

Some of the “Secondary Effects” Involved in the Motion of the Slug for C_p/C_v

http://www.physics.arizona.edu/~haar/ADV_LAB/Gamma_extra.pdf

1. To the first approximation the motion of the slug is assumed to be that of an nondriven, undamped, harmonic oscillator. That is:

$$A = A_0 + A_{MAX} \sin(\omega t) = A_0 + A_{MAX} \sin\left(\frac{2\pi}{T} t\right)$$

$$\omega = k/m \qquad T = \frac{2\pi}{\omega}$$

Where: A is the amplitude,
 A_0 is the initial position
 A_{MAX} is the maximum amplitude,
 t is the time,
 ω is the angular frequency,
 T is the period,
 k is the spring constant, and
 m is the mass .

Figure 1 is a graph of this motion.

2. One correction to this is damping. Damped harmonic motion is described by:

$$A = A_0 + e^{-Bt} A_{MAX} \sin(\omega' t) = A_0 + e^{-Bt} A_{MAX} \sin((\omega^2 - B^2)^{1/2} t)$$

$$\omega' = (\omega^2 - B^2)^{1/2} \qquad B = b/2m$$

Where ω' is the new angular frequency,
 b is the damping constant,
 B is related to the damping constant.

Figure 2 is a graph of this motion.

3. An additional correct may be needed because as air escapes from the system the equilibrium position drops. For undamped harmonic motion this would yield:

$$A = A_0 - vt + A_{MAX} \sin(\omega t) = A_0 - vt + A_{MAX} \sin\left(\frac{2\pi}{T} t\right)$$

Where v is the velocity at which the slug's equilibrium position is dropping .
 Figure 3 is a graph of this motion.

With damping:

$$A = A_0 - vt + e^{-Bt} A_{MAX} \sin(\omega' t) = A_0 - vt + e^{-Bt} A_{MAX} \sin((\omega^2 - B^2)^{1/2} t)$$

Figure 4 is a graph of this motion.

Initial selection of parameters:

Period, T / angular frequency, ω Crudely the undamped period is about 1 second,
Let **$T = 1$ second, then $\omega = 2 \pi = 6.28$ radians/second.**

The initial equilibrium position **$A_0 = 25$ cm** (10 inches)
and the amplitude of oscillations, **$A = 10$ cm** (4 inches)

The amplitude of the oscillations is reduced to about half after one cycles.

$$\text{Let } e^{-B T} = 1/2 \quad \mathbf{B = - \ln(0.5) = 0.7 /second}$$

$$\text{Thus } \omega' = (\omega^2 - B^2)^{1/2} = ((2 \pi)^2 - (0.7)^2)^{1/2}$$
$$\omega' = \mathbf{6.24} \text{ radians/second}$$

The equilibrium position drops about 10 cm (4 inches) in 2 cycles or
 $v 2T = 10$ **$v = 5$ cm/second**

Excel setup

http://www.physics.arizona.edu/~haar/ADV_LAB/CPCV_2.xls

Column A parameters (estimated above)

Column B

Column C time (increments of 0.05 s)

Column D undamped SHO

Column E damped SHO

Column F undamped SHO with dropping

Column G damped SHO with dropping