SELECTIVE EXPERIMENTS IN PHYSICS

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EXP. No. ....

THE MICHELSON INTERFEROMETER

OBJECT: To calibrate a Michelson interferometer and to use the calibration for measuring unknown wavelengths including the wavelength difference between the two sodium D lines.

METHOD: A Michelson interferometer is put into proper adjustment, the movable mirror is displaced, and the resulting changes in the fringe system are observed. The instrument is calibrated using a known wavelength of light, and the interferometer is then used to make wavelength measurements of other "unknown" light sources.

THEORY: Interference is accomplished in the Michelson interferometer by separating a beam of light into two beams which, after having proceeded along different paths, are subesequently united, as shown in Fig. 1. The optical components of such an interferometer consist of two highly polished mirrors, M_1 and M_2, and two glass plates, P_1 and P_2. Plate P_1 is lightly silvered on one surface so that light falling upon it is partially reflected and partially transmitted towards mirrors M_1 and M_2, respectively. After reflection from these mirrors, a portion of the light that follows path (1) will continue on through plate P_1, and a portion of the light that follows path (2) will be reflected by P_1. These two rays will then be travelling in the same direction and thus be in a position to interfere with each other. An eye in the position shown will thus see two images of the source. If the source is monochromatic and extended in area, the resulting field of view will be crossed by bright and dark interference fringes.

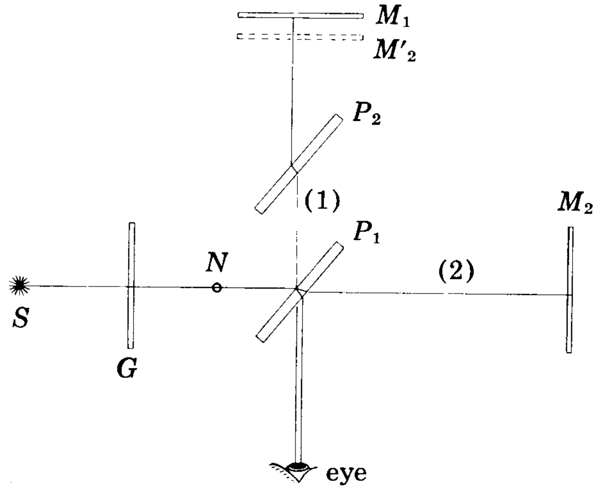

Fig. 1. Optical system of the Michelson interferometer.

When the reflecting surface of P_1 is the front surface, as in Fig. 1, the portion of the light that travels path (2) passes through plate P_1 three times. To compensate for this, a second plate P_2 is inserted in path (1). Without such a compensating plate the optical paths would vary with wavelength because of the dispersion of the glass. This plate must be made of the same glass, have the same thickness, and be oriented at the same angle as P_1. A compensating plate is not needed for monochromatic light, but it is required for viewing fringes in white light. On some interferometers P_2 is mounted in such a manner that it can be rotated slightly to bring it into more accurate parallelism with P_1.

In all Michelson interferometers, mirror M_1 is mounted on a base plate that is supported on well-machined tracks or "ways" so that it can be moved towards or away from the beam splitting plate P_1, and mirror M_2 is mounted on the main frame of the instrument at a fixed distance from P_1. In the Atomic Laboratories (Cenco) M-4 interferometer, the movable plate rests directly on the ways, and it is held securely against them by a lock screw that passes through a slot in the plate. The plate can be moved very slowly in a horizontal direction by means of the micrometer screw that projects from the base of the instrument. This screw acts through an intermediate set of levers, one of which is connected to the lock screw. When the lock screw is loosened the movable plate can be moved to any desired position by hand.

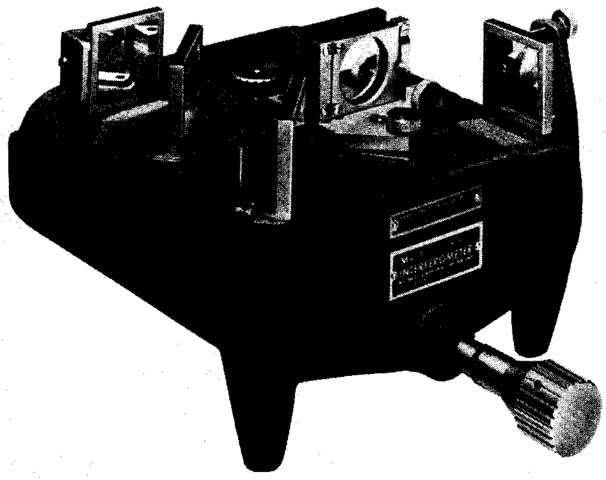

Fig. 2. The Atomic Laboratories M-4 interferometer.

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The nature of the interference pattern produced by a Michelson interferometer can be visualized most simply by observing that the light which follows path (2) can be thought of as being reflected from a partially reflecting mirror M_2' which is the image of M_2 formed by reflection from P_1, as shown in Fig. 1. The equivalent optical system is shown in Fig. 3 where, by omitting the initial reflection from P_1, the extended source of light is in effect behind the observer, and S_1 and S_2 are its images in mirrors M_1 and M_2', respectively. When the mirrors are separated by distance d, the virtual images will be separated by 2d, and the two rays shown will experience constructive or destructive interference, depending on the path difference 2d cos ϕ. If the two mirrors are accurately parallel (i.e., if the real mirrors on the interferometer are accurately perpendicular to each other), interference fringes will consist of concentric circles with bright fringes in those directions for which 2d cos ϕ is an integral number of whole wavelengths—the condition for fringes of equal inclination.

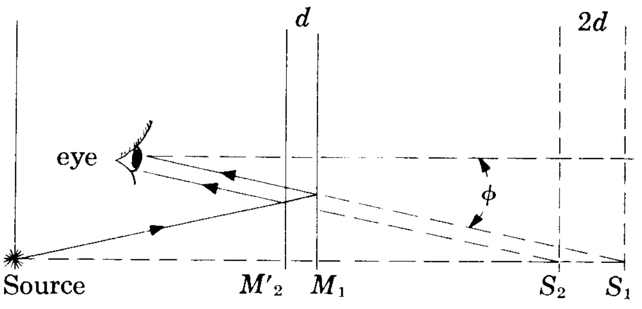

Fig. 3. The equivalent optical system of a Michelson interferometer. When the inteferometer is adjusted for circular fringes, the fringe system is similar to that which would be seen by an observer viewing the reflection of an extended source reflected in two partially reflecting mirrors M_1 and M_2' separated by distance d, where d is the difference in optical paths in the real interferometer and M_2' is the image of the real mirror M_2 reflected in the beam splitting plate P_1 (Fig. 1).

If the separation of the two mirrors in Fig. 3 is a centimeter or so, the fringes will be closely spaced, and quite a large number of circular fringes will be visible in the field of view. If M_1 is moved inwards so that the separation is decreased the fringe system will contract, since, for each fringe, 2d cos ϕ must remain constant. Fringes will thus disappear one at a time at the center of the pattern. Furthermore, as M_1 approaches M_2', the fringes become more widely spaced until, when d = zero, the central fringe will fill the entire field of view. On continuing this motion, fringes will begin to appear at the center and move outwards, and more and more fringes become visible as the separation is increased.

In general, as mirror M_1 is moved a distance d, n fringes will either appear or disappear at the center (or pass a given point near the center) when $n\lambda = 2d$ if λ is the wavelength of the light. Thus, if one counts fringes for a measured motion of the mirror, the wavelength can be calculated. Conversely, the mechanism for moving the mirror can be calibrated in terms of known wavelength.

Fringes in White Light: If an incandescent lamp or some other source of white light is substituted for the monochromatic source, colored fringes are produced, but they can be seen only when the distance d is nearly zero—i.e., only when the two optical paths in the real interferometer are nearly equal in length. This is because white light consists of a continuous distribution of wavelengths, and at anything but a very small value of d the fringe systems for the various components so completely overlap that the field of view is uniformly white.

APPARATUS: Michelson interferometer, sodium arc, mercury arc with green filter, low-power telescope (magnification about one or slightly higher).

Caution: The components of an interferometer are delicate and can easily be damaged; they must be handled with care. Under no circumstances should the mirrors or glass surfaces be touched or the adjustment of any of the parts be forced. Do not attempt to use the instrument until you have studied and thoroughly understand the following material.

PROCEDURE: The interferometer should be mounted on a platform that is as free of vibration as possible and at a height that is convenient for viewing. Using a millimeter rule, adjust the position of the movable mirror M_1 so that it is approximately the same distance from the beam splitting plate as is mirror M_2, and mount a monochromatic source of light in the position S shown in Fig. 1. The monochromatic source might well be a mercury arc fitted with a filter which passes the 5461 Å green line (e.g., a Wratten No. 77 or 77A filter or a thin film interference type filter of appropriate thickness). A frosted glass plate (G in Fig. 1) mounted between the source and the interferometer will serve as an extended source of nearly uniform brightness.

To simplify adjusting the mirrors to perpendicularity, a needle or pin should be mounted between the source and the interferometer, as at N in Fig. 1. The Atomic Laboratories instrument has a pin mounted on the bed of the instrument for this explicit purpose. Two images of the pin will be visible, one reflected from each of the two mirrors. (A weaker set of images may also be visible, due to reflection at the unsilvered surface of the beam splitting plate. These may be disregarded.) By turning the two adjusting screws that control the orientation of the mirrors, one at a time, the two images can be superposed, whereupon fringes appear. Turn the screws slowly; for it will very likely be necessary to bring the two images past the position where they should be superposed several times before fringes are observed. (If a sodium source is used and if fringes cannot readily be obtained, mirror M_1 should be moved a short distance; it may be that the initial setting of the mirror is such that the fringe systems corresponding to the two components of the sodium doublet are out of step as will be explained later.) After fringes have been obtained, the center of the fringe system can be brought into view by further small adjustment of the screws, a little at a time, and in such a direction as to increase the curvature of the fringes.

When an interferometer is properly adjusted the two mirrors are perpendicular to each other and M_2' is parallel to M_1. The adjustment can be checked by moving one's head from side to side. The concentric circular fringes will then move across the field of view, remain the same size, and appear to be at infinity.

Fringes can be viewed either with the unaided eye or with a low-power telescope which has crosshairs in the eyepiece. The magnifying power of the telescope does not need to be more than one. The crosshairs provide a convenient reference system for counting fringes that pass as mirror M_1 is moved.

After having learned how to obtain circular fringes at will, one should become acquainted with the behavior of the fringe system when M_1 is moved. In particular, observe that the direction of motion of the fringes is reversed as M_1 passes through the position of path equality. Then, when the interferometer is adjusted for path equality, substitute a source of white light for the monochromatic source and attempt to obtain colored fringes. It will very likely be necessary to move M_1 slowly one way or the other to bring them into view since the condition for path equality is critical.

I. **Calibration of the Interferometer.** Mount the low-power telescope in viewing position and direct it a little to one side of the center of the circular pattern. It is somewhat easier to count the fringes that pass the crosshairs when the telescope is in this position than to count the fringes as they either appear or disappear at the center. Obtain several values of the change in reading of the micrometer screw which causes, say, 100 fringes to move past the crosshairs. To obtain a number of readings expeditiously, take readings of every tenth fringe for about 190 fringes, and treat the data as follows. Calculate the difference between the readings for the zeroth and the 100th fringes, the 10th and the 110th, etc., on up to the 90th and the 190th fringes. From the average of these data and the known wavelength of the light used, calculate the distance that mirror M_1 moves for each revolution of the micrometer screw using the equation $n\lambda = 2d$.

One can check this result for the Atomic Laboratories M-3 interferometer by careful measurement of the radius r of the spindle which moves M_1 and the length L of the lever which rotates the spindle. Except in extreme positions of the micrometer screw when the cosine of the angle must be taken into consideration, M_1 will move a distance equal to r/L times the distance the micrometer screw moves.

The motion of the base that supports M_1 can also be measured directly by using a micrometer microscope together with a scale accurately graduated to, for example, tenths of a millimeter. The scale is attached to the movable base and the microscope is mounted to one side in such a position that the scale can be read conveniently. The microscope should be clamped firmly in position so that it will not move relative to the instrument while readings are being taken. With the assistance of a second person, the number of fringes which pass the field of view can then be counted while mirror M_1 moves through successive one-tenth millimeter steps.

II. **Determination of Wavelengths of the Sodium D Lines.** The yellow line of sodium consists of two lines, λ_1 and λ_2, separated by a few Angstrom units. Therefore, when a sodium arc is used with a Michelson interferometer, each line produces a separate fringe system. At certain positions of the movable mirror the two sets of fringes fall on top of each other, and sharp fringes are produced. This, of course, occurs when the two optical paths have the same length. If M_1 is then moved either outwards or inwards the two sets move at different rates and thus gradually get out of step until, when the mirror has been moved through distance d such that

$$2d = n_1 \lambda_1 = (n_1 + \tfrac{1}{2})\, \lambda_2$$

they will be exactly interlaced, and the fringe pattern will be least distinct. (The field of view will be almost uniformly bright since the two lines are almost equally intense; the shorter wavelength line is somewhat more intense than the longer.) As the mirror continues to move in the same direction, the two fringe systems will again become superposed when

$$2d = n_2 \lambda_1 = (n_2 + 1)\, \lambda_2 \qquad (1)$$

Thus, by determining the distance between adjacent settings for which the fringes are most distinct (or least distinct), one can calculate the difference between the two wavelengths. From Eq. 1,

$$\lambda_1 - \lambda_2 = \frac{\lambda_2}{n_2} = \frac{\lambda_1 \lambda_2}{2d}$$

or, the difference is given by

$$\Delta \lambda = \frac{\lambda^2}{2d} \qquad (2)$$

where λ is the geometric mean of the two wavelengths.

1. **Average Wavelength of the D lines:** Adjust the interferometer to give distinct fringes when a sodium arc is used as the source of light. Obtain a series of micrometer readings for every 10th fringe that passes the crosshairs for 190 fringes. (Make a preliminary quick run to make sure that this number of fringes will remain satsifactorily visible over the range of motion required.) Average your data as outlined earlier, and use the calibration obtained in Part I for calculating the wavelength of the sodium light. Note that this result should be the mean of the two wavelengths λ_1 and λ_2.

2. **Difference in wavelength between the Sodium D Lines:** Obtain as many settings as the range of the micrometer screw will permit for the positions of most distinct fringes and least distinct fringes. Determine the distance d from your data and calculate the wavelength difference between the two lines of sodium, using Eq. 2.

Considerably more data can be obtained with the Atomic Laboratories instrument by using a micrometer microscope and a scale accurately divided into, say, tenths of a millimeter, mounted as described in Part I. The ways (Fig. 2) permit considerably more motion of mirror M_1 than can be produced by the micrometer screw. However, the base on which the mirror is mounted must then be moved by hand—one hand carefully placed on each side. The scale is read each time fringes become most distinct and also each time they are least distinct, and the data are treated as described for a similar set of data in Part I. If, say, 19 readings are taken, one can calculate the difference between the zero and the tenth readings, the first and the eleventh, and so on. From the final average distance, d can be determined and $\Delta\lambda$ can then be calculated.

QUESTIONS: 1. A student obtains micrometer readings for the setting of every tenth fringe for 190 fringes as suggested in Part I. However, instead of treating these data as suggested, he simply takes the difference between successive readings and averages the resulting differences. Show that he is making use of only the first and the last readings. (The intermediate readings might as well not have been taken.)

2. The index of refraction of a gas can be determined by introducing into one of the arms of an interferometer a gas cell which has plane glass end windows and a side tube for changing air pressure. The number of fringes that pass the field of view is counted as the pressure is changed by one

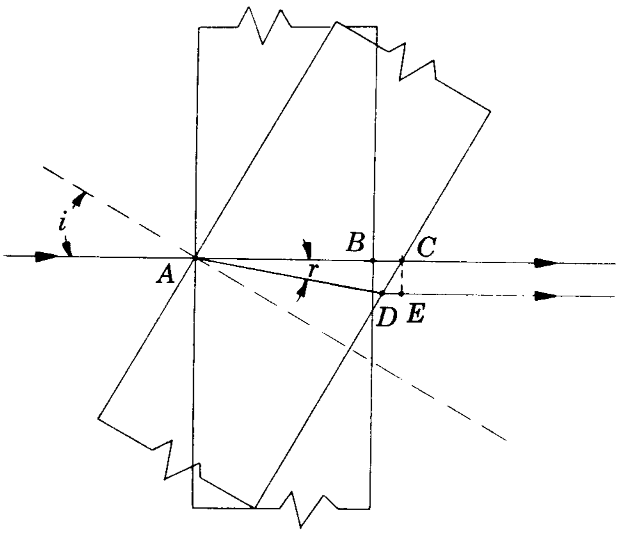

Fig. 4. When a plate of glass is rotated through angle i, the optical path changes from AB in glass plus BC in air to AD in glass plus DE in air.

atmosphere. Calculate this number for a cell of length 2.00 cm when a gas, having a refractive index 1.0003 at the wavelength used, is pumped out of the cell.

3. The index of refraction of a glass plate can be determined by interposing the plate in one of the arms of an interferometer and counting the number of fringes that pass the field of view as the plate is rotated through a measured angle. A certain plate, 0.500 mm thick and having a refractive index of 1.50 at the wavelength used, is inserted and rotated slowly through 30° from its initial position normal to the light. Calculate the number of fringes that pass the field of view. (See Fig. 4. Note that the optical path changes from *AB* in glass plus *BC* in air to *AD* in glass plus *DE* in air.)

4. An interferometer is used to study the separation of the *D* lines of sodium. Calculate the number of fringes that pass the field of view for each of the two wavelengths (5890 A and 5896 A) when the movable mirror is moved between settings for which the two sets of fringes are superposed.