Shot Noise in Semiclassical Chaotic Cavities

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We construct a trajectory-based semiclassical theory of shot noise in clean chaotic cavities. In the universal regime of vanishing Ehrenfest time τ_{E}, we reproduce the random matrix theory result and show that the Fano factor is exponentially suppressed as τ_{E} increases. We demonstrate how our theory preserves the unitarity of the scattering matrix even in the regime of finite τ_{E}. We discuss the range of validity of our semiclassical approach and point out subtleties relevant to the recent semiclassical treatment of shot noise in the universal regime by Braun *et al.* (cond-mat/0511292).

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Introduction.—Quantum transport through chaotic ballistic cavities is often well described by random matrix theory (RMT) [1]. Despite its many successes, or should we say, because of these successes, one might wonder what is the origin of this RMT universality, and under what conditions do system specificities modify the RMT of transport. System specific contributions to transport originate from the underlying classical dynamics, which suggests that one employs semiclassical methods based on classical trajectories [2]. Indeed, the semiclassical program toward a microscopic foundation for the RMT of transport, including explicit bounds for its regime of applicability, is currently on its way to being completed successfully [3–9].

Here we contribute to this program by deriving the zero-frequency shot-noise power S for quantum chaotic systems. The interest in shot noise, the intrinsically quantum part of the fluctuations of a nonequilibrium electronic current, is that it often contains information on the system that cannot be obtained through conductance measurements. For instance, shot-noise experiments have determined the charge and statistics of the charge carriers in superconducting heterostructures or in the fractional quantum hall effect [10]. In this Letter, we consider an open ballistic quantum dot [11] carrying a large number of conducting channels and accordingly neglect electron-electron interactions. We reproduce the RMT result and show how shot noise deviates from RMT predictions in the semiclassical limit. We calculate the Fano factor $F = S/S_{\mathrm{p}}$, given by the ratio of S to the Poissonian noise $S_{\mathrm{p}} = 2e\langle I\rangle$ that would be generated by a current flow of uncorrelated electrons. According to the scattering theory of transport one has $F = \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}(1-\mathbf{t}^\dagger\mathbf{t})]/\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}]$ [10]. If one makes the RMT assumption that the transmission matrix $\mathbf{t}$ is the $N_{\mathrm{L}} \times N_{\mathrm{R}}$ off-diagonal block of a $(N_{\mathrm{L}} + N_{\mathrm{R}}) \times (N_{\mathrm{L}} + N_{\mathrm{R}})$ random unitary scattering matrix, one gets $F = N_{\mathrm{L}}N_{\mathrm{R}}/(N_{\mathrm{L}} + N_{\mathrm{R}})^2$ [1,10], in terms of the number of quantum channels N_{L} and N_{R} carried by the contacts to the left and right leads. Reference [5] carried out the first semiclassical calculation of F for the specific case of quantum graphs. The difficulty is to calculate $\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}] = \sum_{i,j,q}|t_{j,i}|^2|t_{q,i}|^2 + \sum_{i,j,p}|t_{j,i}|^2|t_{j,p}|^2 + \sum_{i\neq p;j\neq q} t^*_{j,i}t_{j,p}t^*_{q,p}t_{q,i}$. Reference [5] employed a diagonal approximation to calculate the first two terms and identified the dominant four-trajectory contributions to the third one. Quantum graphs fundamentally differ from continuum models which we treat here. In our semiclassical derivation we find the dominant contributions to F from the path pairings shown in Fig. 1. These pairings are similar to those considered in Ref. [5] for quantum graphs. However, unlike quantum graphs, chaotic systems have continuous families of scattering trajectories with similar actions, which means, in particular, that we cannot make a diagonal approximation to evaluate the contributions D_2 and D_3 shown in Fig. 1. This important point was not addressed in Ref. [9].

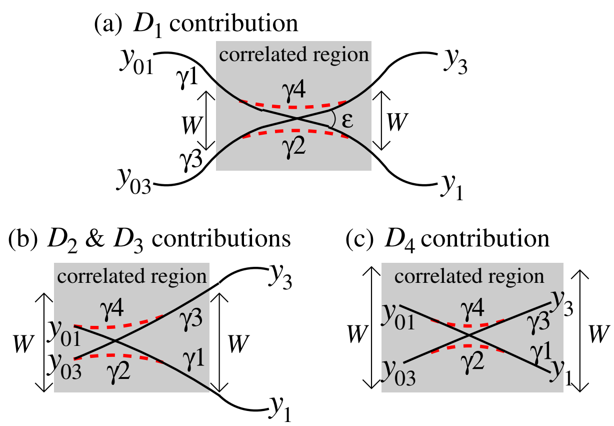

FIG. 1 (color online). The four dominant contributions to $\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}]$. Paths are paired everywhere except at encounters where two of them ($\gamma1$, $\gamma3$) cross each other (solid lines) while the other two ($\gamma2$, $\gamma4$) avoid the crossing (dashed lines). (a) Contribution D_1 has uncorrelated escape on both sides of the encounter. (b) Contributions D_2 and D_3 have correlated escape only on one side of the encounter. (c) Contribution D_4 has correlated escape on both sides.

Exploring the range of validity of RMT for chaotic systems, we find F to be exponentially reduced [12],

$$F = N_\mathrm{L} N_\mathrm{R} (N_\mathrm{L} + N_\mathrm{R})^{-2} \exp[-\tau_\mathrm{E}^\mathrm{o}/\tau_\mathrm{D}], \tag{1}$$

for systems with left (right) lead width, W_L (W_R), such that the width of leads $W_\mathrm{L,R} \gtrsim \hbar_\mathrm{eff}^{1/2} L$. These systems witness the emergence of the new Ehrenfest time scale $\tau_\mathrm{E}^\mathrm{o} = \lambda^{-1} \ln[\hbar_\mathrm{eff}^{-1}(\tau_\mathrm{f}/\tau_\mathrm{D})^2]$, which generically induces significant deviations from the RMT of transport [13]. Here, $\hbar_\mathrm{eff} = \hbar/(p_\mathrm{F} L)$, L is the linear system size, p_F the Fermi momentum of the particle with mass m, τ_f the time of flight, τ_D the dwell time through the system, and λ the Lyapunov exponent of the chaotic classical dynamics.

Our semiclassical calculation correctly captures both the universal regime with $\tau_\mathrm{E}^\mathrm{o}/\tau_\mathrm{D} \ll 1$ and the deep semiclassical regime where $\tau_\mathrm{E}^\mathrm{o}$ becomes comparable to or exceeds τ_D. We reproduce Eq. (1) and explicitly show that the exponential suppression of F is due to paths shorter than $\tau_\mathrm{E}^\mathrm{o}$ which become noiseless [6,7,14]. We demonstrate the unitarity of the theory by calculating both $F = \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}(1 - \mathbf{t}^\dagger\mathbf{t})]/\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}]$ and $F = \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{r}^\dagger\mathbf{r}]/\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}]$. We finally comment on the current limitations of the trajectory-based semiclassical approach.

We consider a two-dimensional chaotic quantum dot ideally connected to two external leads. We require that the size of the openings to the leads is much smaller than the perimeter of the system but is still semiclassically large, $1 \ll N_\mathrm{L}, N_\mathrm{R} \ll L/\lambda_F$. This ensures that the chaotic dynamics inside the dot has enough time to develop. The system's transport properties are given by its scattering matrix $\mathcal{S}$, with an $N_\mathrm{L} \times N_\mathrm{R}$ transmission block $\mathbf{t}$ and an $N_\mathrm{L} \times N_\mathrm{L}$ reflection block $\mathbf{r}$. To calculate the Fano factor, one needs to calculate the conductance $g = \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}]$, as well as $\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}]$.

Semiclassically, the transmission matrix reads [15]

$$t_{ji} = -(2\pi i\hbar)^{-1/2} \int_\mathrm{L} dy_0 \int_\mathrm{R} dy \sum_\gamma (dp_y/dy_0)_\gamma^{1/2} \langle j|y\rangle\langle y_0|i\rangle \times \exp[iS_\gamma/\hbar + i\pi\mu_\gamma/2], \tag{2}$$

where $|i\rangle$ is the transverse wave function of the ith lead mode. This expression sums over all paths γ (with classical action S_γ and Maslov index μ_γ) starting at y_0 on a cross section of the injection (L) lead and ending at y on the exit (R) lead. We approximate $\sum_n \langle y'|n\rangle\langle n|y\rangle \approx \delta(y' - y)$ [16], to write $\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}]$ as a sum over four paths, $\gamma 1$ from y_{01} to y_1, $\gamma 2$ from y_{03} to y_1, $\gamma 3$ from y_{03} to y_3, and $\gamma 4$ from y_{01} to y_3,

$$\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}] = \frac{1}{(2\pi\hbar)^2} \int_\mathrm{L} dy_{01} dy_{03} \int_\mathrm{R} dy_1 dy_3 \times \sum_{\gamma 1, \cdots \gamma 4} A_{\gamma 4} A_{\gamma 3} A_{\gamma 2} A_{\gamma 1} \exp[i\delta S/\hbar]. \tag{3}$$

Here, $A_\gamma = [dp_y/dy_0]_\gamma^{1/2}$ and $\delta S = S_{\gamma 1} - S_{\gamma 2} + S_{\gamma 3} - S_{\gamma 4}$ (we absorbed all Maslov indices into the actions $S_{\gamma i}$). We are interested in quantities averaged over variations in the energy or the system shape. For most contributions, $\delta S/\hbar$ oscillates wildly with these variations. The dominant contributions that survive averaging are those for which the fluctuations of $\delta S/\hbar$ are minimal. They are shown in Fig. 1. Their paths are in pairs almost everywhere except in the vicinity of encounters. Going through an encounter, two of the four paths cross each other, while the other two avoid the crossing. They remain in pairs, though the pairing switches, e.g., from ($\gamma 1; \gamma 4$) and ($\gamma 2; \gamma 3$) to ($\gamma 1; \gamma 2$) and ($\gamma 3; \gamma 4$) in Fig. 1(a). Paths are always close enough to their partner that their stability is the same. Thus, for all pairings in Fig. 1,

$$\sum_{\gamma 1, \ldots \gamma 4} A_{\gamma 4} A_{\gamma 3} A_{\gamma 2} A_{\gamma 1} \to \sum_{\gamma 1, \gamma 3} A_{\gamma 3}^2 A_{\gamma 1}^2. \tag{4}$$

We define $P(\mathbf{Y}, \mathbf{Y}_0; t)\delta y \delta\theta \delta t$ as the product of the momentum along the injection lead, $p_\mathrm{F}\cos\theta_0$, and the classical probability to go from an initial position and angle $\mathbf{Y}_0 = (y_0, \theta_0)$ to within ($\delta y, \delta\theta$) of $\mathbf{Y}$ in a time within δt of t. Then the sum over all paths γ from y_0 to y is

$$\sum_\gamma A_\gamma^2 [\cdots]_\gamma = \int_0^\infty dt \int d\theta_0 \int d\theta P(\mathbf{Y}, \mathbf{Y}_0; t) [\cdots]_{\mathbf{Y}_0}. \tag{5}$$

For an individual system, P has δ functions for all classical trajectories. However, averaging over an ensemble of systems or over energy gives a smooth function

$$\langle P(\mathbf{Y}, \mathbf{Y}_0; t)\rangle = \frac{p_\mathrm{F}\cos\theta_0 \cos\theta}{2(W_\mathrm{L} + W_\mathrm{R})\tau_\mathrm{D}} \exp[-t/\tau_\mathrm{D}]. \tag{6}$$

Using Eqs. (5) and (6) to calculate the conductance within the diagonal approximation directly leads to the Drude conductance $\langle \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}]\rangle \simeq g_\mathrm{D} = N_\mathrm{L} N_\mathrm{R}/(N_\mathrm{L} + N_\mathrm{R})$. This level of approximation for $\langle \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}]\rangle$ is sufficient to obtain F to leading order in $N_\mathrm{L,R}^{-1}$. We now use Eqs. (3)–(5) to analyze the contributions in Fig. 1.

There are two things that can happen to two pairs of paths as they leave an encounter. The first is *uncorrelated escape*. The pairs of paths escape when the perpendicular distance between them is larger than $W_\mathrm{L,R}$, which requires a minimal time $T_W(\epsilon)/2 = \lambda^{-1} \ln[\epsilon^{-1} W/L]$ between encounter and escape. The two pairs of paths then escape in an uncorrelated manner, typically at completely different times, with completely different momenta (and possibly through different leads). The second is *correlated escape*. Pairs of paths escape when the distance between them is less than $W_\mathrm{L,R}$. Then the two pairs of paths escape together, at the same time through the same lead.

Contributions to the Fano factor.—Taking into account the two escape scenarios just described, we write $\langle \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}]\rangle = D_1 + D_2 + D_3 + D_4$. Each of these four contributions, sketched in Fig. 1, can be written as

$$D_i = \frac{1}{(2\pi\hbar)^2} \int_\mathrm{L} d\mathbf{Y}_{01} d\mathbf{Y}_{03} \int_\mathrm{R} d\mathbf{Y}_1 d\mathbf{Y}_3 \times \int dt_1 dt_3 \langle P(\mathbf{Y}_1, \mathbf{Y}_{01}; t_1) P(\mathbf{Y}_3, \mathbf{Y}_{03}; t_3)\rangle \times \exp[i\delta S_{D_i}/\hbar], \tag{7}$$

where subscripts 1, 3 make the connection to Fig. 1. When evaluating Eq. (7) the joint exit probability for two crossing paths has to be computed.

To evaluate D_1, we use the method developed by Richter and Sieber [3], while taking into account that paths in the same region of phase space (shaded areas in Fig. 1) have highly correlated escape probabilities [17]. Here the action difference is $\delta S_{D_1} = E_F\epsilon^2/\lambda$ [3], where ϵ is the crossing angle shown in Fig. 1(a). We write

$$P(\mathbf{Y}_i, \mathbf{Y}_{0i}; t_i) = \int d\mathbf{R}_i \tilde{P}(\mathbf{Y}_i, \mathbf{R}_i; t_i - t_i')P(\mathbf{R}_i, \mathbf{Y}_{0i}; t_i'),$$

where $\tilde{P}$ is the probability for the classical path to exist (not multiplied by the injection momentum), and $\mathbf{R}_i$ is a point in the system's phase space $(\mathbf{r}_i, \phi_i)$ visited at time t_i', with ϕ_i giving the direction of the momentum. We choose $\mathbf{R}_1$ and $\mathbf{R}_3$ as the points at which the paths cross, so $\mathbf{R}_3 = (\mathbf{r}_1, \phi_1 \pm \epsilon)$ and $d\mathbf{R}_3 = v_F^2 \sin\epsilon dt_1' dt_3' d\epsilon$. Thus

$$D_1 = 2(2\pi\hbar)^{-2} \int_L d\mathbf{Y}_{01} d\mathbf{Y}_{03} \int_0^\pi d\epsilon \,\mathrm{Re}[e^{i\delta S_{D_1}/\hbar}] \times \langle I(\mathbf{Y}_{01}, \mathbf{Y}_{03}; \epsilon)\rangle. \tag{8}$$

$I(\mathbf{Y}_{01}, \mathbf{Y}_{03}; \epsilon)$ is related to the probability that $\gamma 3$ crosses $\gamma 1$ at angle $\pm\epsilon$. Its average is independent of $\mathbf{Y}_{01,03}$, so $\langle I(\mathbf{Y}_{01}, \mathbf{Y}_{03}; \epsilon)\rangle = \langle I(\epsilon)\rangle$. For D_1, injections or escapes are more than $T_W(\epsilon)/2$ from the crossing, so

$$\begin{aligned}\langle I(\epsilon)\rangle = 2v_F^2 \sin\epsilon \int_R d\mathbf{Y}_1 d\mathbf{Y}_3 \int d\mathbf{R}_1 \int_T^\infty dt_1 \int_{T/2}^{t_1 - T/2} dt_1' \\ \times \int_T^\infty dt_3 \int_{T/2}^{t_3 - T/2} dt_3' \langle \tilde{P}(\mathbf{Y}_1, \mathbf{R}_1; t_1 - t_1') \\ \times P(\mathbf{R}_1, \mathbf{Y}_{01}; t_1')\tilde{P}(\mathbf{Y}_3, \mathbf{R}_3; t_3 - t_3')P(\mathbf{R}_3, \mathbf{Y}_{03}; t_3')\rangle,\end{aligned} \tag{9}$$

where T is shorthand for $T_W(\epsilon)$. We next note that within $T_W(\epsilon)/2$ of the crossing, paths $\gamma 1$ and $\gamma 3$ are so close to each other that their joint escape probability is the same as for a single path (this was absent from Ref. [3] and was first noted in Ref. [17]). Elsewhere $\gamma 1$, $\gamma 3$ escape independently through either lead at anytime; hence

$$\langle I(\epsilon)\rangle = \frac{p_F^4 \tau_D}{\pi\hbar m} \frac{N_R^2 \cos\theta_{01} \cos\theta_{03} \sin\epsilon}{(N_L + N_R)^3} e^{-T_W(\epsilon)/\tau_D}, \tag{10}$$

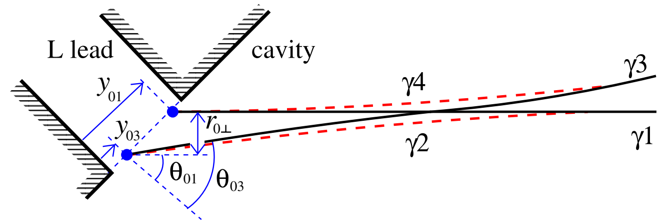

FIG. 2 (color online). Paths for the D_2 and D_4 contributions when they are in the correlated region (close to L lead). Paths $\gamma 1$ and $\gamma 3$ (solid black lines) start on the cross section of the L lead at positions y_{01} and y_{03} with transverse momenta $p_F \sin\theta_{01}$ and $p_F \sin\theta_{03}$, respectively. In the basis parallel or perpendicular to $\gamma 1$, the initial position and momentum of path $\gamma 3$ are $r_{0\perp} = (y_{01} - y_{03})\cos\theta_{01}$, $r_{0\|} = (y_{01} - y_{03})\sin\theta_{01}$, and $p_{0\perp} \simeq p_F(\theta_{01} - \theta_{03})$. Contribution D_3 has exactly the same structure close to the R lead.

where we used $N_R = (\pi\hbar)^{-1} p_F W_R$ and assumed that the probability that $\gamma 3$ is at $\mathbf{R}_3$ at time t_3' in a system of area A is $(2\pi A)^{-1} = m[2\pi\hbar\tau_D(N_L + N_R)]^{-1}$. Then the $\mathbf{Y}_{01,03}$ integral in Eq. (8) gives $(2W_L)^2$, while the ϵ integral is dominated by $\epsilon \ll 1$ and yields a factor of $-\pi\hbar(2E_F\tau_D)^{-1} e^{-\tau_E^o/\tau_D}\{1 + \mathcal{O}[(\lambda\tau_D)^{-1}]\}$ [4]. Thus

$$D_1 = -N_L^2 N_R^2 (N_L + N_R)^{-3} \exp[-\tau_E^o/\tau_D]. \tag{11}$$

The contribution D_2 is shown in Fig. 1(b), with Fig. 2 showing the paths in the correlated region in more detail. Noting that $\gamma 2$ decays exponentially towards $\gamma 1$, we find the action difference between the two paths to be

$$S_2 - S_1 = p_F(y_{01} - y_{03})\sin\theta_{01} + \tfrac{1}{2} m\lambda(y_{01} - y_{03})^2 \cos^2\theta_{01}. \tag{12}$$

The equation for $S_4 - S_3$ has the opposite sign for $(y_{01} - y_{03})$ and θ_{01} replaced by θ_{03}. In terms of $(r_{0\perp}, p_{0\perp})$,

$$\delta S_{D_2} = -(p_{0\perp} + m\lambda r_{0\perp})r_{0\perp}, \tag{13}$$

where we have dropped cubic terms (they give only $\hbar$ corrections to the stationary-phase integral). We next perform the average in Eq. (7). We define $T_W'(r_{0\perp}, p_{0\perp})$ as the time for which $\gamma 1$ and $\gamma 3$ are less than W apart, and insist that the paths are more than W apart before they escape to the right. Hence we must evaluate

$$\int_R d\mathbf{Y}_1 d\mathbf{Y}_3 \int_{T_W'}^\infty dt_1 dt_3 \langle P(\mathbf{Y}_1, \mathbf{Y}_{01}; t_1)P(\mathbf{Y}_3, \mathbf{Y}_{03}; t_3)\rangle = \frac{N_R^2 p_F^2 \cos\theta_{01}\cos\theta_{03}}{(N_L + N_R)^2} \exp[-T_W'(r_{0\perp}, p_{0\perp})/\tau_D]. \tag{14}$$

Inserting this into Eq. (7), we change integration variables using $p_F \cos\theta_{03} d\mathbf{Y}_{03} = dr_{0\perp} dp_{0\perp}$ [15], and then define $\tilde{p}_0 \equiv p_{0\perp} + m\lambda r_{0\perp}$. In the regime of interest $T_W'(r_{0\perp}, p_{0\perp}) \simeq \lambda^{-1} \ln[(m\lambda W)^{-1}\tilde{p}_0]$. Evaluating the integral over $r_{0\perp}$ leaves a $\tilde{p}_0$ integral which we cast as Euler Γ functions. To lowest order in $(\lambda\tau_D)^{-1}$ we find

$$D_2 = N_L N_R^2 (N_L + N_R)^{-2} \exp[-\tau_E^o/\tau_D]. \tag{15}$$

Substituting $N_L \leftrightarrow N_R$ in the derivation of Eq. (15) gives

$$D_3 = N_L^2 N_R (N_L + N_R)^{-2} \exp[-\tau_E^o/\tau_D]. \tag{16}$$

The contribution D_4 is shown in Fig. 1(c), with Fig. 2 showing the paths in detail at the L lead. This contribution can be evaluated in a way similar to D_2, the difference being that the paths escape before time $T_W'(r_{0\perp}, p_{0\perp})$, i.e., before becoming a distance W apart. The paths are always correlated, so the escape probability for the two paths equals that for one. Moreover, both paths will automatically escape through the same lead, hence

$$\int_{\mathrm{R}} d\mathbf{Y}_1 d\mathbf{Y}_3 \int_0^{T'_W} dt_1 dt_3 \langle P(\mathbf{Y}_1, \mathbf{Y}_{01}; t_1) P(\mathbf{Y}_3, \mathbf{Y}_{03}; t_3)\rangle = \frac{N_{\mathrm{R}} p_{\mathrm{F}}^2 \cos\theta_{01}\cos\theta_{03}}{N_{\mathrm{L}}+N_{\mathrm{R}}}(1 - e^{-T'_W(r_{0\perp}, p_{0\perp})/\tau_{\mathrm{D}}}). \tag{17}$$

Performing the same analysis as for D_2 we find that

$$D_4 = N_{\mathrm{L}} N_{\mathrm{R}} (N_{\mathrm{L}} + N_{\mathrm{R}})^{-1} (1 - \exp[-\tau_{\mathrm{E}}^{\mathrm{o}}/\tau_{\mathrm{D}}]). \tag{18}$$

The Fano factor is given by $F = 1 - g_{\mathrm{D}}^{-1}(D_1 + D_2 + D_3 + D_4)$. Our results of Eqs. (15), (16), and (18) show that $D_2 + D_3 + D_4 = g_{\mathrm{D}}$. One hence gets $F = -D_1/g_{\mathrm{D}}$. From Eq. (11), one finally obtains our main result, Eq. (1). The splitting of phase space discussed in Refs. [6,7,14] for $\tau_{\mathrm{E}}^{\mathrm{o}} \gtrsim \tau_{\mathrm{D}}$ naturally emerges here. For paths shorter than $\tau_{\mathrm{E}}^{\mathrm{o}}$, only D_4 is nonzero. This cancels these path's $\mathrm{Tr}[\mathbf{t}^\dagger \mathbf{t}]$ contribution, making them noiseless.

Preservation of unitarity.—The unitarity of the scattering matrix ensures that $\mathbf{t}^\dagger\mathbf{t} + \mathbf{r}^\dagger\mathbf{r} = 1$, and hence the Fano factor can be written as $F = g_{\mathrm{D}}^{-1}\langle \mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{r}^\dagger\mathbf{r}]\rangle$. We calculate this expression to explicitly show that our method preserves unitarity. We first note that there is no contribution D_3 nor D_4 to $\mathrm{Tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{r}^\dagger\mathbf{r}]$. We are left with the calculation of two contributions, D'_1 and D'_2, obtained from D_1 and D_2 shown in Figs. 1(a) and 1(b) with y_{01}, y_{03} and y_3 on the left lead and y_1 on the right lead. The calculation proceeds as for D_1 and D_2, with one factor of $N_{\mathrm{R}}/(N_{\mathrm{L}} + N_{\mathrm{R}})$ replaced by $N_{\mathrm{L}}/(N_{\mathrm{L}} + N_{\mathrm{R}})$ in both contributions. The sum of these two contributions is $D'_1 + D'_2 = e^{-\tau_{\mathrm{E}}^{\mathrm{o}}/\tau_{\mathrm{D}}} N_{\mathrm{L}}^2 N_{\mathrm{R}}^2 (N_{\mathrm{L}} + N_{\mathrm{R}})^{-3}$; the Fano factor is then $F = (D'_1 + D'_2)/g_{\mathrm{D}}$, which reproduces Eq. (1).

Off-diagonal nature of all contributions.—In our analysis we allow for the fact that open chaotic systems have continuous families of paths with highly correlated actions coupling to multiple lead modes. For example, paths $\gamma 1$ and $\gamma 2$ in Fig. 2 have an action difference given in Eq. (12), which does not fluctuate under energy or sample averaging. The stationary-phase integral for $D_{2,3,4}$ over such paths is dominated by paths $\gamma 1$ and $\gamma 3$ with $p_{0\perp} \simeq -m\lambda r_{0\perp}$. Since $r_{0\perp}$ is integrated over the width W of the lead, $p_{0\perp}$ varies over a range of order $m\lambda W$; these contributions are clearly *not* diagonal in the lead mode basis. Upon completion of this Letter, we became aware of Ref. [9], which presents a semiclassical calculation of F for $\tau_{\mathrm{E}}^{\mathrm{o}} = 0$. While their method is superficially similar to ours, they make a diagonal assumption to get the contributions that we call $D_{2,3,4}$. Our analysis shows that this is unjustifiable. Moreover, such an assumption would violate unitarity for finite $\tau_{\mathrm{E}}^{\mathrm{o}}$.

Regime of applicability of these semiclassics.—We appear to be the first to report that all trajectory-based semiclassical methods used so far in the theory of transport (including in the present Letter) are applicable only in the regime $W \geq \hbar_{\mathrm{eff}}^{1/2} L$. Dominant off-diagonal contributions such as those discussed above have encounters of a typical size $\sim\hbar_{\mathrm{eff}}^{1/2} L$. When $W < \hbar_{\mathrm{eff}}^{1/2} L$, the two noncrossing paths (i.e., $\gamma 2$ and $\gamma 4$ in Fig. 1) at an encounter are a distance apart greater than W. The probability that one of the four paths escapes while the other three paths remain in the system is of order $\tilde{\tau}_{\mathrm{E}}/\tau_{\mathrm{D}}$, where $\tilde{\tau}_{\mathrm{E}} \sim \lambda^{-1} \ln[\hbar_{\mathrm{eff}}(L/W)^2]$ is the time over which this path is a distance of order W from any of the other paths. The current methods fail once this is taken into account, suggesting that diffraction effects may become important. We believe that the regime $\hbar_{\mathrm{eff}} < (W/L) \leq \hbar_{\mathrm{eff}}^{1/2}$ is well described by RMT, and thus suspect this diffraction may be the microscopic source of RMT universality in this regime. Clearly this regime merits further study.

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