β held at zero degrees locate the positions of the strong (intense) spectral lines in an unidentified source. Use the calibration curve of part 2 to determine the wavelengths of the lines. Identify the source by comparing these values with those published in spectroscopic tables.

QUESTIONS

1. Distinguish between angular dispersion and resolution (resolving power) of a grating. How is linear dispersion related to angular dispersion?
2. What is the magnitude of the angular dispersion and resolution of the grating used in this experiment "in the region of" the first order green line of mercury?
3. What is the theoretical minimum width of the grating needed to "just resolve" the yellow lines in the mercury spectrum?
4. A given light source contains a prominent blue, a green and a red line. Could the spectrum as seen with this apparatus have the succession of - green, blue and red lines in that order? Explain how this could occur.
5. What change would appear in a spectrum if each of the following changes were independently made:
   a. the width of the grating is doubled?
   b. the number of lines on the grating is doubled?
   c. the distance between the grating grooves (lines) is doubled?
   d. the height of the grating is doubled?
   e. the area of the grating is doubled?
   f. the radius of curvature of the grating is doubled?
   g. the height of the slit is doubled?
   h. the width of the slit is doubled?
   i. the grating were rotated 15 degrees about its <u>optical</u> axis?
   j. the central half of the grating were destroyed?
6. How does the slit-grating-detector assembly assure that they all stay on the Rowland circle
7. Suppose you measure a wavelength to a certain number of significant figures. By what factor must the grating width be increased to get an additional significant figure in this value?

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