Wave Addition of Sodium Doublet on the Michelson Interferometer

Light is associate with a wave (or oscillating, moving electric field, ***E)*** of the form:

$$E = E_0 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi \right) . \qquad \textbf{(1)}$$

In a medium with a refractive, **n**, and the speed of light in vacuum is c, then the angular frequency, ω, and the wavelength, λ, are related by:

$$\lambda \frac{\omega}{2\pi} = \frac{c}{n} . \qquad \textbf{(2)}$$

Of course, what is observed is the intensity, ***I*** which is proportional to E^2 . Finally, it should be realized that what is observed is the time average of the intensity, and this allows one to drop the time factor that is shown in eq.1.

When viewing the sodium doublet, the electric field is the sum of two waves of the form shown in eq.1. (Recall that the rule for finding the resultant intensity of a number electromagnetic waves is to add the electric fields and then square.) After dropping the time factor this yields:

With the assumption that $E_1 \approx E_2 = E_0$ and a little trig (sin A + sin B = 2 sin ½(A+B) cos ½(A-B)), this becomes:

$$E = 2 E_0 \sin\left(2\pi x \frac{\lambda_1 + \lambda_2}{2 \lambda_1 \lambda_2} \right) \cos\left(2\pi x \frac{\lambda_2 - \lambda_1}{2 \lambda_1 \lambda_2} \right) . \qquad \textbf{(3)}$$

If $\lambda_1 \approx \lambda_2$, then $\lambda_1 \lambda_2 \approx \lambda^2_{\mathrm{AVERAGE}}$, where $\lambda_{\mathrm{AVERAGE}} = ½ (\lambda_1 + \lambda_2)$. Let

$$\lambda_{SMALL} = \frac{2 \lambda_1 \lambda_2}{\lambda_1 + \lambda_2} \approx \lambda_{AVERAGE} , \qquad \textbf{(4)}$$

and

$$\lambda_{ENVELOPE} = \frac{2 \lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \approx \frac{2 \lambda^2_{AVERAGE}}{\lambda_2 - \lambda_1} . \qquad \textbf{(5)}$$

$$E = E_1 \sin\left(\frac{2\pi x}{\lambda_1} \right) + E_2 \sin\left(\frac{2\pi x}{\lambda_2} \right) . \qquad \textbf{(6)}$$

$$E = 2\,E_0 \sin\left(\frac{2\pi x}{\lambda_{AVERAGE}}\right) \cos\left(\frac{2\pi x}{\lambda_{ENVOLOPE}}\right) . \qquad \textbf{(7)}$$

With these definitions, eq.4 becomes:

Remember, what is observed is the intensity and this will then have the form

$$I = I_0 \sin^2\left(\frac{2\pi x}{\lambda_{AVERAGE}}\right) \cos^2\left(\frac{2\pi x}{\lambda_{ENVOLOPE}}\right) . \qquad \textbf{(8)}$$

The $\sin^2$ and the $\cos^2$ factors each go to zero twice as their arguments change by 2π. This implies that the small structure of the observed intensity pattern for the sodium doublet has a separation between minima of

$$\Delta\ \textit{Optical Path Length}_{SMALL} = \tfrac{1}{2}\lambda_{AVERAGE} = \tfrac{1}{4}(\lambda_1 + \lambda_2) . \qquad \textbf{(9)}$$

Similarly the separation of minima of the envelope is

$$\Delta\ \textit{Optical Path Length}_{ENVELOPE} = \tfrac{1}{2}\lambda_{ENVELOPE} = \frac{\lambda^2_{AVERAGE}}{\lambda_2 - \lambda_1} . \qquad \textbf{(10)}$$