SELECTIVE EXPERIMENTS IN PHYSICS

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EXP. No. ....

INTERFERENCE OF MICROWAVES

OBJECT: To measure the wavelength of microwave electromagnetic radiation using interference from two sources of waves.

METHOD: Three different experimental methods are used for producing interference between two coherent sources of waves obtained from one microwave generator. In one method the microwave generator is connected to two antennas, thus producing two real coherent sources of radiation. In another method a plane metallic mirror is used in connection with the microwave generator to produce interference between a real and a virtual source. The third method uses two fairly large slits cut in a metallic sheet. This sheet is placed in front of the microwave generator so that the waves emerging from the slits produce interference. By measuring the positions of the maxima and minima of the interference pattern relative to the two sources the wavelength of the microwaves can be obtained.

THEORY: An important property of all kinds of progressive waves is that they can show the phenomenon of interference. To produce interference, two or more similar waves must be superimposed at the same point. The resultant displacement is the vector sum of the displacements of the waves producing the interference. If, at some point, one wave is producing a crest while the other wave is producing a trough, the two waves cancel one another. If both are producing a crest or trough simultaneously, the resultant displacement is the sum of the two separate ones.

Suppose that two coherent sources of waves S_1, S_2, Fig. 1, producing waves of the same wavelength λ and frequency f are in phase; that is, if S_1 is producing a crest at some instant of time, then at this same instant of time the source S_2 is producing a crest. Consider the effect of these two sources

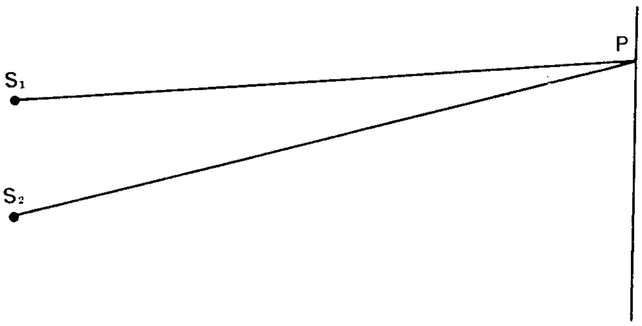

Fig. 1. Two sources of waves producing interference at P.

of waves at some point P. These waves travel different distances to the point P, and the difference in distance determines the relative phases of the waves from S_1 and S_2 arriving at P. This path difference is $(S_2P - S_1P)$. Since the phase of a wave changes by 2π radians in a distance of one wavelength λ, then the path difference $(S_2P - S_1P)$ corresponds to a phase difference at P of $2\pi(S_2P - S_1P)/\lambda$.

If the waves from S_1 and S_2 start out in phase they will arrive in phase at P if the phase difference is some integral multiple n of 2π. Thus there is constructive interference at P if

$$2\pi(S_2P - S_1P)/\lambda = 2\pi n \tag{1}$$

or if

$$S_2P - S_1P = n\lambda \tag{2}$$

Similarly, there is destructive interference at P if the phase difference is an odd number of π radians or if the path difference is an odd number of half wavelengths; that is, if

$$S_2P - S_1P = (2n + 1)\lambda/2 \tag{3}$$

The locus of points showing interference is such that $(S_2P - S_1P)$ is some constant distance. If the constant distance is an integral number of wavelengths, constructive interference takes place. The locus of points having $(S_2P - S_1P)$ equal to a constant is an hyperbola with the points S_1 and S_2 as foci.

A horizontal section of the wave pattern from two similar sources S_1 and S_2 is shown for some particular instant in Fig. 2. In this figure the sources are set apart a distance of

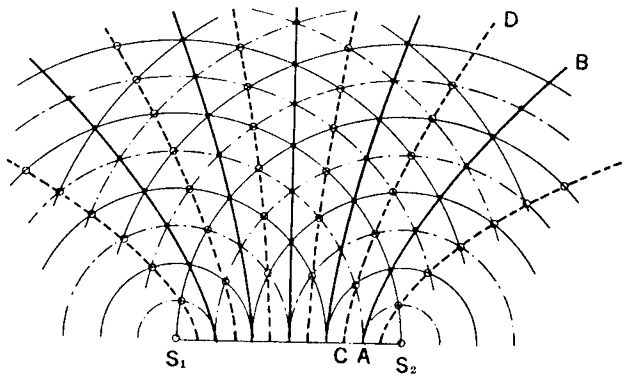

Fig. 2. Interference pattern produced by two sources S_1 and S_2. Maxima or constructive interference along the solid lines, AB etc., while the minima or destructive interference is along the dashed lines, CD etc.

3λ, and the full circles represent the crests while the dot-dash circles represent the troughs at the instant considered. Constructive interferences between the two waves occurs at the intersection of two full or two dot-dash wave patterns as shown by the line AB in Fig. 2. The intersection of a full and a dot-dash circle represents a crest from one source and a trough from the other source arriving together. This would correspond to destructive interference as shown by the line CD. As the waves from the sources move out the points of interference move out along hyperbolic paths.

The fact that electromagnetic radiations can produce in-

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(formerly L69b)

terference shows that these radiations are propagated as a wave motion. All electromagnetic waves have a common speed in a vacuum, namely the speed of light. It was about 1864 that Maxwell announced the electromagnetic theory of light. This theory was developed from the experimental results of Faraday and Ampere. Maxwell expressed these results in terms of electric and magnetic fields. Faraday's law of electromagnetic induction may be stated as follows: *a time rate of change of the magnetic field intensity at a point produces a space rate of change of the electric field.* Ampere's law gives the magnetic field produced by a current. Maxwell extended the concept of current to include a changing electric field, so Ampere's law for a vacuum becomes: *a time rate of change of the electric field intensity at a given point produces a space rate of change of the magnetic field at that point.* The mathematical expression of these statements can be found in texts on electricity and magnetism where it is shown how the speed of light can be obtained from suitable electric and magnetic measurements.

Two coherent sources of waves can be produced from a single microwave generator by connecting the generator to

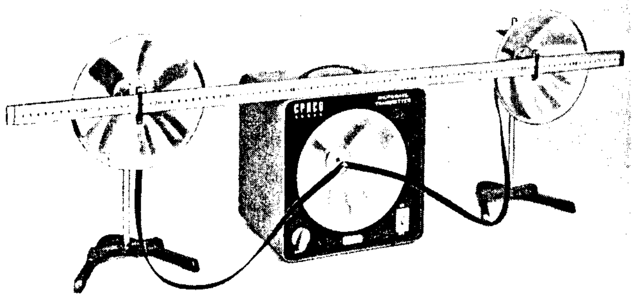

Fig. 3. Double Dipole Antenna shown in use with Transmitter and Reflectors.

two suitable antennas. In Fig. 3 these are shown with reflectors and mounted on a meter stick. Though the reflectors tend to concentrate the radiation in a forward direction, some spreading takes place and interference of the two sets of waves can be observed at a distance from the antennas.

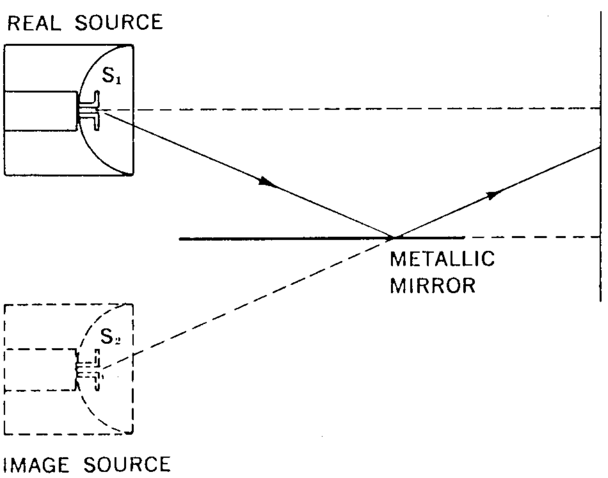

Fig. 4. Lloyd's Mirror.

In order to produce interference close to the antennas, the reflectors should be removed.

Another method of obtaining two coherent sources of waves from a single generator is the use of a metallic mirror as shown in Fig. 4. The corresponding experiment in optics was first performed by Lloyd; for this reason the experiment is called Lloyd's mirror. It was Lloyd who showed that the image source is 180° out of phase with the real source; that is, there is a change of phase of 180° of the electric field of the microwave beam upon reflection. From this it follows that the plane of the mirror is a plane of destructive interference.

A third method for producing two coherent sources of radiation from a single microwave generator uses two slits placed symmetrically in front of the generator as shown in

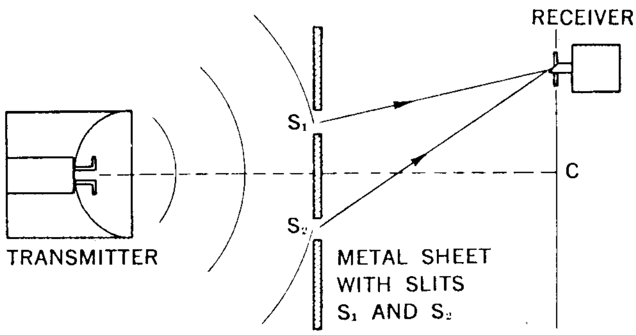

Fig. 5. Young's Two Slit Experiment.

Fig. 5. This double slit interference was first used in optics by Thomas Young about 1812, and the experiment is known as Young's experiment.

The maxima and minima of the interference patterns are located by a halfwave dipole connected to a rectifying crystal. In Fig. 6 this system is shown connected to a micro-

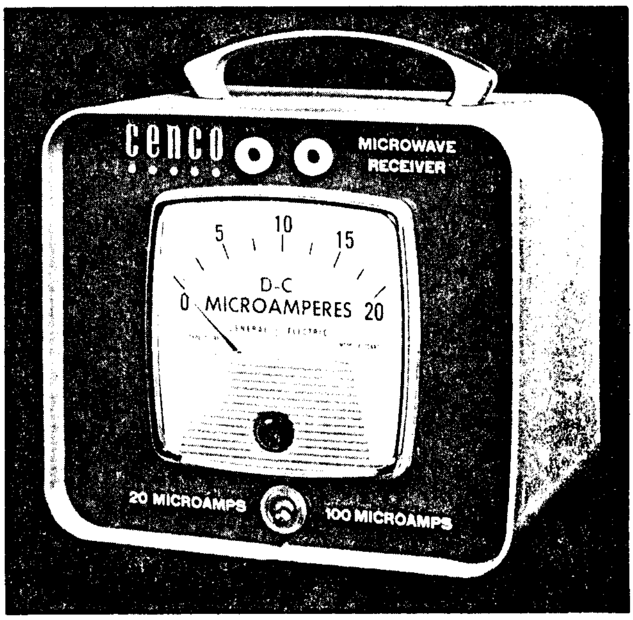

Fig. 6. Microwave Receiver.

ammeter. Figure 7 shows the dipole and rectifying crystal mounted so as to be used with an oscilloscope or earphones. Since the microwave signal is modulated with the 60 cycle/sec ac, the rectified output has the form shown in Fig. 8.

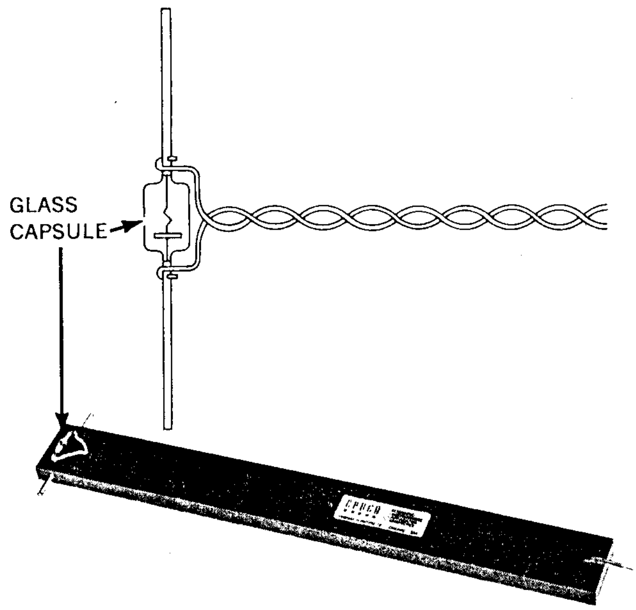

Fig. 7. Probe for the oscillating electric field.

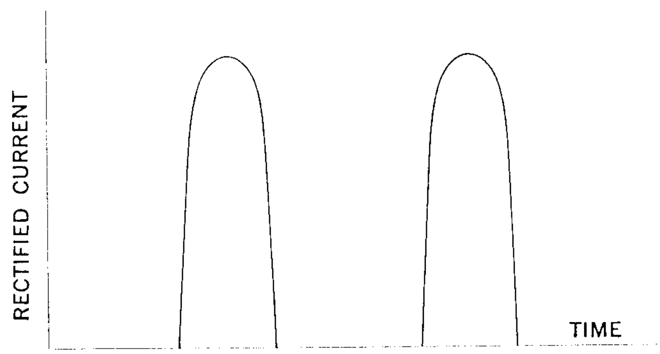

Fig. 8. Graph of rectified current against time.

APPARATUS: The apparatus consists of a microwave transmitter, a double dipole antenna adaptor with reflectors, Fig. 3; a microwave receiver, Fig. 6; a dipole probe for the electric wave, Fig. 7; a plane metallic mirror, a metal sheet with slits cut in, Fig. 9; and a cathode ray oscilloscope or earphones. The following items also are required: a block of wood to raise the receiver above the table top to the height of the antennas, some form of absorber such as a two-inch thick wooden block to screen the receiver from the direct radiation of the transmitter, and a two-meter stick to measure distances.

PROCEDURE, I, Measurement of Wavelength: Set up the microwave transmitter with the antenna adaptor and reflectors as shown in Fig. 3. Place this near the end of a wide wooden table. If desired the two-antenna adaptor can be used without the reflectors. In this case the antennas should be slipped over a meter stick supported by a wooden clamp.

Turn on the microwave transmitter and allow about a minute for the microwave tube to warm up. Then turn the knob on the left clockwise as far as possible. This increases the output of the microwave transmitter.

As detectors of the interference pattern, either the microwave receiver with microammeter or the electric probe with an oscilloscope can be used. The receiver should be placed about a meter from the antennas and should be mounted on a block of wood so that it is at the same height as the antennas. A block of wood about two inches thick should be placed in front of the microwave transmitter so as to shield the receivers from the direct radiation.

Place the receiver in the central position at equal distances from the two antennas. The receiver will show a maximum response if the phases of the radiation from each of the two antennas is the same. If the response is not a maximum, reverse the direction of one of the antennas on the meter stick. Move the receiver along a line parallel to the two antennas and locate the various maxima and minima of the interference pattern. Measure the distances S_2P, S_1P of each maximum and minimum from the two antennas, Fig. 1. Make a column tabulation of the distances S_2P, S_1P and $(S_2P - S_1P)$. The difference in distance $S_2P - S_1P$ is an integer multiple of $\lambda/2$ so that a measure of $\lambda/2$ is obtained from each of these differences. Calculate for each of the maxima and minima measured, the value of the wavelength of the microwave radiation.

(Optional): Reverse one of the antennas on the meter stick and make measurements on the distances of the maxima and minima from the antennas when the radiation from the two sources is out of phase. Trace out two hyperbolic paths of interference, one of maximum and the other of minimum amplitude. From these measurements find the wavelength of the microwave radiation.

II, Lloyd's Mirror Interference: Replace the two-antenna adaptor with the single dipole antenna. Be sure this single dipole makes good contact with the transmitter. This can be checked by noting the response of the receiver as the antenna is pushed in. Place a rigid sheet of metal M about 30 cm from the antenna S_1 of the real source as shown in Fig. 4. (The aluminum mirror used in experiment 71990-L70c, *Standing Electromagnetic Waves*, is suitable). Mark a line on or clamp a two-meter stick to the wooden table in the position APB of Fig. 4 about a meter from the antenna S_1. As shown in the figure the line S_1A is parallel to the plane of the mirror M and at right angles to the line APB. From the measurements of the distances S_1A and AB, where B is the extension of the plane of the mirror to the line APB, find the position of S_2 the image source antenna.

Since the interference is produced by the direct and reflected waves, it is immediately apparent that the interference pattern can only be formed on the real-source side of the mirror. Locate as many maxima and minima as possible

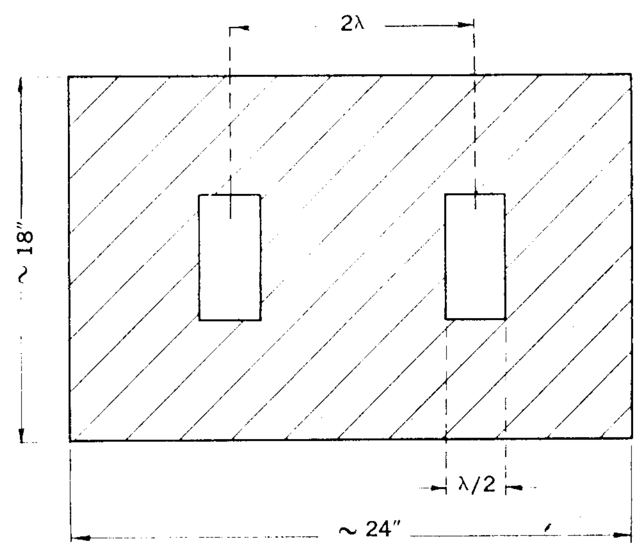

Fig. 9. Screen with Two Slits as Secondary Sources.

as the receiver is moved along the line BA. For each of these positions calculate the distances from the real and image source antennas. From these measurements calculate the wavelength for the microwave radiation. Check the data to show that the mirror causes a change of 180°, i.e. that the radiation from the real and image sources is 180° out of phase.

III, Young's Experiment: A rigid metal sheet, about eighteen inches by twenty-four inches or larger, has two rectangular slits cut out as shown in Fig. 9. There is nothing critical about the width of the slits or their distance apart, though they should be of the order of magnitude of those shown in the figure. Place this metallic sheet so that the midpoint between the slits is directly in front of the antenna of the transmitter and about 30 cm from it.

Place the receiver about a meter from the metallic sheet and locate the central maximum of the interference pattern along the central line to C, as shown in Fig. 5. See that the heights above the table top of the antenna, the centers of the slits and the dipole of the receiver are all the same. Be sure that there are no metallic objects lying on or near the table which can reflect radiation into the receiver. Locate the first minimum on either side of the central maximum as the receiver is moved in a line parallel to the slits. Measure the distances from the centers of the slits to these minima and find the wavelength of the radiation. In a similar manner locate the position of the first maximum on either side of the central maximum and, from the measurements of the distances, find the wavelength of the radiation.

IV, Analysis of data: Find the average value of the wavelength of the microwave radiation as measured in Parts I, II, and III. Given the frequency of the oscillations of the transmitter as 2450 megacycles, find the theoretical value of the wavelength of the radiation. Compare this value with the average measured value and give the per cent difference. Comment on the possible sources of error in the determination of the wavelength.

QUESTIONS: 1. Describe a simple test for determining whether the two antennas in Fig. 3 are radiating in phase.

2. To which component of the microwaves, the electric or magnetic, are the receivers sensitive?

3. If a receiver is placed on a maximum and then is turned at right angles, so that the dipole of the receiver is at right angles to the antennas, would you expect the reading of the microammeter to be a maximum or a minimum? Give reasons for your answer.

4. The transmitter tube in this experiment produces oscillations when its plate potential, which is fed by the 60 cycle ac, reaches a sufficiently high positive voltage. This takes place during one quarter of a cycle. Approximately how many oscillations are produced during each sixtieth of a second?

5. Suppose the single transmitter feeding the two antennas was replaced by two similar transmitters of the same frequency. Would you expect an interference pattern to be produced by these two transmitters? Give reasons for your answer.

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EXP. No. ....

MICROWAVE DIFFRACTION and ZONE PLATE

OBJECT: To investigate the diffraction of a plane electromagnetic wave falling on a circular opening and to measure the wavelength of the incident microwaves.

METHOD: A plane beam of microwaves is incident perpendicularly on a metal plate having a circular opening. After the microwaves pass through the opening their intensity is measured at points along the axis of the circular opening by means of a diode-rectifier receiver. The distances of the positions of maximum and minimum intensity, both from the center and from the edge of the opening, are used to determine the wavelength of the microwaves.

THEORY: The phenomena which characterize traveling waves are interference and diffraction. *Interference* occurs when two or more waves act simultaneously on the same region of space. When the waves are of the same speed and wavelength the interference pattern at any given point remains unchanged. *Diffraction* occurs whenever a wave front is intercepted by an obstacle or is limited by a narrow opening. These phenomena were investigated by a number of physicists, among whom were Huygens, Young, and Fresnel. It was Huygens who, in the seventeenth century, first interpreted the phenomena of interference and diffraction by assuming that each point on a wave front acts as a source of secondary wavelets, the envelope of which forms the new wave front. Fresnel extended the Huygens' principle to include sinusoidal waves, and he was thus able to predict quantitatively the effects of diffraction. This Huygens-Fresnel theory will herein be applied to the diffraction of a plane wave by a circular opening.

Consider a plane wave incident perpendicularly on a metal plate with a circular opening of diameter AB, Fig. 1.

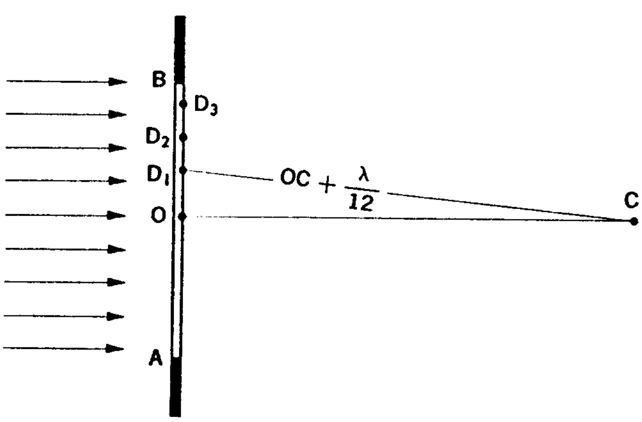

Fig. 1. Plane wave incident perpendicularly on a circular opening of diameter AB.

The problem is to find the amplitude and intensity of the waves at some point C on the axis of the circular opening. According to the Huygens-Fresnel theory every point on the wave front AB emits secondary wavelets. The resultant amplitude produced by these wavelets at any point, such as C, Fig. 1, is the vector sum, added with due regard to phase, of the amplitudes of all the wavelets arriving at C.

Wavelets which start out at the same time and in phase but from different points on the wave front AB arrive at C at different times and in different phases. Thus for two points O and D_1 differing in distance from the point C by a twelfth of a wavelength, $\lambda/12$, the wavelets arriving at C from O and D_1 will have a phase difference of

$$\frac{2\pi}{\lambda}\frac{\lambda}{12} = \frac{\pi}{6} \text{ radians} = 30°$$

This is because in a distance of one wavelength λ there is a change in phase angle of 2π radians, or 360°. For each pair of points D_1, D_2, etc., whose difference in distance to the point C is $\lambda/12$, the wavelets, starting out in phase, will arrive at C with a phase difference of 30°.

For the purposes of analysis the wave front AB may be considered to be broken up into circular zones, a central one of radius OD_1 and rings of width D_1D_2, D_2D_3 etc. The amplitude at C contributed by the wavelets originating in the zones is proportional to the area of the zones. As may be shown by answering question 5 at the end of this experiment, the area of the zones is constant if the wavelength λ is small compared to the distance OC. Theory indicates that, due to a so-called obliquity factor, there is also a progressive decrease in amplitude at C of the wavelets originating at points which are farther from the center O. The amplitude is greatest in the forward direction and gradually decreases to zero in the reverse direction.

As an approximation it may be assumed that all of the wavelets from a given zone will arrive at C in phase; the total amplitude from the wavelets in a zone can be represented by a vector. The vectors representing successive zones have slightly different magnitudes, due to the obliquity factor, and they are at angles of 30° to each other. These are shown in Fig. 2 where the vector marked 1 represents the contribution from the central zone, vector 2 the contribution from the first ring of width D_1D_2, and so on as shown in Fig. 1.

Suppose the circular hole is such that it contains the first five zones. Then the resultant amplitude at C is given by the vector sum of the first five vectors and is represented in magnitude and direction by the vector OP, Fig. 2. The resultant wave at C has a phase difference of 75° with the wavelet from the center O. If the opening is divided into an increasing number of smaller rings the vectors representing the contribution from the wavelets in these rings become

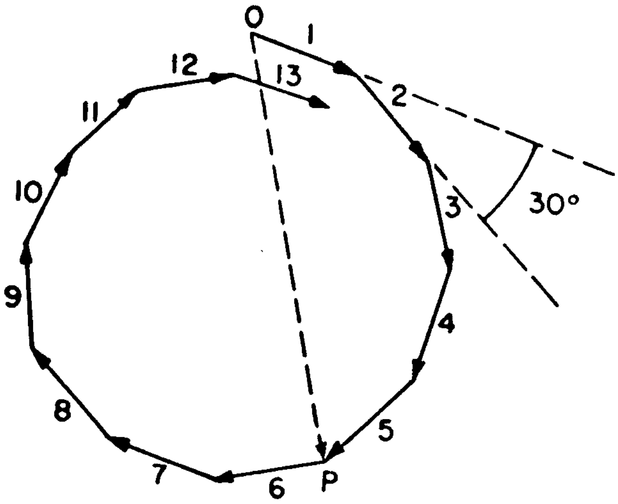

Fig. 2. Vector sum of amplitudes of disturbance from twelve rings.

smaller and smaller, at the same time increasing in number until the twelve-sided figure of Fig. 2 becomes, in the limit, the smooth spiral reproduced in Fig. 3. This spiral is used to predict the amplitude of the wavelets at point C in Fig. 1 from increasingly larger apertures. The intensity at C is proportional to the square of the resultant amplitude produced at C by the secondary wavelets.

In order to make quantitative predictions of the intensity of the radiation reaching point C, Fig. 1, consider the wave front on AB as being divided into zones in the following manner: with C as a center draw a series of spheres having radii of $CO + \lambda/2$, $CO + 2\lambda/2$, $CO + 3\lambda/2$, etc., thus making the radius of each sphere a half-wavelength longer than the preceding one. These spheres intersect the wave front along AB in circles dividing the wave front into rings, or zones, known as Fresnel zones. If only the first, or central, zone of limiting radius $CO + \lambda/2$ is exposed so as to contribute wavelets at C, then the amplitude of the resultant wave at C is given by OO_1 in Fig. 3. Opening the aperture wider so that the limiting radius is $CO + 2\lambda/2$ the resulting amplitude at P is nearly zero, and it is represented by OO_2 in Fig. 3. The effect of opening a second zone and having the intensity decrease can be shown in a very striking manner with the microwave apparatus. It is seen from Fig. 3 that, as the size of the hole increases to add more and more zones, the resulting amplitude alternately increases and decreases: an odd number of zones produces the larger amplitude, an even number of zones produces an almost zero amplitude.

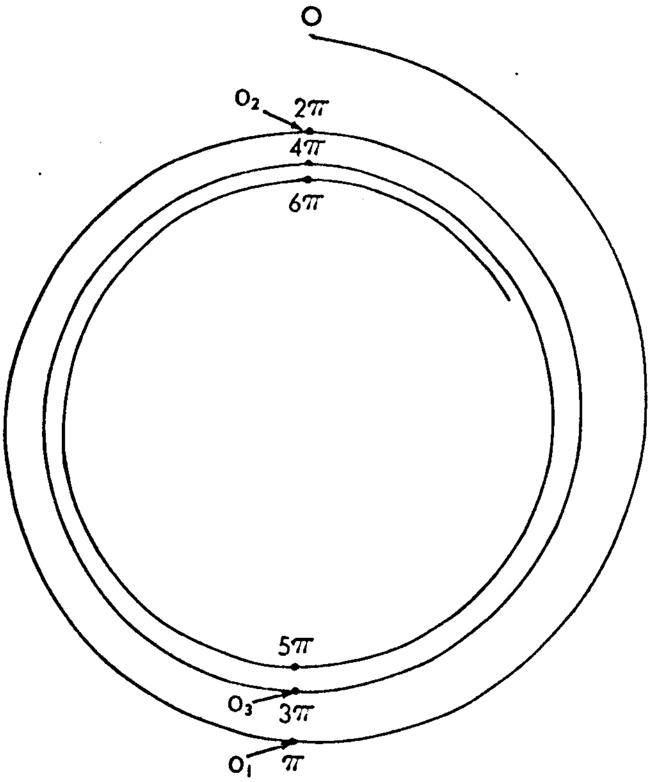

Fig. 3. Spiral for determination of amplitude and phase.

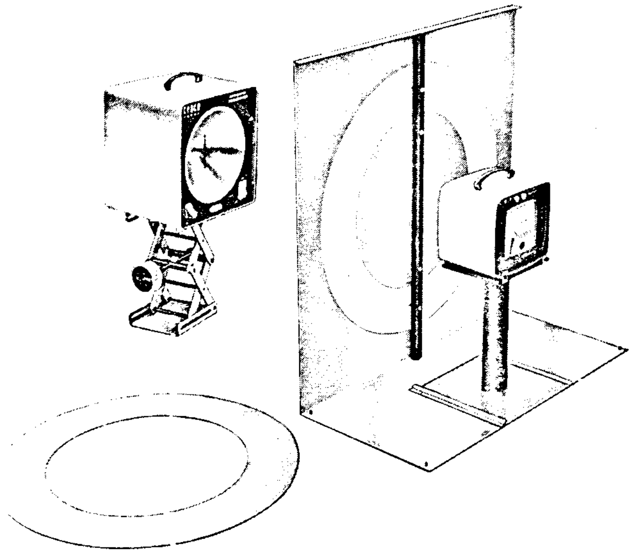

Fig. 4. Arrangement for studying diffraction by a circular aperture.

APPARATUS: A microwave transmitter, a microwave receiver, a zone plate (Fig. 4) and a meter stick are required. The microwave receiver consists of a silicon crystal diode detector, Sylvania 1N82A, sealed in a glass envelope. Rigid metal rods of small diameter are connected to the ends of the crystal diode and are of such a length as to form a half-wave dipole antenna. This antenna is connected to a microammeter mounted in a plastic case, Fig. 5; it may also be connected to a cathode ray oscilloscope. When the dipole is used with the microammeter, it should be left in the plastic case; for a metal one would block off much of the radiation to the zone plate.

PROCEDURE: Place the transmitter ten or more feet from the circular aperture of the zone plate with the centers of both at the same height from the floor. By this means a nearly-plane wave is incident on the zone plate. Mount the dipole receiver on a wooden support on the far side from the transmitter and be sure that its center is on the axis of the circular aperture of the zone plate.

Start with the circular opening at its largest diameter, that is, with both zone plates removed. Move the receiver along the axis until a minimum of intensity is reached. Record the distance from the center of the aperture to the center of the dipole and from the edge of the aperture to the center of the dipole. Keeping the positions of the zone plate and receiver fixed, cover the central portion of the aperture with the aluminum disk provided and record the reading of the microammeter. Measure the distance between the edge of the disk and the center of the dipole receiver. Remove the central disk and put the aluminum ring in position so as to cover the outer zone. Record the reading of the micro-

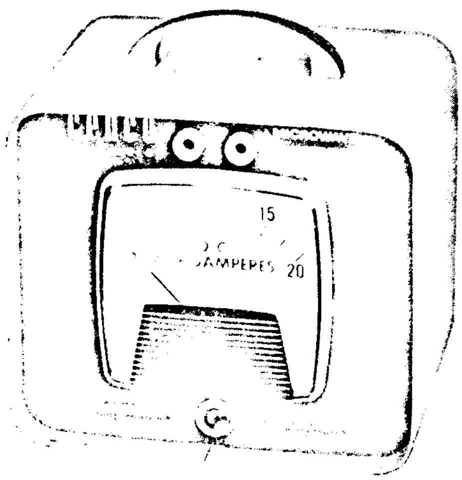

Fig. 5. The microwave receiver in a plastic case.

ammeter. Using the measured distances from the center of the dipole receiver to the edges of the zones, determine the wavelength of the microwaves.

Remove both zone plates and find the positions and intensities of successive maxima and minima as the receiver is moved along the axis of the aperture and away from it. Record the distances between the center of the aperture and the positions of maxima and minima and the corresponding readings of the microammeter. Plot the graph of microammeter reading (intensity) as ordinate against distance as abscissa. From this graph find the wavelength of the microwaves. Estimate and discuss the errors involved in these measurements of wavelength.

QUESTIONS: 1. In a pinhole box camera, the pinhole is at a distance of 10 cm from the back of the camera. Find the diameter of the pinhole that uncovers one Fresnel zone relative to a point at the center of the back of the camera. Consider the wavelength of light to be 0.000055 cm.

2. At a point along the axis of a zone plate there is maximum intensity from a plane wave when only the central aperture is open. If the next Fresnel zone is exposed the intensity goes almost to zero. Explain what has happened to the energy in the wave.

3. A plane monochromatic beam of light has a wavelength of 5500 Å (0.000055 cm). It is incident perpendicularly upon an opaque screen which has a circular opening 0.30 cm in diameter. Find the position of the first maximum and minimum nearest to the screen along the axis of the aperture.

4. A plane monochromatic wave of wavelength λ is incident perpendicularly upon an opaque screen which has a circular opening of radius r. Show that for a point at a distance R_n from the center of the opening and on the axis, the radius r_n of the nth Fresnel zone is given approximately by $r_n = \sqrt{nR_n\lambda}$.

5. Show that to a first approximation the area of the Fresnel zones is constant.

6. A so-called zone plate is a screen having alternate Fresnel zones alternately opaque and transparent. Such a zone plate can simulate a lens. Suppose that there is a point source of light on the axis of the zone plate at a distance r_o from it. If light of wavelength λ is brought to a focus at a point at a distance r_1 from the zone plate show that

$$\frac{1}{r_o} + \frac{1}{r_1} = \frac{\lambda}{R_1^2}$$

where R_1 is the radius of the first Fresnel zone.

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EXP. No. ....

STANDING ELECTROMAGNETIC WAVES

Especially prepared by
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OBJECT: To measure the wavelengths of the interdependent electric and magnetic waves that make up a standing electromagnetic wave.

METHOD: A beam of microwaves is directed upon a plane mirror and reflected to yield superposed standing electric and standing magnetic waves. The positions of the nodes in each wave are measured on the meter stick of an optical bench and the wavelength is calculated. The relative positions of the nodes in the standing electric and standing magnetic waves are observed.

THEORY: *Electromagnetic Waves:* Electromagnetic waves are described in terms of electric and magnetic lines of force. Lines of force were conceived by Michael Faraday as a means of visualizing the strengths and directions of electric and magnetic fields. Fig. 1 indicates the electric field in the neighborhood of an electric dipole consisting of two oppositely charged metal rods.

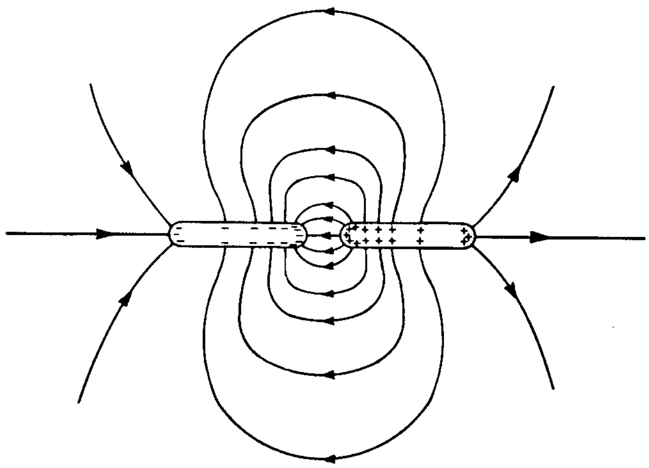

Fig. 1. Electrostatic field of a dipole.

When the potential between the dipoles is alternating at microwave frequencies (10^9 to 10^{11} cps) the electric lines of force are "snapped off" in loops as indicated in Fig. 2. These electrical loops travel outward as waves. When oscillating currents are present in the dipole, oscillating magnetic fields with magnetic lines of force in circles about the axis of the dipole travel out as a part of the electromagnetic wave, as shown in Fig. 2. In this figure, dots indicate magnetic lines directed out of the plane of the paper; crosses represent magnetic lines directed into the plane of the paper. The plane of the paper is the plane of polarization of the electric field.

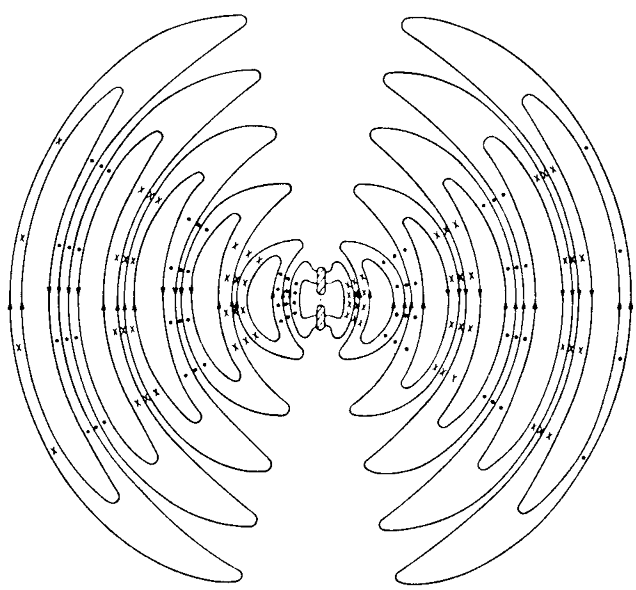

Fig. 2. A cross section of the instantaneous electric and magnetic fields originating from the electrical oscillations of the dipole. The closed loops show the electric lines of force while the dots and crosses show respectively the magnetic lines of force out of and into the plane of the figure.

The explanation of how the interdependent, changing electric and magnetic fields travel outward as waves was made by James Clerk Maxwell in 1864. Maxwell's purpose was to express in rigorous mathematical form the known laws of electricity and magnetism. Maxwell expressed Faraday's law of induced emf in terms of fields. *A time rate of change of magnetic field intensity at a point gives rise to a space rate of change of electric field intensity.* Likewise, broadening the concept of current density to include the time rate of change of electric field intensity, Maxwell expressed Ampere's law in terms of fields. *A time rate of change of electric field intensity at a point is accompanied by a space rate of change of magnetic field intensity at that point.*

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Combining these field forms of Faraday's and Ampere's laws, Maxwell showed that electric and magnetic fields do not spread out at infinite speed. They spread out in a vacuum with a speed equal to the ratio of the sizes of a charge measured in the electrostatic system of units to the same charge measured in the electromagnetic system of units. Maxwell noted that this fundamental constant in electricity was, within the limits of existing measurements, equal to the speed of light. From this he concluded that light is an electromagnetic wave.

In the recent meter-kilogram-seconds-ampere rationalized system of units, empty space is assigned two properties called the magnetic permeability of free space, μ_o, and the electric permittivity of free space, ϵ_o. The value of the product of these two constants was arbitrarily chosen so that the speed of light in a vacuum, c, is given by

$$c = \frac{1}{\sqrt{\mu_o \epsilon_o}} \qquad (1)$$

In any medium the speed, u, of an electromagnetic wave is

$$u = \frac{1}{\sqrt{\mu \epsilon}} \qquad (2)$$

where μ and ϵ are respectively the permeabilities and permittivities of the medium. The arbitrary constants μ_o and ϵ_o were chosen so as to simplify Maxwell's equations.

Graphical Description of Plane Traveling Waves. Since this experiment is concerned with plane waves, a graphical interpretation of Maxwell's wave equations will be made for a plane wave. Fig. 3(a) is a graphical indication of a plane electromagnetic wave traveling to the right. Since the electric and magnetic fields and the direction of propagation are mutually perpendicular, three dimensional Cartesian coordinates are employed. The instantaneous plots of electric field strength, E, and magnetic field strength, H, of Fig. 3(a) are proportional to the line densities of Fig. 2. The traveling electric and magnetic waves are in phase. Maxwell's equations yield a simple rule for the direction of propagation of a traveling electromagnetic wave. *If the fingers of the right hand are closed from the direction of the electric field to the direction of the magnetic field at a given point and given instant, the extended thumb points at right angles to the plane of the two fields in the direction of propagation of the wave.* If the wave is reflected back along the same path by any kind of plane mirror, one of the two waves, either the electric or magnetic wave, must undergo a change of phase of 180° upon reflection in order to satisfy the above rule of directions. Fig. 3(b) is a graph of the two interdependent waves traveling in the negative x direction. A purpose of this experiment is to determine which of the two waves undergoes the 180° phase change at a metal mirror.

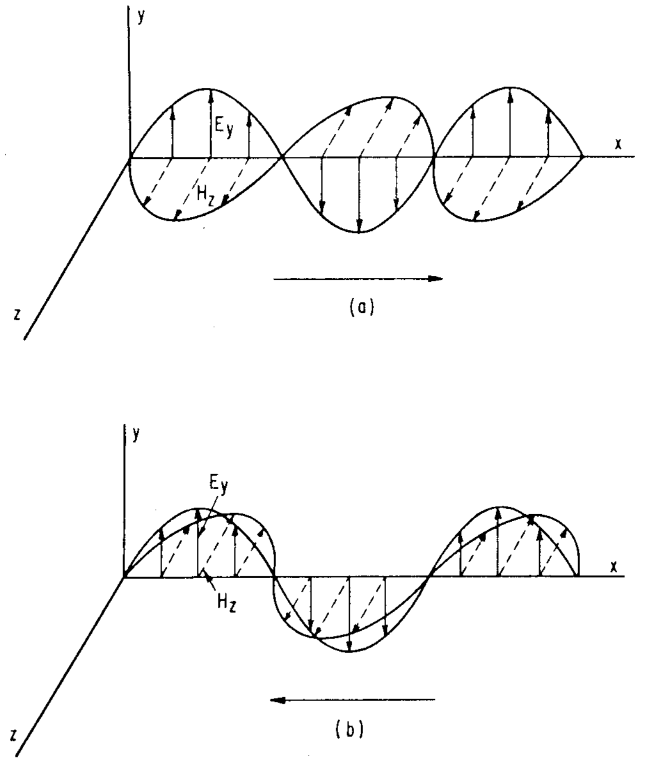

Fig. 3. Graphs of the electric and magnetic fields of a plane electromagnetic wave (a) traveling to the right, (b) traveling to the left.

Equations of the Traveling Waves. The equations of the traveling electric wave polarized in the y direction and the traveling magnetic wave polarized in the z direction, each traveling in the positive x direction, may be written

$$E_y = E_{yo} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \qquad (3)$$

and

$$H_z = H_{zo} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \qquad (4)$$

The symbols E_y and H_z represent the variable electric and magnetic fields which are dependent on the two independent variable times, t, and position, x. The symbols E_{yo} and H_{zo} represent the maximum values of E_y and H_z, the amplitudes of the two waves. Position and time are expressed as unitless fractions of wavelength, λ, and period, T.

When these waves are reflected by a conductor so as to travel in the negative x direction, the equations for the reflected waves are

$$E'_y = -E_{yo} \sin 2\pi \left(\frac{x}{\lambda} + \frac{t}{T} \right) \qquad (5)$$

and

$$H'_z = H_{zo} \sin 2\pi \left(\frac{x}{\lambda} + \frac{t}{T} \right) \qquad (6)$$

When a wave is traveling in the negative x direction the signs of the two independent variables are the same. The negative sign in Eq. (5) is used to indicate that this particular one of the two waves underwent the 180° phase change upon reflection.

Equations of the Standing Waves: By Thomas Young's principle of interference the electric field intensity at any position and time is the resultant of the electric field intensities of the component waves. By the expansion of the sines of the sums and differences of the position terms and time terms and the summation of the two electric field intensities, the equation for the standing electric wave becomes

$$E_y = 2E_{yo} \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T} \qquad (7)$$

Similarly by addition of the magnetic field intensities the equation for the standing magnetic wave becomes

$$H_z = 2H_{zo} \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi t}{T} \qquad (8)$$

It may be noted that there are positions $\frac{x}{\lambda}$ for which E_y is zero for all values of time. These are *nodes* in the electric field and occur when $\frac{x}{\lambda} = \frac{1}{4}(2n + 1)$, where n has the values of any positive integer and zero. The nodes in the magnetic field of Eq. 8 occur when $\frac{x}{\lambda} = \frac{n}{2}$. Midway between the nodes are antinodes with amplitudes equal to twice the amplitudes of the component waves.

Figs. 4(a) and (b) respectively are plots of the standing electric and standing magnetic waves at a given time when the fields are a maximum. The dotted lines indicate the displacements a half period later.

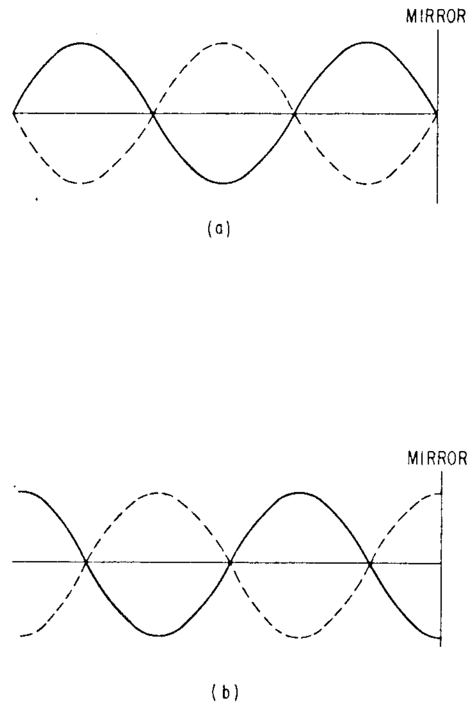

Fig. 4. Plots of (a) a standing electric wave and (b) the corresponding standing magnetic wave when the fields are maximum. The dotted lines indicate the field strengths a half period later.

When there is a node of electric field intensity there is an antinode of magnetic field intensity and where there is an antinode of electric field intensity there is a node of magnetic field intensity. There is no net propagation of energy in a standing wave.

Intensity of Radiation: The rate of energy flow per unit area, when the area is taken at right angles to the direction of flow, is called the *intensity of radiation, I,* of the wave. In mks units I is expressed in watts per square meter. In an electric field at any point in a dielectric of permittivity ϵ the energy density is $\frac{1}{2}\epsilon E^2$ and in a magnetic field of permeability μ the energy density is $\frac{1}{2}\mu H^2$.

Maxwell's equations reveal that for the traveling plane wave of Eqs. (5) and (6) the amplitudes H_{z_0} and E_{y_0} are related by

$$\mu H^2_{z_0} = \epsilon E^2_{y_0} \qquad (9)$$

Since μ and ϵ are constants and since the traveling electric and magnetic waves are in phase with each other, we may relate the instantaneous values of the fields at a point in the plane-traveling wave by

$$\mu H^2 = \epsilon E^2 \qquad (10)$$

Thus for the combined electric and magnetic waves the energy density is

$$\frac{\mu H^2 + \epsilon E^2}{2} = \mu H^2 = \epsilon E^2 \qquad (11)$$

The energy density is equally divided between the electric and magnetic waves.

The rate of flow of energy per area, where the area is taken at right angles to the velocity, is the product of the energy density and the velocity. Thus for a traveling wave

$$I = v\mu H^2 = v\epsilon E^2 \qquad (12)$$

where v is the wave velocity.

However, in a standing wave, or indeed in any interference pattern, the intensities of radiation due to the electric and magnetic fields are *not* equal. Nodes of electric and magnetic fields do not occur at the same points. Separate probes will be used to measure the intensities of radiation of the electric and magnetic waves.

This experiment is concerned, not with the absolute intensity, but with the ratio of the intensity at a point in the interference pattern to the intensity of the unperturbed traveling wave. Thus for a point in the standing electric wave the relative intensity is

$$\left(\frac{I}{I_0}\right)_E = \frac{\overline{E^2}}{\overline{E^2_0}} \qquad (13)$$

where $\overline{E^2_0}$ is the time average of the square of the electric field in the unperturbed plane wave and $\overline{E^2}$ is the time average of the square of the electric field at a point in the interference pattern. Similarly,

$$\left(\frac{I}{I_0}\right)_H = \frac{\overline{H^2}}{\overline{H^2_0}} \qquad (14)$$

where $\overline{H^2_0}$ is the time average of the square of the magnetic field in the unperturbed plane wave and $\overline{H^2}$ is the time average of the square of the magnetic field at a point in the interference pattern. The detectors for measuring the relative values of $\overline{E^2}$ and $\overline{H^2}$ are described in the following sections.

APPARATUS: The apparatus consists of a microwave transmitter, a mirror, a wooden optical bench with meter stick, a dipole probe for the electric wave, a loop probe for the magnetic wave, and an indicator such as a low sensitivity galvanometer, a cathode-ray oscilloscope or earphones. The arrangement is shown in Fig. 5. If the transmitter is placed farther from the mirror, the beam becomes more nearly parallel in the region of measurements. A distance of 5 feet will be satisfactory for the following studies.

Transmitter: The transmitter consists of two coaxial resonant cavities with a disk-seal microwave triode amplifier as an integral part of the two cavities. The grid-cathode cavity is the input cavity and the grid-plate cavity the output cavity. When sufficient power is fed back from the output to input cavity of the amplifier, it becomes an oscillator. The two resonant cavities are of the same frequency having lengths of one wavelength or three quarters of a wavelength. The study of standing waves in resonant transmission lines and coaxial resonant cavities normally follows the study of standing waves in free space made in this experiment.

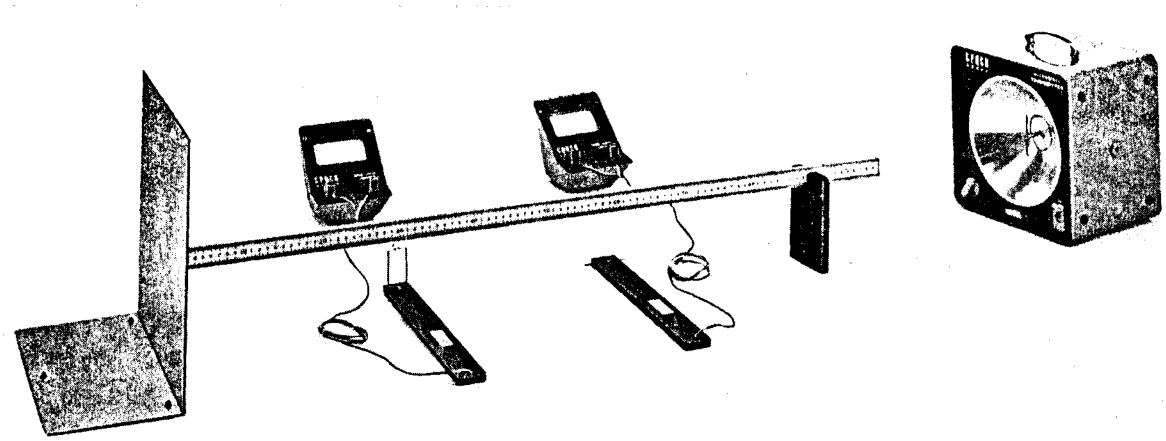

Fig. 5. Arrangement of transmitter, optical bench, mirror, and probe for measuring standing waves.

The plate potential for the oscillator tube is 350 volts at a frequency of 60 cycles/sec. Oscillation takes place during about one quarter of a cycle when the plate potential is sufficiently positive. During that time there are about ten million oscillations so that we may think of the wave as continuous as far as any studies in interference are concerned.

The antenna is a half-wave resonant dipole coupled by a half-turn loop with the oscillating magnetic field at the shorted end of the output cavity. The antenna lies at the principal focus of a parabolic reflector with a focal length of one quarter wavelength. The parabolic reflector is less than two wavelengths in diameter. To produce a sufficiently narrow beam for point-to-point communication in a microwave relay system over distances of 50 miles, the reflector must be of the order of 20 wavelengths in diameter. However, the aperture of such a reflector would have a complicated Fresnel diffraction pattern longer than a laboratory table top. Parallelism of the beam must be sacrificed in order to obtain a uniform phase front near the source.

Optical Bench and Mirror: The optical bench of Fig. 5 consists of wooden supports and a wooden meter stick without metal ends. The mirror is a sheet of aluminum 18 inches square. It need not be polished. Peaks and pits of the order of one millimeter are less than a hundredth of a wavelength and are unobservable by microwaves.

Diode Detectors: The simplest detector of microwaves is a crystal rectifier consisting of a fine wire, the end of which is pressed against particular faces of certain crystals, usually silicon. Fig. 6 is a typical graph of current through a crystal plotted against potential difference across the crystal. If the currents are sufficiently small, of the order of 100 μa or less, the rectified current is proportional to the *square* of the potential difference across the crystal. Thus the microammeter reading is proportional to the power received and therefore to the intensity of radiation, I.

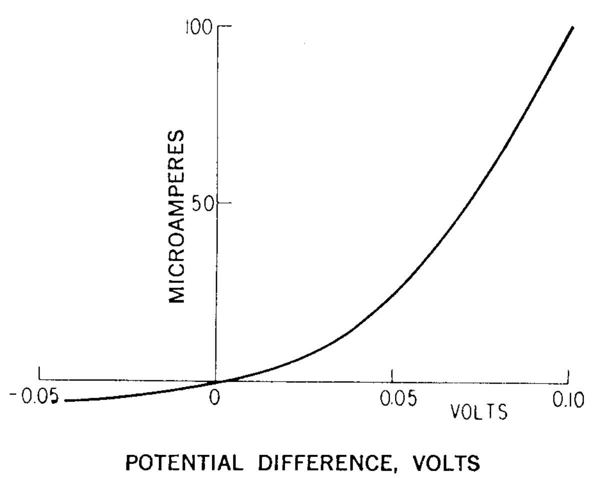

Fig. 6. Graph of current in a crystal detector against potential difference across the crystal.

The Electric Probe: The electric probe of Fig. 7 consists of a silicon crystal detector, Sylvania 1N82A, sealed in a glass capsule. The wire leads serve as a half-wave dipole antenna.

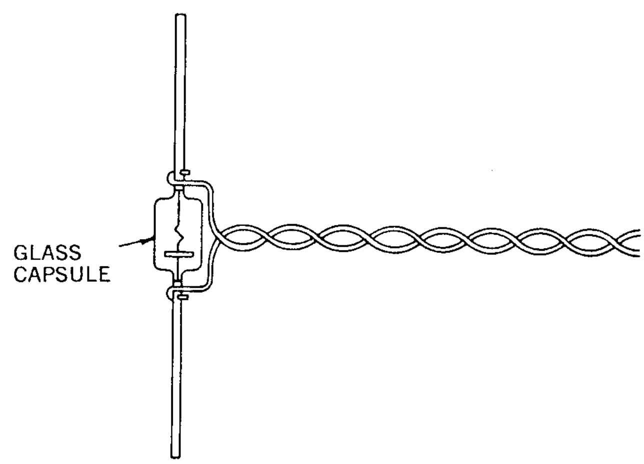

Fig. 7. Probe for the oscillating electric field.

A twisted lead is used to prevent picking up the microwave signal except at the antenna. This lead extends from the detector-antenna at right angles to the electric field for 12 inches in order not to disturb the pattern that is being

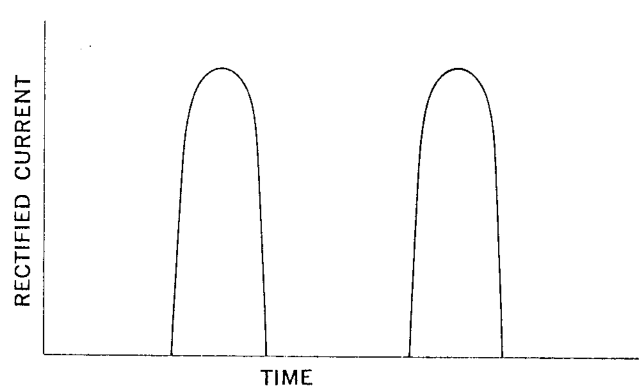

Fig. 8. Graph of rectified current against time.

measured. Since the microwave signal is modulated with 60 cycles/sec, the detected current contains both a dc component and a 60-cycle component. If the output of the crystal is fed to a cathode-ray oscilloscope, the rectified current will have the form indicated in Fig. 8, which shows that the oscillations exist for somewhat less than a half cycle.

The Magnetic Probe: The magnetic probe (Fig. 9) is held above the meter stick so that it will detect only the magnetic wave without detecting the electric wave. From Faraday's law the emf induced in the loop is proportional to the rate of change of linkage of magnetic lines of force with the loop.

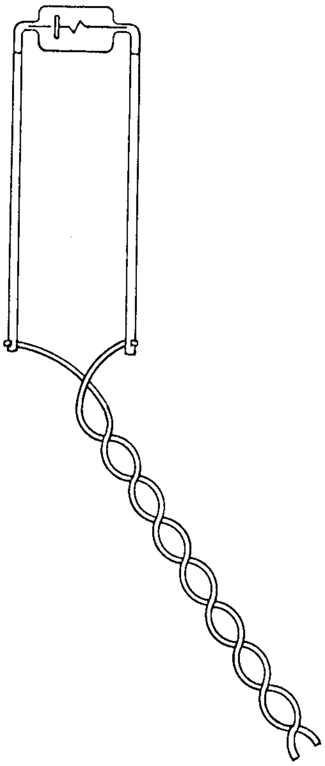

Fig. 9. Probe for the oscillating magnetic field.

The rectified current by the crystal is proportional to the average of the square of the magnetic field intensity. The plane of the rectangular loop is perpendicular to the magnetic field and the side of the rectangle that contains the crystal is perpendicular to the electric field. The magnetic probe would be more sensitive if the loop were larger. However, if the two sides of the loop which are parallel to the electric field are too far apart they will lie in appreciably different electric fields permitting a resultant emf in the loop due to the electric wave. Thus we must sacrifice some sensitivity in the magnetic probe in order to avoid detecting the electric wave. The red wire of the twisted wire lead is connected to the high-potential side of the detector diode crystal.

Since the output of the crystal has both a dc and an ac component, the output may be fed to a low sensitivity galvanometer, or earphones, or a cathode-ray oscilloscope. The galvanometer should have a sensitivity of about 50 μa full scale.

PROCEDURE: *Experimental:* Hold the probe for the electric wave above the wooden optical bench at the zero line of the meter stick so that the detector is at the surface of the aluminum reflector. Moving the probe slowly away from the mirror, estimate the positions of the nodes to within a millimeter and record each position corresponding to the number of the node. Record the positions of at least ten nodes. Because the beam is not quite parallel but divergent, the intensity readings will not be zero at those nodes which are farthest from the mirror. A traveling wave is superposed on the standing wave in that region. However, the nodes can be located at positions of minimum intensity. As an additional study, record the intensity of radiation at intervals of 5 mm from the mirror outward for two wavelengths.

Hold the magnetic probe on the optical bench with the loop close to the mirror. Because it has finite width, the probe can not be used to determine the intensity of radiation at the surface. However, the intensity of the magnetic field at the surface of the mirror may be determined by recording the intensity at a few points near the mirror and extrapolating to the surface. Is there a node or antinode of magnetic field intensity at the surface? Record the positions of at least ten nodes counted from the mirror outward.

Analysis and Calculations: Plot graphs of the positions of the nodes of the electric standing wave and the magnetic standing wave against the number of the node. Since the distance between consecutive nodes is a half wavelength, the slope of the curve is in centimeters per half wavelength. Determine the wavelengths of the electric and magnetic waves. In finding the slope use two points far apart on the curve which are not points of data.

As a check on the above determination find the slope using all points of data once. Record in a column the difference in position of nodes 10 and 5, 9 and 4, and so on to 6 and 1. In another column record the number of corresponding half wavelengths. From the ratio of the sums determine the half wavelength.

Plot a graph of the relative intensities of radiation $\frac{I}{I_0}$ of the electric wave against position for a distance one and one-half wavelengths from the mirror. Compare this plot with a calculated plot of

$$\frac{I}{I_0} = 4 \sin^2\left(\frac{2\pi x}{\lambda}\right) \qquad (15)$$

QUESTIONS: 1. On the basis of your observations of the standing electric wave at the surface of the metal mirror, how great was the change in phase of the traveling wave at the plane of reflection? Explain.

2. On the basis of your observations of the magnetic standing wave at the surface of the mirror, what was the change of phase of the traveling wave at the plane of reflection?

3. Explain why the intensity minima increased with distance from the mirror.

4. The antenna of the microwave transmitter is placed about a quarter wave from the back of the parabolic reflector. Compare the phase of a wave that has traveled from the antenna to the back of the parabolic reflector and returned to the antenna with the phase of the wave that is about to leave the antenna.

5. In finding the wavelength directly from the data, why would it not have been as precise to take the position of the second node minus the first, the third minus the second and so forth to the tenth minus the ninth and average the ten determinations?

6. From the wavelength of the microwaves that you have measured and the speed of light in a vacuum ($c = 3.0 \times 10^8$ m/sec) find the frequency of the waves.

7. What is the wavelength associated with an ac source of 60 cycles/sec? With a dc source? What frequency corresponds to waves of length of 1.00 mm?

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EXP. No. 2

ELECTRIC and MAGNETIC COMPONENTS of MICROWAVES

OBJECT: To investigate the electric and magnetic components of standing electromagnetic microwaves; to measure the wavelength of these microwaves; and to demonstrate the polarization of microwaves coming from a transmitter.

METHOD: A plane beam of microwaves is reflected from a metallic mirror so as to produce standing electric and magnetic waves. The distance between nodes in each of these waves is measured and the wavelength of the microwaves determined. A wire framework is used as an analyzer to demonstrate the polarization of the microwaves.

THEORY: The name *microwave* refers to electromagnetic radiation whose wavelength is of the order of a few centimeters, as contrasted with a broadcast radio beam for which the wavelength is measured in kilometers. Ordinary visible light is electromagnetic radiation having wavelengths in the region of one-hundred-thousandth of a centimeter. Because of its convenient wavelength the microwave region of electromagnetic radiation is commonly used in making measurements of the electric and magnetic field which make up electromagnetic radiation.

A number of important discoveries led James Clerk Maxwell, in 1864, to predict the electromagnetic character of light waves. The first in this line of discoveries was made by the Danish physicist Hans Christian Oersted, who, in 1820, showed that electric currents produce magnetic effects. In 1831 Faraday showed that a changing magnetic field produces electric effects. Another important advance was made by Maxwell when he considered a changing electric field to have magnetic effects equivalent to those of an electric current. Thus electric and magnetic fields are interrelated: a changing magnetic field produces an electric field (Faraday) and a changing electric field produces a magnetic field (Oersted, Maxwell). By rigorous mathematical reasoning Maxwell was able to show that interdependent electric and magnetic fields are propagated in a vacuum with the speed of light (approx. 3.00×10^8 m/sec.).

Electromagnetic waves are produced when electric charges, such as electrons, are accelerated. This is accomplished by means of a suitable electronic transmitter circuit used to make electrons rush rapidly back and forth in a broadcasting antenna. The resulting electric and magnetic fields leave the antenna and are propagated as waves through space. In the microwave apparatus used in this experiment, the small dipole antenna is placed at the focus of a parabolic metal reflector so that there is a very nearly plane electromagnetic wave coming from the transmitter. This wave is plane-polarized with the electric field vector parallel to the transmitting antenna A shown in Fig. 1. Such a wave, Fig. 1a, of wavelength λ, traveling to the right along the positive X axis has the electric field of intensity E in the XY plane while the magnetic field of intensity H is in the XZ plane. Figure 1b shows a similar plane-polarized wave traveling to the left along the negative X axis. In these traveling waves, the electric and magnetic fields are in phase, that is, where there is a maximum of the one there is a maximum of the other.

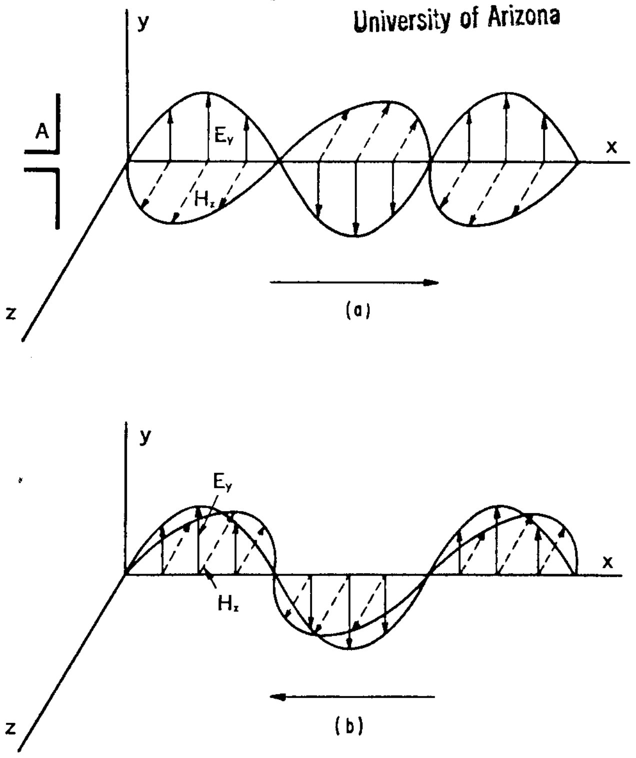

Fig. 1. Graphs showing the strengths of the electric and magnetic fields of a plane electromagnetic wave (a) traveling to the right, (b) traveling to the left. A represents the transmitting antenna.

The direction of travel of the waves shown in Fig. 1 can be determined by a simple rule: close the fingers of the right hand along the direction of the electric field to the magnetic field, through an angle of 90°, then the outstretched thumb indicates the direction in which the wave is traveling. (Using this rule check the directions of travel of the waves shown in Fig. 1.)

Suppose the wave traveling to the right is reflected by a plane metallic mirror so as to produce a second wave traveling to the left. On reflection there is a change in phase of 180° of either the electric or magnetic field; that is, the reflected electric or magnetic field at the mirror is the reverse of the incident field at the mirror. This result may be found experimentally by noting whether there is a node or loop in the waves representing the intensity of the electric or magnetic fields at the surface of the mirror. In the region between the microwave transmitter and the metallic mirror there are two waves: the incident wave and the reflected wave. These waves interfere so as to produce standing waves in both the electric and magnetic fields. Owing to the phase change at reflection, the nodes of the standing waves of the electric field do not occur at the same positions as those of the magnetic field. This phenomenon is illustrated in Fig. 2.

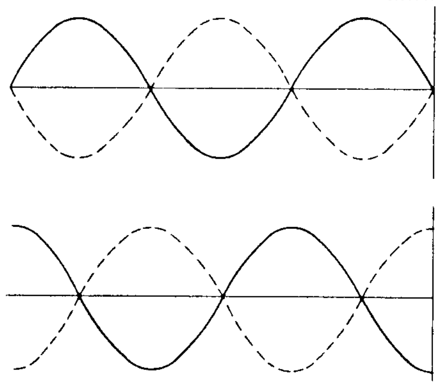

Fig. 2. Plots of standing waves, electric and magnetic. The full and dotted lines represent the strengths of the fields half a period apart.

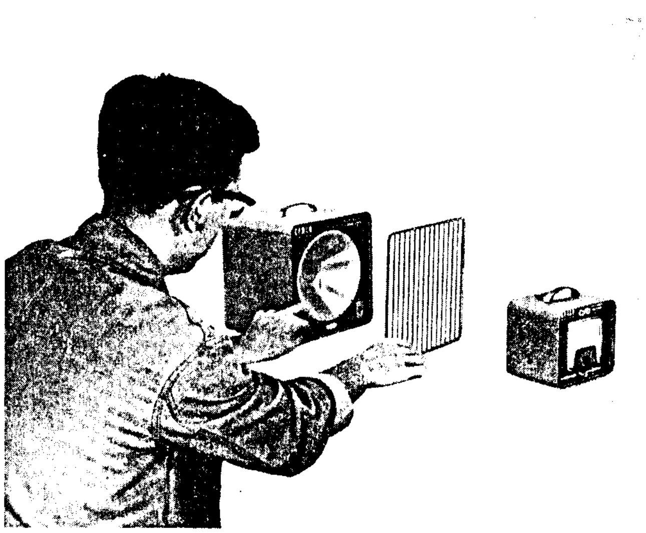

Fig. 4a. The microwave equipment being used to demonstrate polarization of microwaves.

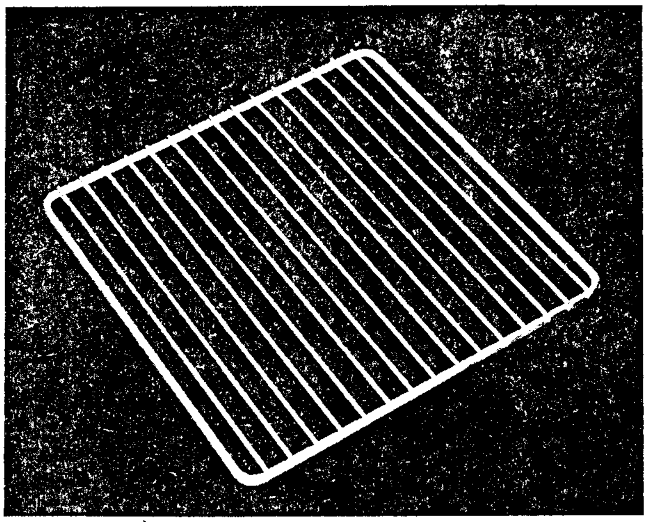

Fig. 4b. Polarization analyzer grid for demonstrating the polarized nature of the radiation emitted by the transmitter.

APPARATUS: The apparatus (Fig. 3) consists of a microwave transmitter, a metallic mirror, a meter stick, a dipole probe for the electric waves, a loop probe for the magnetic waves, and a low-sensitivity galvanometer. The galvanometer, which measures the strength of the signal from the probes, may be replaced by an oscilloscope or by earphones. A wire grid, Fig. 4, for demonstrating polarization is needed.

The electric probe consists of a silicon crystal detector 1N82A sealed in a glass capsule and electrically connected to two thin metal rods which form a half-wave dipole. Since the microwave output is modulated by the 60 cps alternating voltage, the signal which appears on an oscilloscope screen can have the form shown in Fig. 5. The magnetic probe consists of a silicon crystal detector forming part of a rectangular loop. The two probes are shown in Fig. 3.

The magnetic probe is oriented so that it registers the intensity of the magnetic wave with relatively little effect from the electric wave. Its mode of operation depends on Faraday's law for the emf induced in a closed loop by a changing magnetic field. The electric probe must be held parallel to the antenna of the transmitter so as to register maximum effect.

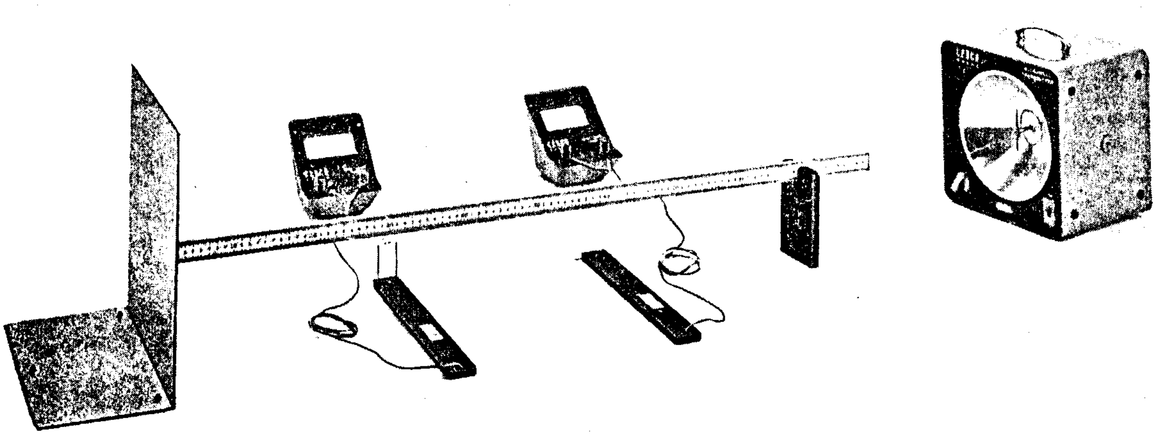

Fig. 3. Arrangement of transmitter, optical bench, mirror, and probes connected to galvanometers, for measuring standing waves.

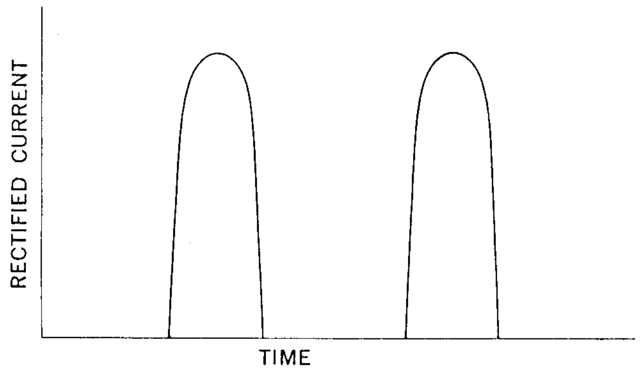

Fig. 5. Graph of rectified current as a function of time.

EXPERIMENTAL PROCEDURE: Set up the apparatus as shown in Fig. 3. If an oscilloscope is used, place the side of the oscilloscope at the back of the mirror so that it adds to the reflection. Connect the leads from the probe to the vertical input terminals of the oscilloscope. Adjust the amplification to a suitable level, using the gain control of the transmitter and/or that of the oscilloscope. Place the electric probe under the meter stick and parallel to the antenna. Note the reading of the measuring instrument, galvanometer or oscilloscope. Move the transmitter a few centimeters toward or away from the reflector and place the electric probe at the position of maximum reading.

Place the electric probe as close as possible to the metal mirror at the zero mark on the meter stick. Note whether this is a node or loop of the electric field. Move the probe slowly away from the mirror and record the positions of ten successive nodes to the nearest millimeter.

Replace the electric probe with the magnetic one and repeat the above procedure to determine the positions of the nodes in the magnetic field. In this case, the center of the loop must be estimated as closely as possible since it is an average value taken over the area of the loop which is obtained.

For the polarization demonstration, place the polarizing grid in front of the transmitter. Use an electric dipole receiver. Start with the grid wires parallel to the transmitter antenna and note the reading of the instrument connected to the electric dipole receiver. Rotate the grid wires through 90° and again note the reading of the instrument connected to the dipole.

ANALYSIS AND CALCULATIONS: Arrange the data for the positions of the nodes of the standing electrical wave in a column and number them from one to ten. Subtract the positional values of node 6 from node 1, of node 7 from node 2 and so on until all the readings are included. Average these values to give the distance between five nodes. From this value find the wavelength of the microwaves. Repeat this procedure for the data on the positions of the nodes in the standing magnetic wave and again find the wavelength of the microwaves.

Discuss the effect of the wire grid on the transmission of the microwaves. How do these observations show that the microwaves are plane-polarized?

QUESTIONS:

1. Using the value of the wavelength of the microwaves found in this experiment, calculate the microwave frequency. (Assume the microwave speed to be 3.00×10^8 m/sec.).
2. If the dipole electric receiver were held at right angles to the antenna of the transmitter, would the receiver instrument register any signal from the transmitter? Explain your answer.
3. If the magnetic loop receiver is held so the plane of its loop is perpendicular to the meter stick, can the receiver register any signal from the transmitter? Explain your answer.
4. Using your observation on the standing electric wave at the surface of the metal mirror, state and discuss what change in phase of the traveling electric wave took place at the plane of reflection.
5. Apply Question 4 to your observations on the standing magnetic wave.
6. What is the wavelength associated with an AC source having a frequency of 60 cps?