

Astronomy 582

High-Energy Astrophysics

Solution Set 1

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Solution 1

(a) A 6 keV X-ray photon has an energy E of about 10^{-15} Joules, so it will raise the temperature of a penny containing 2.5 grams of zinc by an amount $dT = E/C \approx 10^{-15}$ K. At room temperature (300 K), this represents a fractional change of about 3 parts in 10^{18} , a number far beyond any temperature measuring capability available today. For one gram of silicon, however, a fractional change in temperature of 4 parts in 10^5 may be obtained for a silicon semiconductor with a specific heat capacity of $\approx 2.6 \times 10^{-10}$ Joules $\text{g}^{-1} \text{K}^{-1}$.

(b) A 3.65 keV photon will produce roughly 1,000 free electrons (CCD's do not work at photon energies much higher than this). The uncertainty in this number¹ is about $\sqrt{1,000} \approx 30$, so the accuracy of a typical CCD detector at this energy is $\sim 30/1,000$, or $\approx 3\%$ —roughly ten times worse than a microcalorimeter.

Solution 2

(a) Any detector that records the arrival of individual photons becomes inaccurate if the arrival rate is too high. This situation can be handled by assuming a time interval Δt , during which the detector is “dead.” If R is the observed count rate, then during a time T the average number of observed counts is RT , and the average time during which the detector is dead must be $RT\Delta t$. The actual count rate R' is larger than R by the ratio of T to the time during which the detector was “live” during T . That is,

$$R' = \frac{T}{T - RT\Delta t}$$

¹This is essentially the standard deviation for a Poisson distribution.

$$= \frac{R}{1 - R\Delta t}. \quad (0.1)$$

Thus, if $R = 100$ counts s^{-1} and $\Delta t = 1$ ms, then $R' = 111$ counts s^{-1} .

(b) We begin by converting the energy flux into a photon flux by dividing the former by the energy of a photon near the middle of the energy bandpass. (A more accurate method would be to calculate the average photon energy in each bandpass, but this would require knowledge of the source spectrum.) For Sco-X1, the photon flux in the 2–10 keV bandpass is therefore $(3 \times 10^{-7} \text{ erg cm}^{-2} \text{ s}^{-1}) / (6.41 \times 10^{-9} \text{ erg}) = 47$ photons $\text{cm}^{-2} \text{ s}^{-1}$.

Thus, multiplying this flux by the effective area of the *Uhuru* detector, we get the expected count rate of 1.3×10^4 counts s^{-1} . The same calculation for one unit of the proportional counter array on RXTE gives a rate of 4.3×10^4 counts s^{-1} . These results, together with the expected count rates (all in photons s^{-1}) for the other sources, are listed together below:

- Sco-X1: *Uhuru* rate = 1.3×10^4 , RXTE rate = 4.3×10^4 .
- Crab SNR: *Uhuru* rate = 8.7×10^2 , RXTE rate = 2.9×10^3 .
- Perseus Cluster: *Uhuru* rate = 44, RXTE rate = 150.
- QSO 3C273: *Uhuru* rate = 3.4, RXTE rate = 11.

The important quantity for assessing deadtime is $R\Delta t$. For Sco-X1 and RXTE, this product is $(1.3 \times 10^4 \text{ photons s}^{-1})(8.8 \times 10^{-6} \text{ s}) = 0.38$. This is a significant fraction of 1.0, so the answer is yes—deadtime corrections are important for observing Sco-X1.

(c) If the source rate is R and the background rate is B , then the signal-to-noise ratio for an observation of length t is

$$\frac{S}{N} = \frac{Rt}{\sqrt{Rt + Bt}} = \frac{R\sqrt{t}}{\sqrt{R + B}}, \quad (0.2)$$

and the length of time needed to achieve a particular S/N is

$$t = \frac{R + B}{R^2} \left(\frac{S}{N} \right)^2. \quad (0.3)$$

Thus, with $R = 3.4$ counts s^{-1} for *Uhuru* observing 3C273, and $B = 0$ counts s^{-1} , the time to achieve a $S/N = 100$ is $t = 2.9 \times 10^3$ s.

However, with a real background of 10 counts s^{-1} , the time is instead $t = 1.2 \times 10^4$ s. The background greatly increases the amount of observing time needed.

(d) With a photon flux of 0.012 photons $\text{cm}^{-2} \text{s}^{-1}$ from 3C273, and an effective mirror area of 175 cm^2 for the *Einstein* IPC, the expected rate is 2.1 counts s^{-1} . If the background of 10 counts s^{-1} is uniformly spread over the $75' \times 75'$ area of the imaging IPC detector, then only a tiny fraction of the background counts will be near the $1'$ diameter source. Quantitatively, the background rate for the source will be 10 counts s^{-1} times the fractional area $\pi(0.5')^2 / (75' \times 75') = 1.4 \times 10^{-4}$, or approximately 1.4×10^{-3} counts s^{-1} . This rate is negligible compared to the source count rate, demonstrating the advantage of using imaging detectors to greatly reduce the effects of background.

Solution 3

As discussed in the textbook, this exercise amounts to calculating the cumulative number of nuclei and atoms per unit area along the photon's path, starting from the top of the atmosphere. On average, when this column depth times the energy-dependent cross section equals one, the incoming photon will have its first interaction. From the data, one finds that a 10 keV photon should penetrate down to roughly 30 km from Earth's surface, a 1 MeV photon will make it to about 20 km, and a 100 MeV photon can reach as far as 12 km from the ground. (Compare your answers with figure 1.4 in the textbook.)

Solution 4

The mean-free-path is

$$\begin{aligned}\langle l \rangle &= \frac{1}{n_e \sigma_T} \\ &= (2.0 \times 10^{-7} \text{ cm}^{-3})^{-1} (6.66 \times 10^{-25} \text{ cm}^2)^{-1} \\ &\approx 2.4 \times 10^3 \text{ gigaparsecs} .\end{aligned}\tag{0.4}$$

By comparison, light has traveled a distance $\sim c \times (13.5 \text{ Gyrs})$ since the big bang. In gigaparsecs, this distance is roughly 4.5 gigaparsecs. Of course, the density increases significantly as we look back towards the beginning, but it would have to be greater by a factor ~ 533 before $\langle l \rangle \sim 1$, so the intergalactic medium is transparent to X-rays, at least out to a redshift $z > 8$.