PHYSICS 515B

ELECTROMAGNETIC THEORY

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Section VII Problems (due Wednesday, December 2)

Problem 1: The vector potential due to a source $\vec{J}(\vec{x},t)$ may be written

$$\vec{A}(\vec{x},t) = \frac{1}{c}\int d^3x' \int dt' \frac{\vec{J}(\vec{x}',t')}{|\vec{x}-\vec{x}'|}\delta(t'-t+\frac{|\vec{x}-\vec{x}'|}{c}) \; .$$

Assume that the source is contained within a region of linear size $\sim L$. Show that at field points $|\vec{x}| \gg L$, the Fourier transform of $\vec{A}$ may be expanded as

$$\vec{A}_\omega(\vec{x}) = \frac{e^{ik|\vec{x}|}}{c|\vec{x}|}\Sigma_{n=0}^{\infty}\frac{1}{n!}\int \vec{J}_\omega(\vec{x}')(-ik\frac{\vec{x}\cdot\vec{x}'}{|\vec{x}|})^n \, d^3x' \; ,$$

where $\vec{J}_\omega(\vec{x})$ is the corresponding Fourier Transform of $\vec{J}(\vec{x},t)$, and $k \equiv \omega/c$. Assume $kL \ll 1$.

Problem 2: Consider a charge q moving around a circle of radius r_0 at frequency ω_0. By consideration of the current density and its Fourier Transform, show that the Fourier Transform of the vector potential $\vec{A}_\omega(\vec{x})$ is nonzero only at $\omega = \omega_0$ in the dipole approximation, nonzero only at $\omega = 2\omega_0$ in the quadrupole approximation and so on. You may want to use the results of Problem 1 above—but be careful in your choice of multipole components!

Problem 3: An almost spherical surface, defined by

$$R(\theta) = R_0\left[1+\beta P_2(\cos\theta)\right] \; ,$$

has inside of it a uniform volume distribution of charge totaling Q. The small parameter β varies harmonically in time at frequency ω. This corresponds to surface waves on a sphere. Keepingonly lowest-order terms in β and making the long-wavelength approximation, calculate the nonvanishing multipole moments, the angular distribution of radiation, and the total power radiated.

Problem 4: Jackson 13.5