

The Feynman

LECTURES ON PHYSICS

MAINLY ELECTROMAGNETISM AND MATTER

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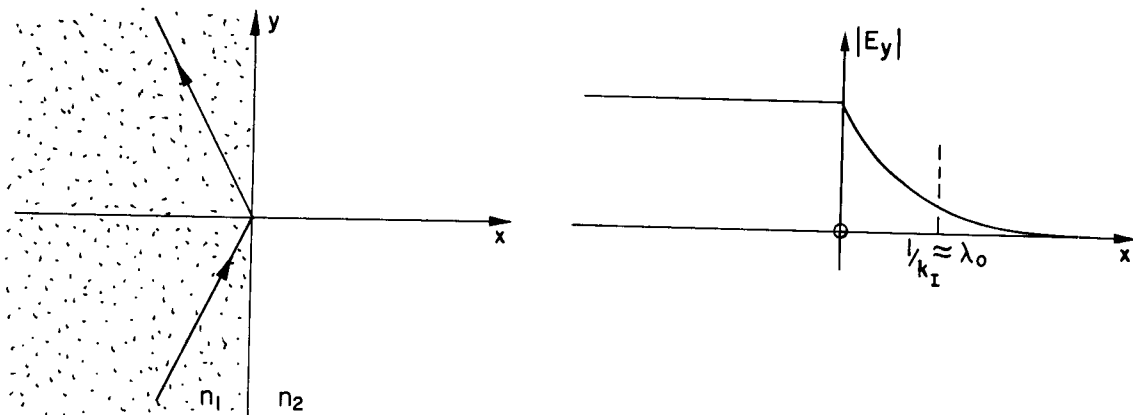


Fig. 33-9. Total internal reflection.

33-6 Total internal reflection

If light goes from a material like glass, with a real index n greater than 1, toward, say, air, with an index n_2 equal to 1, Snell's law says that

$$\sin \theta_t = n \sin \theta_i.$$

The angle θ_t of the transmitted wave becomes 90° when the incident angle θ_i is equal to the "critical angle" θ_c given by

$$n \sin \theta_c = 1. \quad (33.59)$$

What happens for θ_i greater than the critical angle? You know that there is total internal reflection. But how does that come about?

Let's go back to Eq. (33.45) which gives the wave number k_x'' for the transmitted wave. We would have

$$k_x''^2 = \frac{k^2}{n^2} - k_y^2.$$

Now $k_y = k \sin \theta_i$ and $k = \omega n/c$, so

$$k_x''^2 = \frac{\omega^2}{c^2} (1 - n^2 \sin^2 \theta_i).$$

If $n \sin \theta_i$ is greater than one, $k_x''^2$ is *negative* and k_x'' is a pure imaginary, say $\pm ik_I$. You know by now what that means! The "transmitted" wave (Eq. 33.34) will have the form

$$E_t = E_0'' e^{\pm k_I x} e^{i(\omega t - k_y y)}.$$

The wave amplitude either grows or drops off exponentially with increasing x . Clearly, what we want here is the negative sign. Then the *amplitude* of the wave to the right of the boundary will go as shown in Fig. 33-9. Notice that k_I is of the order ω/c —which is λ_0 , the free-space wavelength of the light. When light is totally reflected from the inside of a glass-air surface, there are fields in the air, but they extend beyond the surface only a distance of the order of the wavelength of the light.

We can now see how to answer the following question: If a light wave in glass arrives at the surface at a large enough angle, it is reflected; if another piece of glass is brought up to the surface (so that the "surface" in effect disappears) the light is transmitted. Exactly when does this happen? Surely there must be continuous change from total reflection to no reflection! The answer, of course, is that if the air gap is so small that the exponential tail of the wave in the air has an appreciable strength at the second piece of glass, it will shake the electrons there and generate a new wave, as shown in Fig. 33-10. Some light will be transmitted. (Clearly, our solution is incomplete; we should solve all the equations again for a thin layer of air between two regions of glass.)

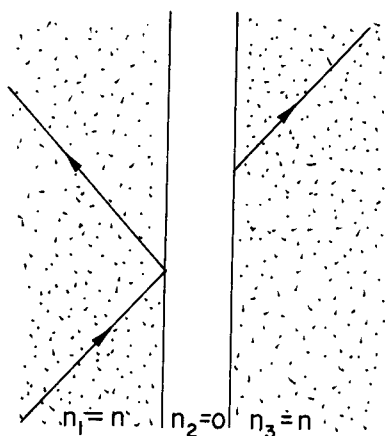


Fig. 33-10. If there is a small gap, internal reflection is not "total"; a transmitted wave appears beyond the gap.

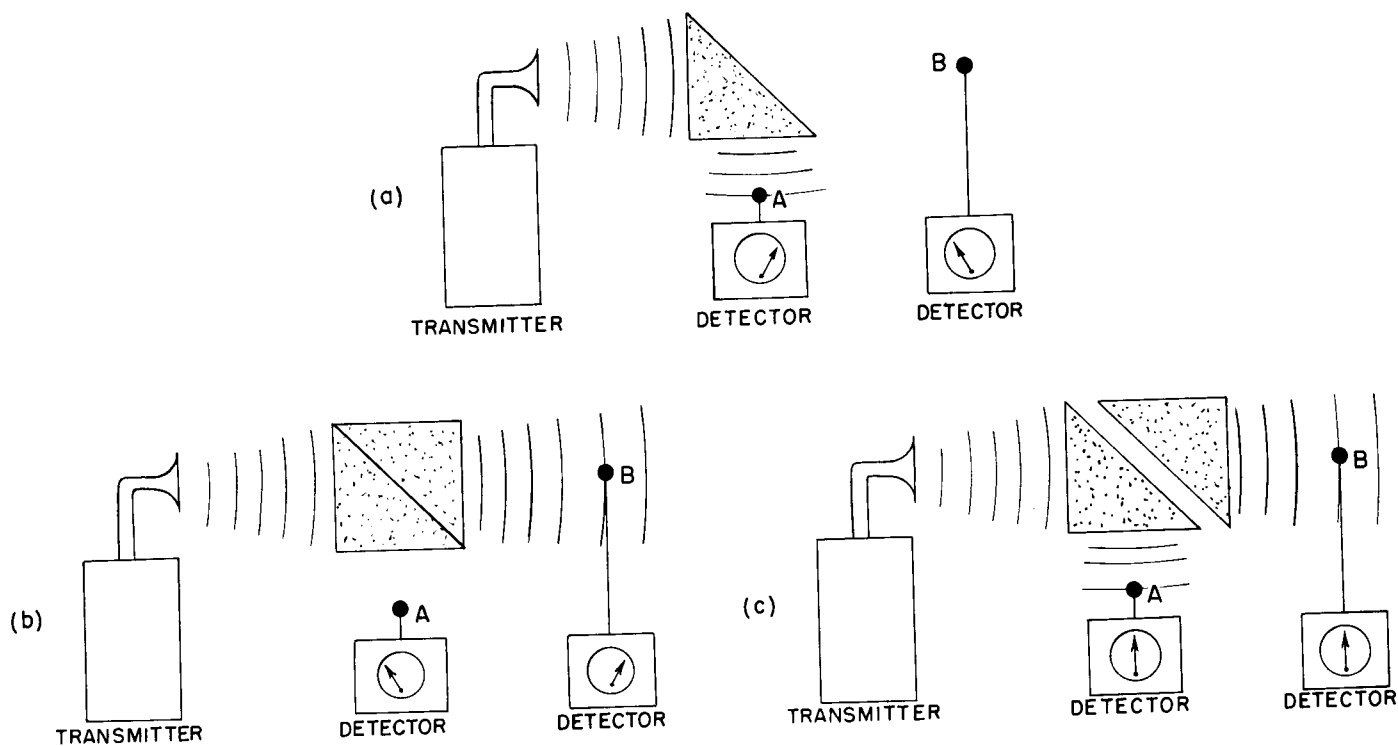


Fig. 33-11. A demonstration of the penetration of internally reflected waves.

This transmission effect can be observed with ordinary light only if the air gap is very small (of the order of the wavelength of light, like 10^{-5} cm), but it is easily demonstrated with three-centimeter waves. Then the exponentially decreasing field extends several centimeters. A microwave apparatus that shows the effect is drawn in Fig. 33-11. Waves from a small three-centimeter transmitter are directed at a 45° prism of paraffin. The index of refraction of paraffin for these frequencies is 1.50, and therefore the critical angle is 41.5° . So the wave is totally reflected from the 45° face and is picked up by detector *A*, as indicated in Fig. 33-11(a). If a second paraffin prism is placed in contact with the first, as shown in part (b) of the figure, the wave passes straight through and is picked up at detector *B*. If a gap of a few centimeters is left between the two prisms, as in part (c), there are both transmitted and reflected waves. The electric field outside the 45° face of the prism in Fig. 33-11(a) can also be shown by bringing detector *B* to within a few centimeters of the surface.