

**The Fast Fourier Transform**  
**Sine, Square, Triangle Waves, Filters and the Acoustical Spectrum**  
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This experiment will introduce you to the science of acoustical waves that make up complex sounds/signals. Fourier ( ) showed that any complex sound (well behaved) could be represented by a sum of sines and cosine waves with proper amplitudes, frequencies and phases. This remarkable discovery has contributed to virtually every field of science and technology. To show how this is done and why it is important we will start with simple basic acoustical equipment, some basic wave forms and analyze them with an oscilloscope and a frequency spectrum analyzer (on a computer). You will be using the following equipment:

1. Oscilloscope Tektronix Type 561A  
 Type 2A63 Differential Amplifier (single input) Type 2B67 Time base
2. Two frequency generators: WaveTek 1 - 1Mhz, (0.5 - 4.0) dial giving sin, square and triangle waves from 0 - 40,000Hz
3. Spectrum Analyzer (computer) that will display on the screen the frequencies and amplitudes that make up the complex tone.
4. Filter Sets (computer) You can choose a selection of high pass, low pass, band pass and band stop filters to remove (absorb) desired frequencies.
5. Frequency Counter to measure the frequency exactly.
6. Long Resonant Tube, assorted pipes, loud speaker and microphone = sound level meter = (SLM) to generate those waves that a pipe can support. For this experiment you will determine the speed of sound in air as a function of frequency.
7. Tuning fork 440 Hz

The experiments should be done in the following order.

1. Hook up a signal generator to the scope and display the wave form from a sin, square and triangle wave. From the display calculate the frequency  $f$  and amplitude  $A$  of the wave (What are the units of  $f$  and  $A$ ?). Do scope measurements compare with the dial on the  $f$ -generator? Are the instruments calibrated? Which one would give a more precise reading if both were calibrated? How would you calibrate the instruments?

2. Hook two frequency generators to the scope and display various combination wave forms. Start with sin waves with

a.  $f_1 = f_2$     $A_1 = A_2$

b.  $f_1 \approx f_2$     $A_1 = A_2$

c.  $f_1 > f_2$     $A_1 = A_2$  but with  $f_1/f_2$  as the ratio of small integers - i.e. 2:1, 3:2, 4:3 etc..

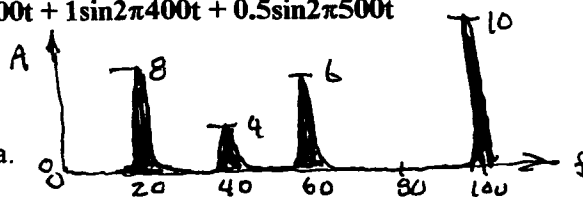
Are they hooked in series or in parallel? Explain the appearance and behavior of the waves forms for each case. For fun switch to square and triangle waves but don't waste time here.

3. Send the signal from one  $f$ -generator into the spectrum analyzer (computer). Read the instructions to learn how to do this. What is happening will be explained later. Pick  $f = \omega/2\pi = 440$  Hz = frequency of the international note A. Display this single frequency in the spectrum analyzer. Your frequency scale should go from 0 - 1000 Hz and amplitude selected to keep the signal on scale. Of course you should see a single peak at 440 Hz --- BUT what is the shape of the peak? What is its FWHM and why does it have a width if

the single  $f = 440.000\text{Hz}$ ? Now run the frequency up and down from  $f = 0$  to  $f = 1000$  Hz and record what you see. The computer = FFT = spectrum analyzer has converted the pure sine wave (a pure tone), which is a (V-t) trace to an (A-f) trace. This is neat.

4. Now put two sin waves into the FFT. Chose frequencies  $f_1 = f_2$   $f_1 \approx f_2$   $f_1 > f_2$  and  $f_1/f_2$  ratios of integers. Observe the results and explain what you see. Adjust the amplitude and phase of the frequency generators and record what happens. At this point we know what the oscilloscope and FFT machine does to pure sin waves and combinations of pure sin waves. What would happen if we added a third frequency generator? Complete the following exercises:

A. Five frequency generators are connected to the FFT machine. Together they put out a wave form given by:  $V(t) = V_s \sin 2\pi f t = 8\sin 2\pi 100t + 4\sin 2\pi 200t + 2\sin 2\pi 300t + 1\sin 2\pi 400t + 0.5\sin 2\pi 500t$   
Plot the amplitude frequency (A-f) spectrum you would see.



B. On the FFT machine you see the spectrum at right  
Write the composite wave form (A-t) that produced this spectra.

We are now ready for the final part of the experiment. We might wonder how the f-generator produces the square and triangle waves. Does it add sin waves with proper A, f and phases? To see, switch a f-generator to produce a 440 Hz square wave and observe the signal on the FFT machine. Describe what you see.

Measure the amplitudes and frequencies seen. What is their relationship. You should now be able to write the equation for a square wave in closed form - as an infinite series of waves. Try it.

Now change f to values above and below 440 Hz and observe the results. What are the frequencies amplitudes and widths (FWHM) of all the peaks.

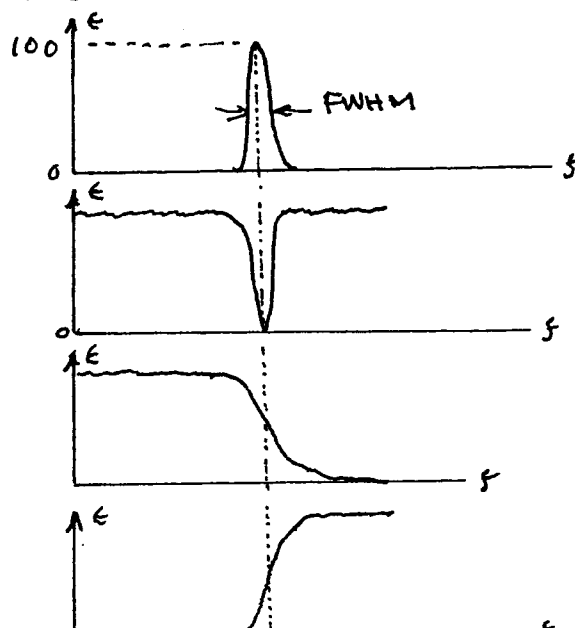
You can experiment a bit by switching to triangle wave. What is the relationship between the f, A and phases? Compare them to the square wave. You can also plug in the second f-generator with a sin or square wave and display a more complex wave form. There should be no surprises - if there are you are missing something.

Always display on the scope all signals you feed into the FFT. This will help you become familiar with certain composite wave forms.

## Filters

There are four kinds of filters used in optics, acoustics, engineering, chemistry etc. They are shown at the right.

What parameters characterize the filters. Check the computer instructions for inserting these filters in the input line to the FFT machine. You should be able to choose the type of filter its center frequency  $f_0$  by proper selection of parameters. Demonstrate that the filters work by feeding into the FFT machine a pure sine wave or a square wave and noting the response of various frequencies. From an EE, or basic electronics or em book find the simple LRC circuits that make these filters. How does the computer do it?



### Microphone (SLM) Loud Speaker and Hollow Tube

Create a pure tone in the loud speaker with  $f = 440$  Hz. Put the speaker at one end of the tube and the SLM at the other end. Record the loudness heard by the SLM as  $f$  varies above and below 440 Hz. To do this scan the frequency  $f$  from 44 Hz to 4400 Hz. Note the frequency of each resonance and relate this to the length of the tube and the speed of sound in air.

Show graphically how three sin waves with proper  $f$  and  $A$  start to generate a square wave. From this deduce the Fourier spectrum of a square wave.

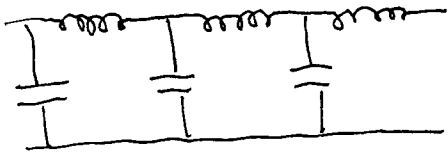
### Timing

Throughout these experiments we have used the following concepts. Explain what they are and their importance to the measurement of time varying signals.

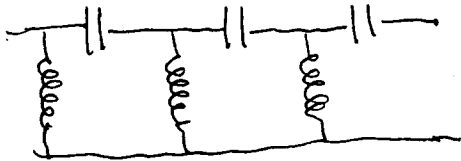
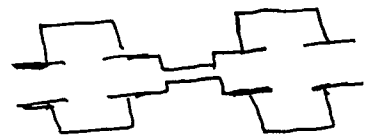
Periods, sweep trace on oscilloscope, duration of sounds, sample time, dead time, time/channel, duty cycle, dwell time, round trip time, reaction time, decay times, attack time, time to make a FFT

teaching/fflab/lab 03/12/02

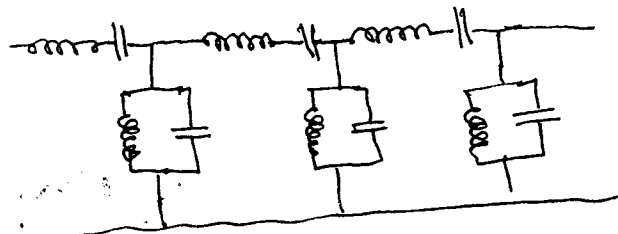
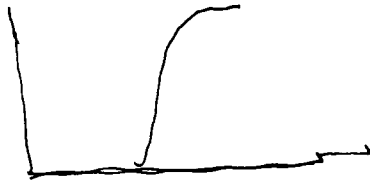
- ① Experiments with the SLM    Vowels A E I O U  
White noise
- ② Draw the standing waves that can exist in an open tube, closed tube and vibrating string. Relate the "segment lengths"  $\equiv$  wave lengths to the round trip times  
RTT  $\equiv$  speed of sound in air ~~or~~ or on the string.  
Calculate the allowed frequencies.



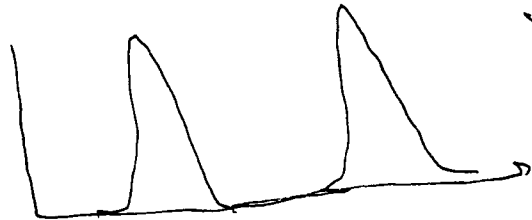
Low pass



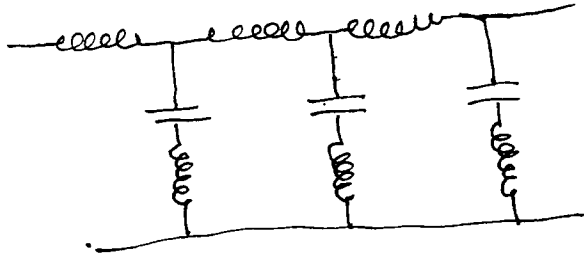
Hi Pass



Band Pass



Pg 124  
Acoustics  
Joseph Hunder



Band Stop.

