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EXP. No. ....

THE CHARGE OF THE ELECTRON

OBJECT: To investigate the motion of a charged oil drop, falling vertically in air and rising in a vertical electric field, and from this to determine the charge of the electron.

METHOD: A small, electrically charged, drop of oil from an atomizer moves vertically between two horizontal metal plates. If there is no electric field between the plates the oil drop falls with constant speed. From the measurement of this speed of fall and known physical constants the radius of the drop is found.

When the plates are connected to a battery, so as to establish a vertical electric field between the plates, the drop is made to move upwards with constant speed. The charge on the oil drop is then obtained from the speeds of rise and fall and known physical constants. If the air between the plates is ionized then the charge, but not the mass, of the oil drop is changed. This new charge is then found. By repeating the process of changing the charge a number of values for the different charges on the oil drop is obtained. These charges are found to be integral multiples of a common unit of charge known as the charge of an electron.

THEORY: It was Michael Faraday who first intimated that electricity has a particle nature for he wrote about 1834 ". . . if we adopt the atomic theory or phraseology, then the atoms of bodies which are equivalent to each other in their ordinary chemical action, have equal quantities of electricity associated with them." The eminent scientist Clerk Maxwell doubted the validity of the concept of an elementary unit of charge. However there were a number of scientists who were convinced that Faraday's laws of electrolysis and the atomic theory necessitated the elementary unit of charge. The German physicist H. von Helmholtz expressed this idea in the Faraday lecture in 1881 when he said: "Now the most startling result of Faraday's law is perhaps this: If we accept the hypothesis that the elementary substances are composed of atoms, we cannot avoid concluding that electricity also, positive as well as negative, is divided into definite elementary portions which behave like atoms of electricity." It was the Irish physicist G. Johnstone Stoney, an ardent proponent of the atomic theory of electricity, who proposed in 1891 the name *electron* for the elementary unit of charge.

Confirmation of this concept came with the investigations of the conductivity of gases which led indirectly to the famous oil drop experiments of R. A. Millikan between 1910 and 1917. These experiments left no doubt that there is an elementary unit of charge and that all charges are integral multiples of this unit charge. This unit of charge, known as the electron, is denoted by the symbol e and has a value of 4.803×10^{-10} esu units of charge or 1.602×10^{-19} coulomb. Its rest mass or inertia m is 9.108×10^{-28} grams or about 1/1836 of the mass of the hydrogen atom. An interesting account of Millikan's experiments is given in his book "Electrons (+ and −), Protons, Photons, Neutrons, and Cosmic Rays" (Univ. of Chicago Press 1935). The present experiment is modeled directly on Millikan's method.

Consider an oil drop of mass m and charge q, falling under gravity between two horizontal plates. In falling the oil drop is subjected to an opposing force due to air resistance. The speed of the oil drop increases until a constant terminal speed is reached at which the weight of the oil drop is exactly equal to the air resistance force. This situation is similar to that of a parachute falling through air. The value of the air-resistance force was first derived by Sir George Stokes and is given as $6\pi\eta r v_g$, where η is the coefficient of viscosity of the air, r is the radius of the drop and v_g its terminal speed. Thus the equation of motion of the drop is

$$mg = 6\pi\eta r v_g \qquad (1)$$

This expression is not precisely correct for the effective value of the coefficient of viscosity depends to a small extent on the radius of the drop. A correction is applied later in Eq. (11).

The mass of the oil drop can be expressed in terms of its density as

$$m = 4\pi r^3 \rho/3 \qquad (2)$$

In this discussion the buoyant force of the air has been neglected.

Substituting for m in Eq. (1) it follows that

$$r = \sqrt{\frac{9 v_g \eta}{2 g \rho}} \qquad (3)$$

Suppose that the metal plates are connected to a battery establishing an electric field ε between the plates such that an oil drop of charge q is made to move upwards. The direction of the electric field must depend on the sign of the charge q which may be either positive or negative. The resultant upward force on the charge is $\varepsilon q - mg$ and this causes the drop to move upwards with a constant speed v. For this motion

$$\varepsilon q - mg = 6\pi\eta r v \qquad (4)$$

Dividing Eq. (4) by Eq. (1) gives

$$\frac{\varepsilon q - mg}{mg} = \frac{v}{v_g}$$

or

$$\frac{\varepsilon q}{mg} = \frac{v + v_g}{v_g} \tag{5}$$

Suppose the charge q on the oil drop is changed to q' by ionizing the air between the plates with a radioactive source. This action results in the oil drop capturing a positive or negative ion of the ionized air. Thus if the charge changes to q' the terminal speed of the drop changes to some value v' such that

$$\frac{\varepsilon q'}{mg} = \frac{v' + v_g}{v_g}$$

By subtracting this equation from Eq. (5) it follows that

$$\frac{\varepsilon\,(q - q')}{mg} = \frac{v - v'}{v_g} \tag{6}$$

In Eq. (6) the quantities ε, m, g, and v_g are all constants so that the change in charge $(q - q')$ is proportional to the change in speed $(v - v')$. If the charges are integral multiples of some elementary unit of charge then the changes in speed should also be integral multiples of some unit of speed. This conclusion is subject to experimental verification and careful work by Millikan showed this to be the case.

Likewise the sum of the speeds of rise in the electric field v and of fall v_g have a common factor. From Eqs. (5), (1) and (3) it follows that:

$$q = \frac{v + v_g}{v_g}\,\frac{mg}{\varepsilon} = \frac{(v + v_g)}{\varepsilon}\,6\pi r \eta_r$$
$$= \frac{(v_g + v)}{\varepsilon}\,6\pi\eta_r\sqrt{\frac{9\eta_r v_g}{2g\rho}} \tag{7}$$

In this equation the symbol η_r has been used for the coefficient of viscosity in order to indicate that it is a function of the radius of the drop.

Suppose that the charge q is made up of n electronic charges such that

$$q = ne \tag{8}$$

and that the minimum change in speed of $(v_g + v)$ for a change of one electronic charge e is v_o or

$$v_o = \frac{v_g + v}{n} \tag{9}$$

then Eq. (7) may be written as:

$$e = 6\pi\eta_r\sqrt{\frac{9\eta_r v_g}{2g\rho}}\,\frac{v_o}{\varepsilon} \tag{10}$$

In Eq. (10) all the quantities are known or can be measured so that the unit charge e_1 can be evaluated.

From his experiments, Millikan found that the unit charge e depended on the size of the oil drop used. This he traced to a failure of Stokes' law which occurred when the radius of the drop became comparable to the mean free path of the air molecules through which the drop was falling. Careful experiments showed that a consistent set of values for the electronic charge on any size oil drop could be obtained by introducing a small correction factor to the coefficient of viscosity which depended on the pressure of the air and the radius of the drop. The coefficient of viscosity η_r for a drop of radius r is given in terms of the generally accepted value η for air as

$$\frac{\eta}{(1 + b/pr)} \tag{11}$$

where b is a constant found experimentally by Millikan to be 0.000625 when the pressure p of the air is measured in centimeters of mercury and the radius r of the drop is measured in centimeters.

The corrected form of Eq. (10) for the electronic charge e is

$$e = 6\pi\left(\frac{\eta}{1 + b/pr}\right)^{3/2}\sqrt{\frac{9v_g}{2\rho g}}\,\frac{v_o}{\varepsilon} \tag{12}$$

since η_r appears to the three-halves power in Eq. (10). The value of η for air is 1.819×10^{-4} dyne sec cm^{-2}.

If the cgs system of units is used for η, p, and r, it is necessary to use this system throughout in Eq. (12). Thus the electric field intensity ε must be expressed in electrostatic units. From the definition of potential difference V and electric field intensity ε it follows that for a uniform field,

$$V = \varepsilon d \quad \text{or} \quad \varepsilon = V/d \tag{13}$$

In Eq. (13) the potential difference V must be expressed in electrostatic units of potential difference and these are related to volts by

$$V\ (\text{esu}) = V\ (\text{volts})/300 \tag{14}$$

where the factor 300 is the conversion factor between the two systems of units. Thus ε in esu is

$$\varepsilon\,(\text{esu}) = \frac{V\ (\text{volts})}{300\,d\ (\text{cm})} \tag{15}$$

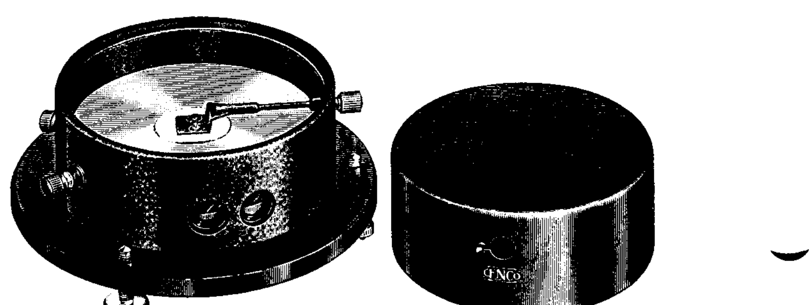

Fig. 1. Oil Drop Apparatus, Millikan. Palmer Design

The density ρ of the oil used can be found by a separate experiment or if a known kind of oil is used its density may be found in suitable reference books. The distance d between the two plates is given for this apparatus as 0.80 cm or 0.30 cm for the two possible positions. For the larger separation of 0.80 cm a potential difference of the order of 2000 volts is used whereas in the smaller separation about 300 volts is suitable. If much less than

300 volts is used then the suitable oil drops may be so small that Brownian movements of the drop occur and the velocities of fall and rise may appear quite inconsistent.

In order to ionize the air between the plates a small radioactive source is attached to a movable pin in the lower plate. The source is exposed by pushing the movable pin upwards. If the oil drop should capture or lose charge during its upward motion then its speed would be inconsistent with any of the other measured speeds. Such inconsistent readings are readily recognized and should be discarded.

The speed of the oil drop is measured with a telemicroscope which is essentially a microscope of fairly long focal length. A scale is mounted in the eyepiece and the time of travel between two lines on the scale can be measured with a stopwatch. An oil drop which has a time of fall of the order of twenty seconds should be used and this time is a constant independent of the charge on the drop. The time of rise, with the electric field on, depends on the charge on the drop and this is changed during the experiment by the ionizing source. A special switch is used so that the electric field can be easily reversed and also be made zero.

For a successful experiment a single drop should be observed for about half an hour. During this time a sufficient number of readings of times of rise and fall can be made with three or four changes of charge. It should be noted that the microscope reverses the apparent direction of travel so that the fall under gravity appears in the eyepiece scale as an upward motion.

APPARATUS: The oil-drop apparatus is shown in Fig. 1. Constant supply voltages, either dry batteries or an electronically regulated source with a high resistance in series to prevent serious short circuits, a reversing switch, a high resistance dc voltmeter, a stopwatch, an atomizer, and a telemicroscope with eyepiece scale and a calibrated millimeter scale for standardizing the scale in the eyepiece are required. For illuminating the drops the following may be used: an incandescent lamp (6-8 volts) with suitable socket, a transformer or battery for use with the lamp, a small converging lens of 5 cm focal length.

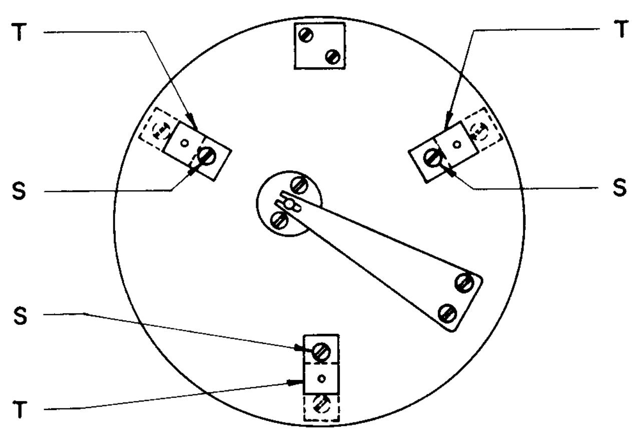

Fig. 2. View of lower side of lower plate. Dotted lines show positions of spacers for low voltage operation. Solid lines show positions for high voltage operation.

PROCEDURE: Having decided whether the high or low voltage is to be used assemble the apparatus in the correct manner. Remove the two plates and for the high voltage the three spacers T on the lower plate should be in positions shown by the solid lines, Fig. 2. For low voltage operation these spacers should be in the positions indicated by the dotted lines. To change the spacers from one position to the other, remove the screws S and lift the spacers about their locating pins. The screws should then be placed in the new position. The button control through the bottom of the apparatus must also be properly assembled for the particular arrangement in which the apparatus is to be used. For the low voltage operation the collar C should be in the position shown in Fig. 3, and for high voltage operation should be as shown in Fig. 4. To change the collar from one position to the other, loosen the knurled nut N, withdraw the button and replace it with the collar in the desired position.

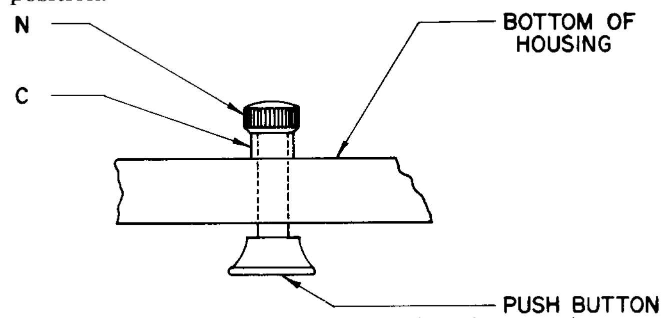

Fig. 3. Showing position of collar (C) for low voltage operation.

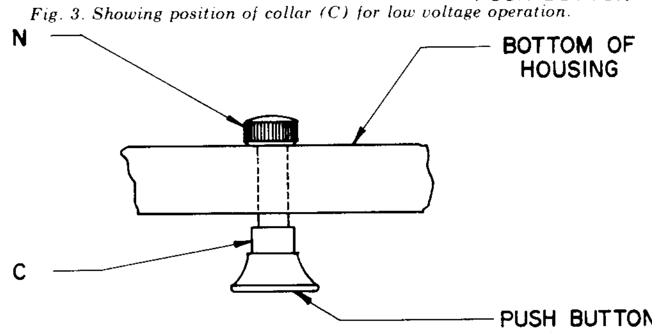

Fig. 4. Showing position of collar (C) for high voltage operation.

When the lower plate and button control have been suitably set, place this plate in the case and orient it so that the binding post can be passed through the case and screwed into the edge of the plate. This can be done by holding the plate inverted (ground surface down) and horizontal on the finger tips of one hand and lowering the inverted case over the plate so that it fits into the lower recess. Place the apparatus right side up and screw the binding post into place. Holding the upper plate with the shutter lower this plate into place and connect the binding post into its edge. Screw the shutter handle into position.

Set the apparatus so that the viewing telemicroscope is at a convenient position for making observations. Place a spirit level on the upper plate and carefully level the apparatus. This is to ensure that the electric field is vertical otherwise the drops may move out of focus in their up and down motions.

The illumination of the oil drops is critical and usually requires considerable care. A microscope light source with adjustable lens and shutter has been found satisfactory. If this is not available mount the small, 5 cm focal length, converging lens over the single window with wax or plasticene. A 6 or 8 candle power lamp at about 8 cm from this lens provides a converging beam which should pass through one of the two openings on the other side of the apparatus. Place a fine wire through one of the holes in the upper plate and view this in the telemicroscope. By raising and lowering this wire the upper and lower plates can be located in the field of view. At this point it may be necessary to change the height of the light source so as to avoid annoying reflections from the surfaces of the plates. Having obtained satisfactory illumination of the wire on a dark background remove the wire and connect the voltage source. It is necessary that the scale in the eyepiece be visible and some compromise for this may be necessary. Spray oil drops with the atomizer in the region above the upper plate. If the adjustments have been satisfactorily made then a cloud of drops is seen in the field of view. These drops will change direction of motion with reversal of the electric field if they are charged. Choose a drop whose motion can be readily followed when the electric field is reversed. The cloud can be cleared by suitable reversals of the electric field. The chosen drop is then observed and the time for the drop to fall through a selected number of divisions of the scale in the eyepiece is measured. Similarly the time is measured for the charged drop to move upwards in the electric field, through the same number of divisions. After a consistent set of readings has been taken, change the charge on the drop and repeat the time measurements using the same voltage between the plates. Make measurements on at least three different charges. Record the voltage applied to the plates.

OPTIONAL PROCEDURE: Vary the voltage and the electric field such that the electric force εq on the drop exactly balances its weight mg. This can be used as an independent method for measuring the charge on the drop.

Having completed a satisfactory set of measurements on the oil drop it is necessary to determine the actual distance over which the time measurements were taken. To do this set up the calibrated scale and view it with the telemicroscope without changing the eyepiece distance. The markings on the calibrated scale should be clearly seen in the eyepiece. Thus the distance d, over which the drop was timed, can be obtained.

Calculate the speed of fall under gravity, $v_g = d/t_g$, and the speeds of rise in the electric field; $v = d/t$. The speeds of rise will occur in groups corresponding to the different values of the charge on the oil drop. Denoting two such groups as v and v' their difference $v - v'$ is taken and these differences have a common factor. The mean value of this common factor gives a value of v_o, the change in speed corresponding to unit change in charge.

Tabulate also the values of $v_g + v$ and these have a common factor depending on the number n of electronic charges on the drop. The number n can be found by inspection and dividing $(v_g + v)$ by the appropriate value of n a constant set of values should be obtained. Take the average of the values found for $(v_g + v)/n$ and from Eq. (9); this gives v_o. Take the grand average of the two values found for v_o.

Determine the density of the oil used in the experiment. Calculate the electric field ε from the measured voltage V and the distance between the plates. Measure the barometric pressure p in centimeters of mercury. From these measurements calculate the value of the electronic charge e in esu and coulombs, using Eq. (12). Give the per cent error of your measurement from the accepted value for e.

QUESTIONS: 1. Estimate the error introduced in Eq. (2) by neglecting the buoyancy of the air on the oil drop.

2. Is it necessary that the correction to Stokes' law or to the value of the coefficient of viscosity be small, in order to obtain a correct value for the electronic charge e? Explain your answer.

3. If any of your data were discarded presumably because the oil drop captured a charge during its upward motion, state whether the captured charge was positive or negative and justify your answer.

4. From the definition of the coefficient of viscosity find its dimensions and give its units and value for air in the mks system of units.

5. Show that Eq. (12) is dimensionally correct.

6. Describe any other methods which have been used to determine the value of e.

7. Given the value of the Faraday is 96,522 coulombs per gram-mole, find the value of Avogadro's number or the number of molecules per gram-mole.

8. Make all the calculations in the mks system of units and find the electronic charge in coulombs.

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