Phys 332
Electricity and Magnetism II
Prof. Fulvio Melia
Homework 3

Problem 1: By explicit differentiation, show that the functions f_1, f_2, and f_3 satisfy the wave equation, but that f_4 and f_5 do not.

$$f_1(z,t) = Ae^{-b(z-vt)^2} \qquad f_2(z,t) = A\sin[b(z-vt)]$$
$$f_3(z,t) = \frac{A}{b(z-vt)^2+1} \qquad f_4(z,t) = Ae^{-b(bz^2+vt)}$$
$$f_5(z,t) = A\sin(bz)\cos(bvt)^3$$

Problem 2: Show that the standing wave $f(z,t) = A\sin(kz)\cos(kvt)$ satisfies the wave equation and express it as the sum of a wave traveling to the left and a wave traveling to the right.

Problem 3: Wangsness 24-5.

Problem 4: Wangsness 24-12.

Problem 5: Wangsness 24-13.

Problem 6:

(a) Suppose you embed some free charge in a piece of glass. About how long would it take for the charge to flow to the surface? Assume that $n_{\rm glass} = 1.5$ and that $\sigma_{\rm glass} \approx 10^{-12}\ \Omega^{-1}\ {\rm m}^{-1}$.

(b) Silver is an excellent conductor, but it's expensive. Suppose you were designing a microwave experiment to operate at a frequency of 10^{10} Hz. How thick would you make the silver coatings? Use $\sigma_{\rm Ag} = 6.29\times10^7\ \Omega^{-1}\ {\rm m}^{-1}$.

(c) Find the wavelength and propagation speed in copper for radio waves at 1 MHz. Compare your results with the corresponding values in air (or vacuum). Use $\sigma_{\rm Cu} = 5.95\times10^7\ \Omega^{-1}\ {\rm m}^{-1}$.

Problem 7:

(a) Show that the skin depth in a poor conductor ($\sigma \ll \omega\epsilon$) is $2/\sigma\sqrt{\epsilon/\mu}$ (independent of frequency). Find the skin depth (in meters) for (pure) water. Use $\epsilon = 80.1\epsilon_0$, $\mu = \mu_0$ and $\sigma = 4\times10^{-6}\ \Omega^{-1}\ {\rm m}^{-1}$.

(b) Show that the skin depth in a good conductor ($\sigma \gg \omega\epsilon$) is $\lambda/2\pi$ (where λ is the wavelength in the conductor). Find the skin depth (in nanometers) for a typical metal ($\sigma \approx 10^7\ \Omega^{-1}\ {\rm m}^{-1}$) in the visible range ($\omega \approx 10^{15}$ rad/s), assuming $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$). Why are metals opaque?

(c) SHow that in a good conductor the magnetic field lags behind the electric field by 45° and find the ratio of their amplitudes. For a numerical example, use the 'typical metal' in part (b).

Problem 8: Calculate the (time-averaged) energy density of an electromagnetic plane wave in a conducting medium. Show that the magnetic contribution always dominates. You may start from

$$\mathbf{E}(z,t) = E_0 e^{-\beta z}\cos(\alpha z - \omega t + \delta_E)\hat{x}$$
$$\mathbf{B}(z,t) = B_0 e^{-\beta z}\cos(\alpha z - \omega t + \delta_E + \Omega)\hat{y}$$

Recall that $\mathbf{B} = (|\mathbf{k}|/\omega)\hat{k}\times\mathbf{E}$ and you will certainly need the expressions for α and β from class.

Problem 9: Find all the elements of the Maxwell stress tensor for a monochromatic plane wave traveling through the vacuum in the z-direction and linearly polarized in the x-direction:

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\hat{x}$$
$$\mathbf{B}(z,t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)\hat{y}$$

Does your answer make sense? Remember that T_{ij} represents the momentum flux density. How si the momentum flux density related to the energy density in this case?

Problem 10: The function

$$\mathbf{f}(z,t) = Ae^{i(kz-\omega t)}\hat{n} \tag{1}$$

describes the most general linearly polarized wave on a string. Linear (or 'plane') polarization results from the combination of horizontally and vertically polarized waves of the same phase:

$$\mathbf{f}(z,t) = A\cos\theta e^{i(kz-\omega t)}\hat{x} + A\sin\theta e^{i(kz-\omega t)}\hat{y} \ . \tag{2}$$

If the two components are of equal amplitude, but out of phase by 90°, the result is a circularly polarized wave. In that case,

$$\mathbf{f}(z,t) = Ae^{i(kz-\omega t)}\hat{x} + Ae^{i(kz-\omega t+\pi/2)}\hat{y} \ . \tag{3}$$

In each equation above, you may assume that A is real and that the real part of the expression is what actually matters.

(a) At a fixed point z, show that the string moves in a circle about the z-axis. Does it rotate clockwise or counterclockwise as you look down the axis towards the origin? How would you construct a wave rotating the other way? (In optics, the clockwise case is called right circular polarization and the counterclockwise case left circular polarization.)

(b) Sketch the string at time $t = 0$.

(c) How would you shake the string in order to produce a circularly polarized wave?