22.6

(a) $\phi = \vec{E} \cdot \vec{A} = EA\cos\theta$ where $\vec{A} = A\hat{n}$.

$\hat{n}_{s_1} = -\hat{j}$ (left) $\quad \phi_{s_1} = -(4\times 10^3 \, N/C)(0.1m)^2 \cos(90^\circ - 36.9^\circ)$
$= -24 \; N \cdot m^2/C$

$\hat{n}_{s_2} = +\hat{k}$ (top) $\quad \phi_{s_2} = -(4\times 10^3 \, N/C)(0.1m)^2 \cos 90^\circ$
$= 0$

$\hat{n}_{s_3} = +\hat{j}$ (right) $\quad \phi_{s_3} = +(4\times 10^3 \, N/C)(0.1m)^2 \cos(90^\circ - 36.9^\circ)$
$= 24 \; N \cdot m^2/C^2$

$\hat{n}_{s_4} = -\hat{k}$ (bottom) $\quad \phi_{s_4} = (4\times 10^3 \, N/C)(0.1m)^2 \cos 90^\circ = 0$

$\hat{n}_{s_5} = \hat{i}$ (front) $\quad \phi_{s_5} = +(4\times 10^3 \, N/C)(0.1m)^2 \cos 36.9^\circ$
$= 32 \; N\,m^2/C$

$\hat{n}_{s_6} = -\hat{i}$ (back) $\quad \phi_{s_6} = -(4\times 10^3 \, N/C)(0.1m)^2 \cos 36.9^\circ$
$= -32 \; N m^2/C$

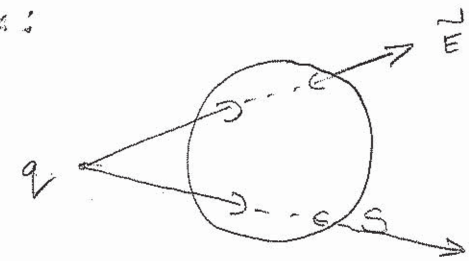
(b) Total flux must be zero. Net flux in equals net flux out.

Q22.15

(a) Uniform $\vec{E} \Rightarrow$ net flux through a closed surface $= 0$. $\quad \phi = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int \rho \, dV = 0$

$\Rightarrow \int \rho \, dV = 0$. But V is arbitrary, so this can only be true if $\rho = 0$.

(b) No. $\vec{E}$ can be divergent, while still providing zero flux:



$\vec{E}$ not uniform.

22.14

(a) $E(r = 0.450\,m + 0.1\,m) =$

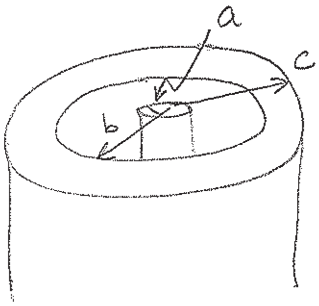
$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(2.50\times10^{-10}\,C)}{(0.550\,m)^2}$$

$$= 7.44 \quad N/C$$

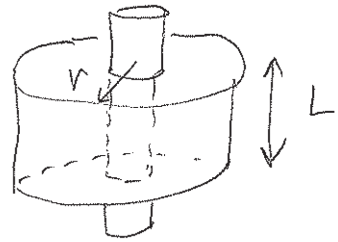
(b) $\vec{E} = 0$ inside a conductor, else free charges move to cancel it.

22.33

(a)



Consider a Gaussian box (cylinder) with radius $\underline{a<r<b}$:



By symmetry, E points radially. Thus

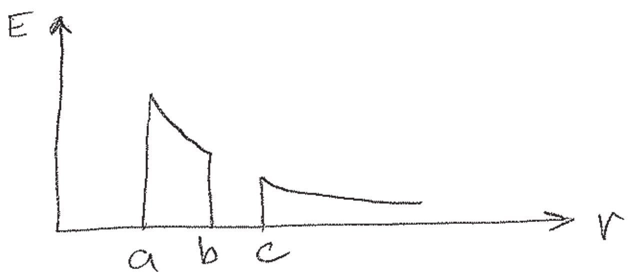
$$\Phi_E = \oint_S \vec{E}\cdot d\vec{A} = E(r)\,2\pi r L$$

$$= \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} L\cdot\lambda$$

So $E(r) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\lambda}{r}$, radially outward

(b) $\underline{r>c}$: $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\lambda}{r}$, radially outward.

(c)



(d) The inner and outer surfaces must have the same amount of charge, though opposite signs:

$$\lambda_{inner} = -\lambda$$

$$\lambda_{outer} = +\lambda$$

22.39

(a) $\underline{r < a}$ $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

since enclosed charge = Q.

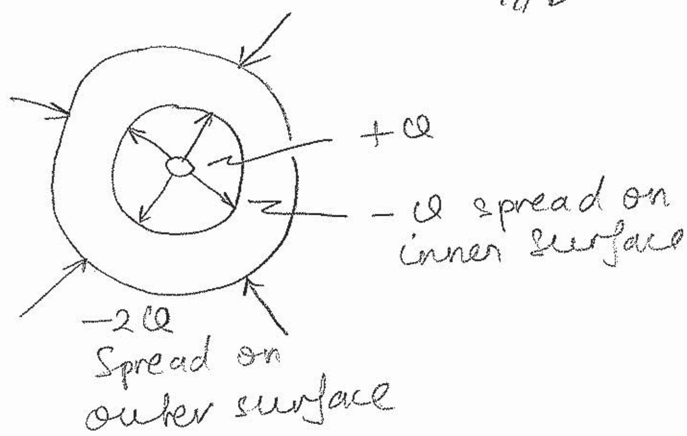
$\underline{a < r < b}$ $E = 0$, since $-Q$ on inner surface of shell.

$\underline{r > b}$ $E = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2}$, since total enclosed charge $= -2Q$.

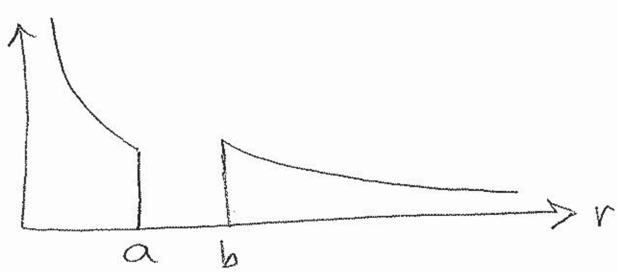
(b) Inner surface $\sigma = -\frac{Q}{4\pi a^2}$

(c) Outer surface $\sigma = -\frac{2Q}{4\pi b^2}$

(d)



(e)



22.61

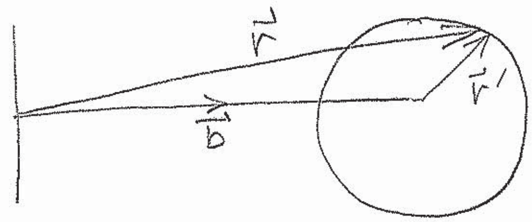
(a) Put $\vec{r}' = \vec{r} - \vec{b}$

$$\phi = 4\pi r'^2 E = \frac{Q_{enc}}{\varepsilon_0}$$

$$= \frac{\rho}{\varepsilon_0} \frac{4\pi r'^3}{3}$$

$$E = \frac{\rho r'}{3\varepsilon_0} \text{ in the } r'\text{-direction}$$

$$\Rightarrow \vec{E} = \frac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0}$$



(b) $$\vec{E}_{hole} = \vec{E}_{sphere} - \vec{E}_{(a)}$$

$$= \frac{\rho\vec{r}}{3\varepsilon_0} - \frac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0}$$

$$= \frac{\rho\vec{b}}{3\varepsilon_0}$$

$\vec{E}$ is uniform.