SELECTIVE EXPERIMENTS IN PHYSICS

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EXP. No. ....

THE CONCAVE GRATING SPECTROMETER–ROWLAND MOUNTING

OBJECT: To make a study of the concave-grating spectrometer with Rowland mounting; to plot the calibration curve; and to use the instrument for the measurement of spectral lines.

METHOD: A Rowland-mounting spectrometer equipped with a concave grating is adjusted to produce a sharp image of the source slit on the cross hair of the eyepiece. The scale positions of the prominent lines in the spectrum of a mercury-arc source are determined. A calibration curve is plotted using these positions and the known wavelengths of these lines. The spectrum of an unidentified source is then studied and the wavelengths of its lines determined from the calibration curve. The source is identified by comparing these wavelengths with values given in spectroscopic tables.

THEORY:

The Diffraction Grating: The diffraction grating, devised by Fraunhofer (1787-1826), is an excellent light dispersing device for the study of spectra. It is made by ruling equally-spaced parallel lines or grooves on a polished reflecting or transmitting surface. These lines are ruled with a fine diamond point and number usually 10,000 to 15,000 lines per inch. The reflection or transmission of light by the ruled surface sets up diffraction and interference effects which are utilized to form the spectrum of a light source. Original gratings are expensive. Replica gratings are made by pouring dilute collodion or cellulose acetate on an original grating surface. After this solution has dried the thin film with the grating impression is detached and mounted on a glass plate. Owing to shrinkage and distortion in handling the replica, it is less perfect than the original but it may satisfactorily be used when high accuracy is not required.

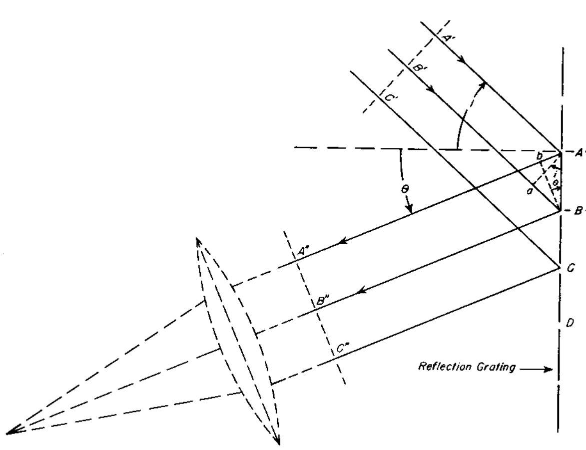

Fig. 1. Theory of the diffraction grating

The action of the grating can be described best by accepting the fundamental principle of wave motion advanced by the scientist Huygens. According to Huygens' principle, a wave may be regarded as propagating itself by an infinite number of secondary wavelets. Points in the existing wave front act as new sources and the envelope of the wavelets at any time constitutes the advancing wave front. Diffraction phenomena are due to interference between wavelets from different portions of the same wave front.

In Fig. 1 let A′ B′ C′ represent a plane wave front in a monochromatic beam of light advancing, at an incident angle i, towards the reflection grating at the right. Positions A, B, C, D, etc., are the equally-spaced reflecting sections between lines of this grating which are shown in cross section. Each of these reflecting areas, with centers separated by the grating-space distance $\triangle$, will start a wavelet and reflect some of the incident light energy towards the left.

There are certain specific reflection angles θ for which the reflected light from all the reflecting lines is in phase to form a wave front such as A″ B″ C″. This light may be brought to focus with a suitable lens. The necessary condition for constructive interference in the wave front is that the path lengths of adjacent narrow beams (rays) differ by an integral number of wavelengths λ. Expressed as an equation,

$$B'\,B\,B'' - A'\,A\,A'' = n\lambda$$

Constructing the lines Aa and Bb perpendicularly to the rays A′ A and B B″ respectively, makes it evident that the path difference in successive pairs of rays is also

$$a\,B - b\,A = n\lambda$$

Since $a\,B = \triangle \sin i$ and $bA = \triangle \sin \theta$

$$\triangle\ (\sin i - \sin \theta) = n\lambda$$

Equation (1) is called the grating equation. The *order* of the spectrum is expressed as the numerical value of n. Thus when the path difference in successive pairs of rays is two wavelengths of the light, the spectrum line is in the second order. Theoretically there is constructive interference at many angles θ. Actually the intensity in increasing orders falls off so rapidly that only a few orders are visible.

When the incident light is not monochromatic, but contains several different wavelengths (colors) of light, each wavelength forms an image at a different angle of reflection in each order. The resulting separation of colors gives the spectrum of the light source.

The Concave Reflection Grating: For many years gratings were ruled only on flat surfaces and lenses were required to bring the diffracted rays, either reflected or transmitted, to a focus. In 1882 Rowland invented the concave grating which focused the rays and eliminated the need of lenses. Thus chromatic aberration and the absorption of energy by the lenses was eliminated. Glass lenses, for example, exhibit color aberration and do not effectively transmit light in the infrared and ultra violet region. The lines or grooves on this type of grating are ruled on a concave spherical mirror at equidistant intervals along a chord.

Professor Rowland discovered that the concave grating has a unique relationship in the relative position of the source slit, the grating and its center of curvature, and the focusing point of the spectrum. These are always located on a circle drawn tangent to the midpoint of the grating with a diameter equal to the radius of curvature of the grating. This circle is called the *Rowland circle.*

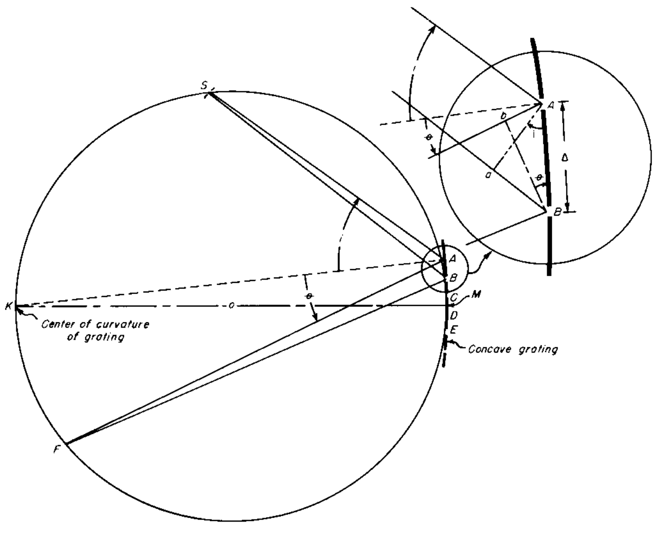

Fig. 2. Rowland circle for a concave grating

In Fig. 2, let AE be a concave grating of radius of curvature KM. This figure is highly exaggerated since the radius of curvature KM is normally several hundred thousand times the length of the grating space Δ. A circle, called the Rowland circle, is drawn having the diameter KM. On this circle is located the source of light to be analyzed at position S and the dispersed light is brought to a focus at some point, or points, such as F also on the circle. To prove that S, F and M lie on the circle proceed as follows.

Draw rays SAF and SBF where A and B are two successive grating spaces as shown in Fig. 2. For constructive interference the path difference of the two rays must satisfy Eq. (1), namely,

$$SAF - SBF = n\lambda$$

With S as center and radius SA describe the arc Aa. With F as center and radius FB describe the arc Bb. The lines Bb and Aa are sensibly straight and perpendicular to SB and FA, respectively, hence

$$aB - bA = n\lambda$$

or

$$\Delta\ (\sin i - \sin \theta) = n\lambda$$

Consider next rays from the slit S which strike any other two consecutive reflecting spaces of the grating, say D and E. These rays have incident and reflecting angles i' and θ' respectively. For reinforcement they must also satisfy the condition

$$\Delta\ (\sin i' - \sin \theta') = n\lambda$$

Since the grating surface is very small in comparison to the radius of curvature, all elements of the grating lie very nearly on the circle KSMF. The angles SAB $= i$ and SDK $= i'$ are subtended by the same arc KS, therefore, these angles are equal $(i = i')$. Similarly $\theta = \theta'$. It thus follows that the light from all grating spaces is in constructive interference at F when

$$\Delta\ (\sin i - \sin \theta) = n\lambda$$

This equation for the concave grating is identical with Eq. (1) for a plane grating.

For any given wavelength, say monochromatic light, there will be constructive interference at angles θ for which n = 0, 1, 2, 3, etc. The n values again designate the order number of the spectrum. When the light contains several different wavelengths, each of these orders contains the full spectrum formed on the Rowland circle.

There is some astigmatism if the mirror is used off axis. Each part of the slit is brought to a focus in two lines, one of them on the Rowland circle perpendicular to its plane, the other beyond the circle in the plane.

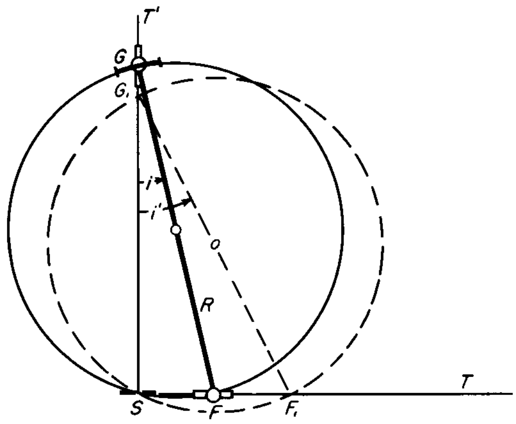

Fig. 3. Rowland Mounting

The Rowland Mounting: The Rowland mounting, Fig. 3, has the grating G and the eyepiece F fixed at opposite ends of a rigid support rod R. The separation distance G to F is equal in length to the diameter of the Rowland circle. The ends of the support slide over a pair of tracks TS and T' S which intersect at a right angle. The slit S is mounted on the intersection of the two tracks. The dash lines in the figure show another possible position for the support rod. In all positions of the rod the grating, eyepiece and source slit are on a Rowland circle.

By sliding the support R on the tracks T the angle of incidence i is changed. Thus the several wavelengths and orders of the spectrum may be viewed successively.

Spectral Study Holding Angle θ Constant and Equal to Zero: When the mounting is used as shown in Fig. 3 the reflection angle θ is a constant. The instrument is usually adjusted with this angle θ equal to zero. For this case Eq. (1) reduces to the form

$$\triangle \sin i = n\lambda$$

For a given spectral order

$$\frac{\triangle}{n} = \text{const.} = \frac{\lambda}{\sin i} = \frac{\lambda}{SF/R}$$

$$\text{Hence,} \quad \frac{\lambda_1}{SF_1} = \frac{\lambda_2}{SF_2} \qquad (2)$$

Equation (2) states that the ratio of wavelengths of spectral lines in a given order is equal to the ratio of the distances on the scale S to F. This is called a *normal spectrum.* When one order of the spectrum is observed the scale may be directly graduated to wavelength readings.

The zero of the linear scale may not coincide with the slit by a scale displacement of length K. The value of SF is then X + K and the grating equation gives,

$$\triangle\ (X + K)/R = n\lambda$$

from which it follows that

$$\frac{d\lambda}{dx} = \frac{\triangle}{nR} \qquad (3)$$

Thus if the values of λ as ordinates are plotted against values of x as abscissas each order will graph as a straight line, with a slope inversely proportional to the order of the spectrum.

Spectral Study with Incident Angle *i* Constant: When the incident angle i is fixed, the spectrum on the Rowland circle may be viewed (with the apparatus used) by moving the eyepiece along this circle to a new position E, as shown in Fig. 4. The several spectral lines will each have their specific reflection angle θ for any given value of angle i.

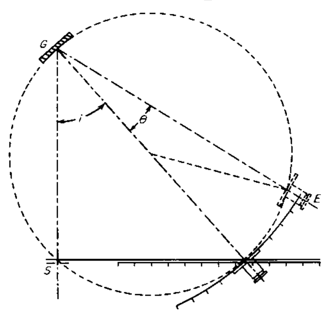

Fig. 4. Rowland mounting showing incident and reflection angles

Differentiating Eq. (1), assuming i = const., gives

$$n\, d\lambda = -\triangle \cos\theta\, d\theta$$

$$\frac{d\lambda}{d\theta} = -\frac{\triangle \cos\theta}{n} \qquad (4)$$

When θ is restricted to small angles $\frac{d\lambda}{d\theta}$ is nearly constant. Therefore, graphs of λ as ordinates against θ as abscissas, for each order, are straight lines with negative slopes. As before the slopes vary inversely as the spectral order n.

Dispersion and Resolving Power: The angular *dispersion* of a grating, i = const., is $\frac{d\theta}{d\lambda}$, the reciprocal value of Eq. (4). It is noted that the dispersion is inversely proportional to the spacing of the rulings on the grating, and directly proportional to the order.

The *resolving power* is defined as the quotient of the wavelength to the smallest change in wavelength that can be detected, $\lambda/d\lambda_{min}$. It depends on the product of the order number and the total number of rulings, N, on the grating surface, that is

$$\lambda/d\lambda_{min} = nN \qquad (5)$$

APPARATUS: The principal piece of apparatus required for this experiment is a Rowland-mounting Spectrometer,

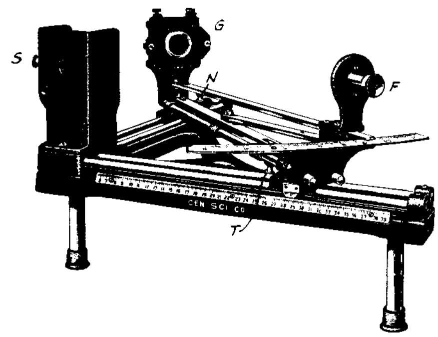

Fig. 5. Rowland-mounting spectrometer

Fig. 5, equipped with a concave reflection grating at position G. The grating must have a radius of curvature equal to the diameter of the Rowland circle of the mounting. Gratings with 15,000 lines per inch are suitable. The track scale is a linear calibration, the circular scale for measuring θ is graduated in degree divisions.

Any convenient spectrum sources may be used. Since the spectrum of mercury is particularly suitable, the directions specifically call for its measurement. The Pluecker Spectrum Tube and power supply, shown in Fig. 6, serve well in this experiment.

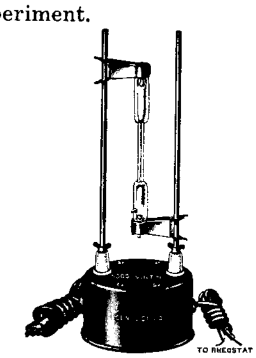

Fig. 6. Spectrum tube and power source

PROCEDURE:

Adjustment of the Spectrometer. The spectrometer should be used in a dark or semi-dark room. It must be carefully adjusted to focus a sharp image of the source slit on the cross hair of the eyepiece. Proceed in the following manner:

1. Carefully center the concave grating in the holder G of Fig. 5. Do not touch the grating surface. Adjust the three screws at the back of the grating holder to place the center of the grating surface directly above the center of the track. The spacing produced by the three screws should be uniform.

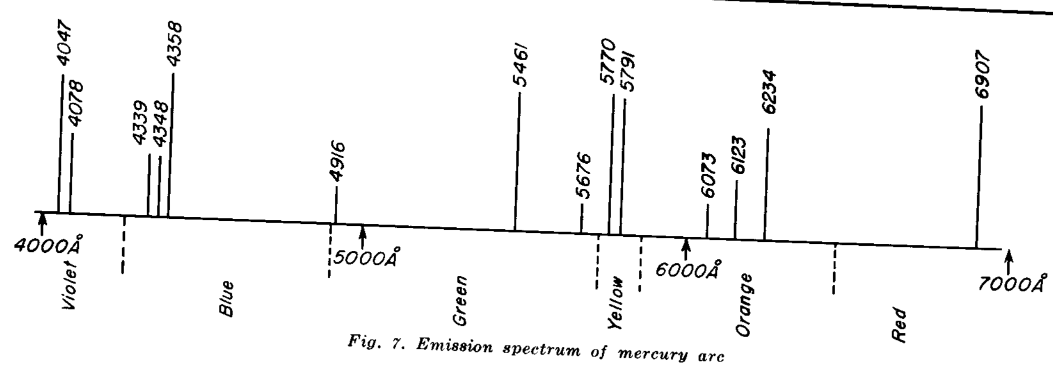

Fig. 7. Emission spectrum of mercury arc

2. Release the thumb nut screw N of Fig. 5. Set the distance from the grating to the cross hair in F to equal the radius of curvature of the grating. For the apparatus shown in Fig. 5 this distance is approximately 42.5 cm.

3. Illuminate the cross hair with a small light (10 or 15 watts) placed near it. The light shield on the cross hair is temporarily removed for this operation and the one following. Looking into the eyepiece, adjust the eyepiece to put the cross hair in sharp focus.

4. The next task is to focus the image of the cross hair produced by the concave grating on the cross hair in the eyepiece. Set the eyepiece on the zero of the circular scale. Illuminate the cross hair as in 3 above. Now, looking through the eyepiece, slightly adjust the three screws at the back of the grating holder until the cross hair image is visible. The white dot and cross are helpful in locating this image. Sharpen the image by releasing the screw N to adjust for the best G to F distance and then reclamp N.

Make a final adjustment of the mirror for perfect concidence of the cross hair and its image.

5. Open the light slit, S, to a width of about a half millimeter and rotate the slit to a vertical position. Place a light source, preferably a mercury arc, in front of the slit so that the light passing through the slit will illuminate the grating. Locate a spectral line on the cross hair. The 5461 Å green line of mercury is very suitable. Turn the screws at the top of the grating holder to produce the sharpest image. It may be necessary also to change slightly the slit width and the angle of the slit. The apparatus is then in adjustment for taking data.

Experimental:

(A) With the reflecting angle θ equal to zero, obtain the positions of the visible lines in the spectrum of mercury. The prominent lines in this spectrum are shown in Fig. 7. Traces of impurity in the source may give rise to additional emission lines. Some of them may not be visible under the conditions of the experiment. The height of each line in Fig. 7 represents the relative intensity of the line. For example, the green 5461Å line and the two yellow lines are strong lines.

Plot wavelengths λ as ordinates against scale positions as abscissas. Draw the curve for each order. Compute the slopes and relate slope values to the order of the spectrum.

(B) With the reflecting angle θ equal to zero, set the cross hair on the first order green line of mercury. Release the movable arm of the spectrometer by pulling down and turning the screw T of Fig. 5. The eyepiece will then swing along the Rowland circle for an examination of the spectrum. Keeping i constant, record θ for all visible lines on the circular scale. Repeat for the other orders of the green line.

Plot λ as ordinates against θ as abscissas for each order. Relate the slope of these lines to the order of the spectrum.

(C) Using your data of part (A) above, plot a full page calibration curve, wavelength versus scale reading, for the first order of the mercury spectrum. Cover the range 4000 Å to 7000 Å and scale readings 10 to 18.

(D) With angle θ held at zero degrees, locate the positions of the visible spectral lines in an unidentified source. Use the calibration curve of part (C) to determine the wavelengths of the lines. Identify the source by comparison of these wavelengths to the values published in spectroscopic tables.

QUESTIONS:

1. Distinguish between angular dispersion and resolving power of a grating.

2. What is the magnitude of the angular dispersion and of the resolving power of the grating used in this experiment in the region of the first order green line of mercury?

3. Theoretically, what is the minimum width of the grating used in this experiment to just resolve the yellow lines of the mercury spectrum?

4. Construct a drawing similar to Fig. 3. Let F and F_1, in this figure, be the scale positions of the first and second orders of a given spectral line. Locate the positions of the third and fourth orders and construct their Rowland circles.

5. Draw a line in the figure of problem (4) connecting the centers of all the Rowland circles. What is the form of this curve?

6. A given light source contains a prominent blue, a green, and a red line. Could the spectrum, as seen with this apparatus, have the succession—green, blue and red lines? Explain.

7. What would be the change seen in a spectrum if each of the following changes were independently made:
(a) Area of grating surface doubled?
(b) Number of lines on grating doubled?
(c) Distance between grating lines doubled?

8. Figure 7 states wavelength values to the nearest Angstrom unit. By what factor must the grating width be increased to require an additional significant figure in these values?

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