PHYSICS 511

ANALYTICAL CLASSICAL MECHANICS

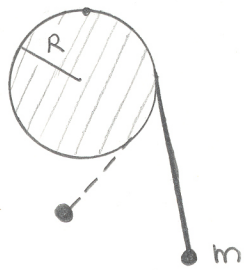
Homework 2

Problem 1:

(a) A point particle slides without friction down a cylinder, starting from rest at the top. Show that the particle leaves the cylinder when the motion has gone through a polar angle equal to $\cos^{-1}(2/3)$.

(b) Suppose that the point particle is replaced by a cylinder that rolls without slipping, starting again from the top. Show that the cylinder will leave the supporting cylinder after the motion has gone through a polar angle equal to $\cos^{-1}(4/7)$.

Problem 2: A pendulum is constructed by attaching a mass m to an extensionless string of length L. The upper end of the string is attached to the uppermost point on a vertical disk of radius R (with $R < L/\pi$), as in the figure. Find the equation of motion of the pendulum.



Problem 3: A massless tube is hinged at one end. A uniform rod of mass m, length L, slides freely in it. The axis about which the tube rotates is horizontal, so that the motion is confined to a plane. Choose a suitable set of generalized coordinates, one for each degree of freedom, and set up the Lagrange equations.

Problem 4: A pendulum bob of mass m is suspended by a string of length L from a car of mass M that moves without friction along a horizontal overhead rail. The pendulum swings in a vertical plane containing the rail. Set up the Lagrange equations.

Problem 5: In geometric optics, the trajectory of a light ray is given by Fermat's principle, which states that a ray travels between two fixed points in such a way that the time of transit is stationary with respect to small variations in the path. Consider propagation in a horizontal plane, assuming that the refractive index, n, is a function only of the distance

from the origin. Express Fermat's principle in integral form. Derive the equation for the ray:

$$\frac{dr}{d\phi} = r(kn^2r^2 - 1)^{1/2} ,$$

where k is a constant and (r, ϕ) are 2-dimensional polar coordinates.