# Astronomy 582

High-Energy Astrophysics

**Problem Set 2** (Due Wednesday, February 20)

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## Problem 1

When inertial reference frames $L$ and $L'$ coincide, let a flash of light be produced at the common origin. Each observer is justified in considering him(her)self at the center of an expanding sphere of light. Experiment has revealed that each obtains the same value $c$ for the speed of light. The Galilean transformation, $x' = x - vt$, does not give this result. Therefore, try a modification $x' = \gamma(x - vt)$, where $\gamma$ is to be determined. The postulates of special relativity require that this equation hold for the inverse transformation $x = \gamma(x' - v't') = \gamma(x' + vt')$. In this equation, we use the assertion that $v' = -v$. But for generality, the possibility has been allowed that $t'$ may be different from $t$. If $x$ and $x'$ are the intersections of the sphere with the axis at times $t$ and $t'$, respectively:

(a) to what is $x'/t'$ equal?

(b) to what is $x/t$ equal?

(c) use the results of parts (a) and (b) to eliminate $x$ and $x'$ in the transformation equations and thus to determine $\gamma$.

## Problem 2

An astronomer observed that a group of protons from the Sun (part of the solar wind) passed Earth at time $t_1$. Later, (s)he discovers that Jupiter has emitted a large burst of radio noise at time $t_1 + \Delta t$. A second astronomer $O'$ riding in a rocket traveling from Earth to Jupiter at speed $V$, observes the same two events. Assume that Earth is directly between the Sun and Jupiter, $6.3 \times 10^8$ km from Jupiter. Let $V = 0.50\,c$ and $\Delta t = 900$ s. Calculate the time interval $\Delta t'$ measured by observer $O'$ in the rocket. Could the protons from the Sun have triggered the radio burst from Jupiter?

## Problem 3

In Newtonian mechanics, the relation $dE/dt = \mathbf{F} \cdot \mathbf{v}$ is valid, where $E$ is the total energy of a particle that is moving with velocity $\mathbf{v}$ and is acted on by a net force $\mathbf{F}$. Show that this relation is also valid in relativistic mechanics. (Assume that Newton's second law is valid under special relativity.)

## Problem 4

Using the idea that dimensionless quantities (such as the total number of photons in a given volume) are Lorentz invariants, find the transformation law for the Intensity $I(\nu)$ (erg cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ steradian$^{-1}$) of radiation from one inertial frame $x^{\alpha}$ to another