HIGH-ENERGY ASTROPHYSICS (582)

(Spring 2006: Fulvio Melia)

Problem Set 1

Problem 1.1: When inertial reference frames L and L' coincide, let a flash of light be produced at the common origin. Each observer is justified in considering him(her)self at the center of an expanding sphere of light. Experiment has revealed that each obtains the same value c for the speed of light. The Galilean transformation, $x' = x - vt$, does not give this result. Therefore, try a modification $x' = \gamma(x - vt)$, where γ is to be determined. The principle of equivalence requires that this equation hold for the inverse transformation $x = \gamma(x' - v't') = \gamma(x' + vt')$. In this equation, we use the assertion that $v' = -v$. But for generality, the possibility has been allowed that t' may be different from t. If x and x' are the intersections of the sphere with the axis at times t and t', respectively: (a) to what is x'/t' equal? (b) to what is x/t equal? (c) use the results of parts (a) and (b) to eliminate x and x' in the transformation equations and thus to determine γ.

Problem 1.2: An astronomer observed that a group of protons from the Sun (part of the solar wind) passed the Earth at time t_1. Later, (s)he discovers that Jupiter has emitted a large burst of radio noise at time $t_1 + \Delta t$. A second astronomer O' riding in a rocket traveling from Earth to Jupiter at speed V, observes the same two events. Assume that the Earth is directly between the Sun and Jupiter, 6.3×10^8 km from Jupiter. Let $V = 0.50\,c$ and $\Delta t = 900$ s. Calculate the time interval $\Delta t'$ measured by observer O' in the rocket. Could the protons from the Sun have triggered the radio burst from Jupiter?

Problem 1.3: In Newtonian mechanics, the relation $dE/dt = \vec{F} \cdot \vec{v}$ is valid, where E is the total energy of a particle that is moving with velocity $\vec{v}$ and is acted on by a net force $\vec{F}$. Show that this relation is also valid in relativistic mechanics. (Assume that Newton's second law is valid under special relativity.)

Problem 1.4: (a) For the reaction $ab \to 12$, we may define the following Lorentz invariant variables

$$s = (p_a + p_b)^2$$

$$t = (p_a - p_1)^2$$

$$u = (p_b - p_1)^2$$

where $p^2 = p^\alpha p_\alpha$. Show that only two of these variables are independent by finding the constant (C) in the equation

$$s + t + u = C \ .$$

(b) Relate t and u to $\cos\theta^*$, the angle in the Center of Momentum Frame between the incident particle a and the produced particle 1. Express your answer in terms of the kinematical function

$$\lambda(e, f, g) = (e - f - g)^2 - 4fg \ .$$

Find the values of $\cos\theta^*$ that correspond to the maximum and minimum allowed values of t and u.

Problem 1.5: Using the idea that dimensionless quantities (such as the total number of photons in a given volume) are Lorentz invariants, find the transformation law for the Intensity $I(\nu)$ (ergs cm^{-2} s^{-1} Hz^{-1} steradian^{-1}) of radiation from one inertial frame x^α to another x'^α.