

Birth of the Ehrenfest time

SOVIET PHYSICS JETP

VOLUME 28, NUMBER 6

QUASICLASSICAL METHOD IN THE THEORY OF SUPERCONDUCTIVITY

A. I. LARKIN and Yu. N. OVCHINNIKOV

Institute of Theoretical Physics, USSR Academy of Sciences

Submitted June 6, 1968

Zh. Eksp. Teor. Fiz. 55, 2262–2272 (December, 1968)

It is shown that replacement of quantum-mechanical averages by the average values of the corresponding classical quantities over all trajectories with a prescribed energy is not valid in the general case. The dependence of the penetration depth on the field is found without making any assumptions about the weakness of the interaction between the electrons and the field of the impurities; the case of very dirty films is also considered.

An estimate of the validity of the resulting formulas may be obtained from the condition that an initial displacement of a particle of the order of its wavelength $h p_0^{-1}$ must lead at a moment of time t to a displacement which is smaller than the interaction radius $\sim \sigma^{1/2}$:

$$\lambda^2 Y(t) \ll \sigma \quad \text{or} \quad h^2 e^{t/t_0} \ll \sigma p_0^2. \quad (31)$$

$$h^2 e^{t/t_0} \ll \sigma p^2$$



Quantum Chaos in Mesoscopic Superconductivity



Philippe Jacquod
U of Arizona



I. Adagideli (Regensburg)
C. Beenakker (Leiden)
M. Goorden (Delft)
H. Schomerus (Lancaster)
J. Weiss (Arizona)



Outline

- Mesoscopic superconductivity - Andreev reflection
- Density of states in ballistic Andreev billiards
- Transport through ballistic Andreev interferometers
- Symmetries of multi-terminal transport in presence of superconductivity



Outline

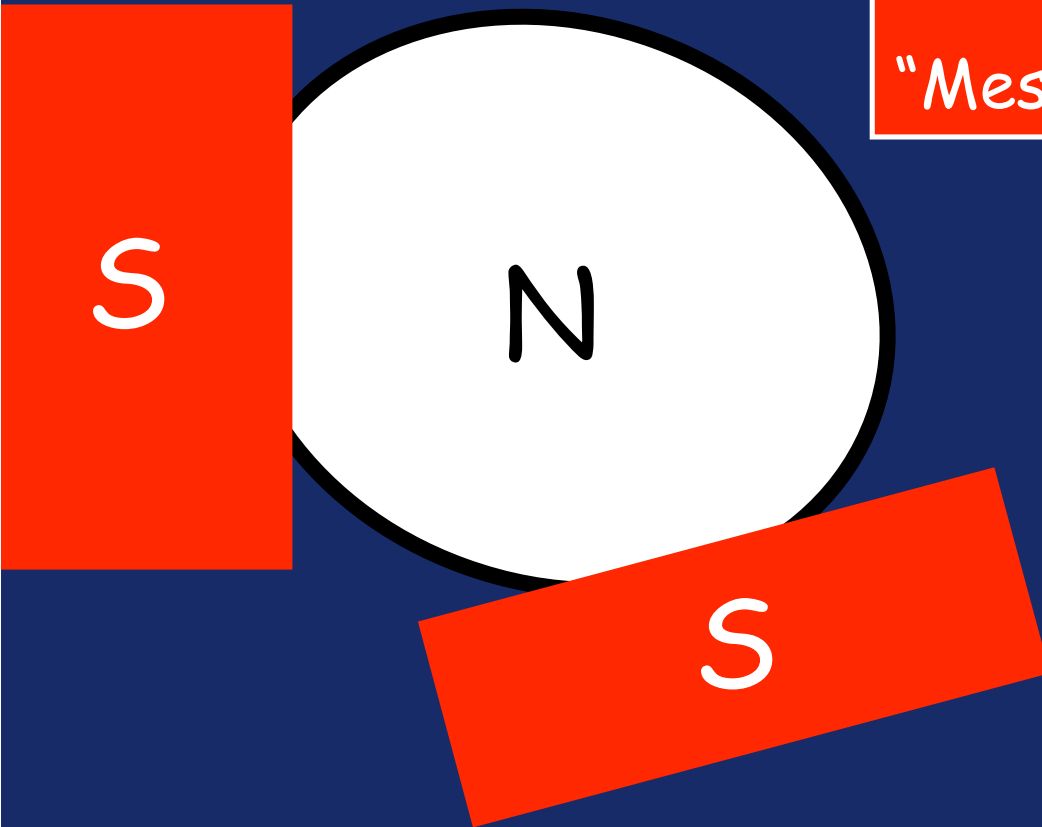
- Mesoscopic superconductivity - Andreev reflection
- Density of states in ballistic Andreev billiards
- Transport through ballistic Andreev interferometers
- Symmetries of charge transport in presence of superconductivity



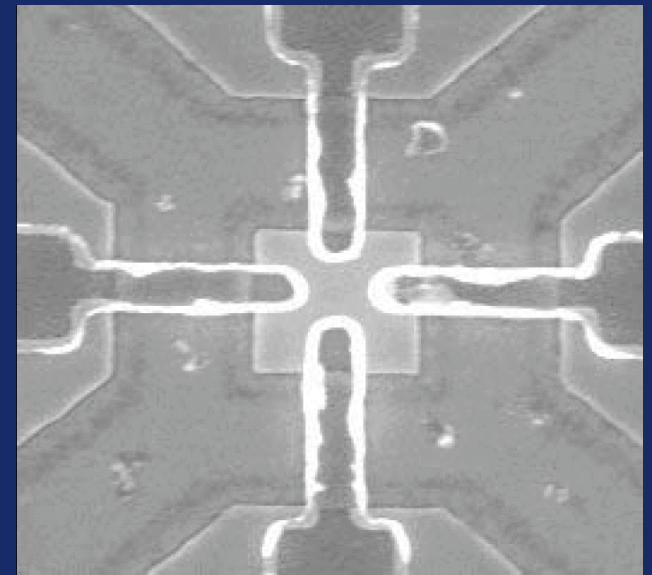
Mesoscopic Superconductivity

Mesoscopic metal (N) in contact with superconductors (S)

$\xi \ll L$



S invades N
"Mesoscopic proximity effect"

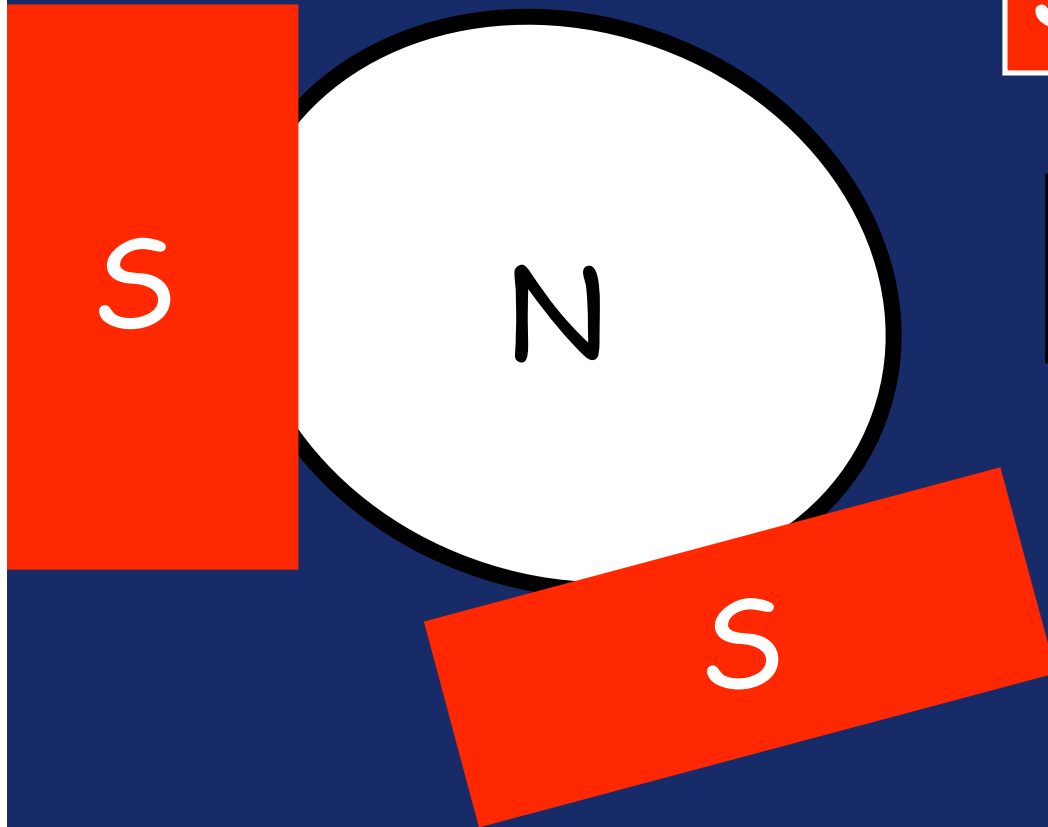


Device by AT Filip, Groningen

Mesoscopic Superconductivity

Mesoscopic metal (N) in contact with superconductors (S)

$\xi \ll L$

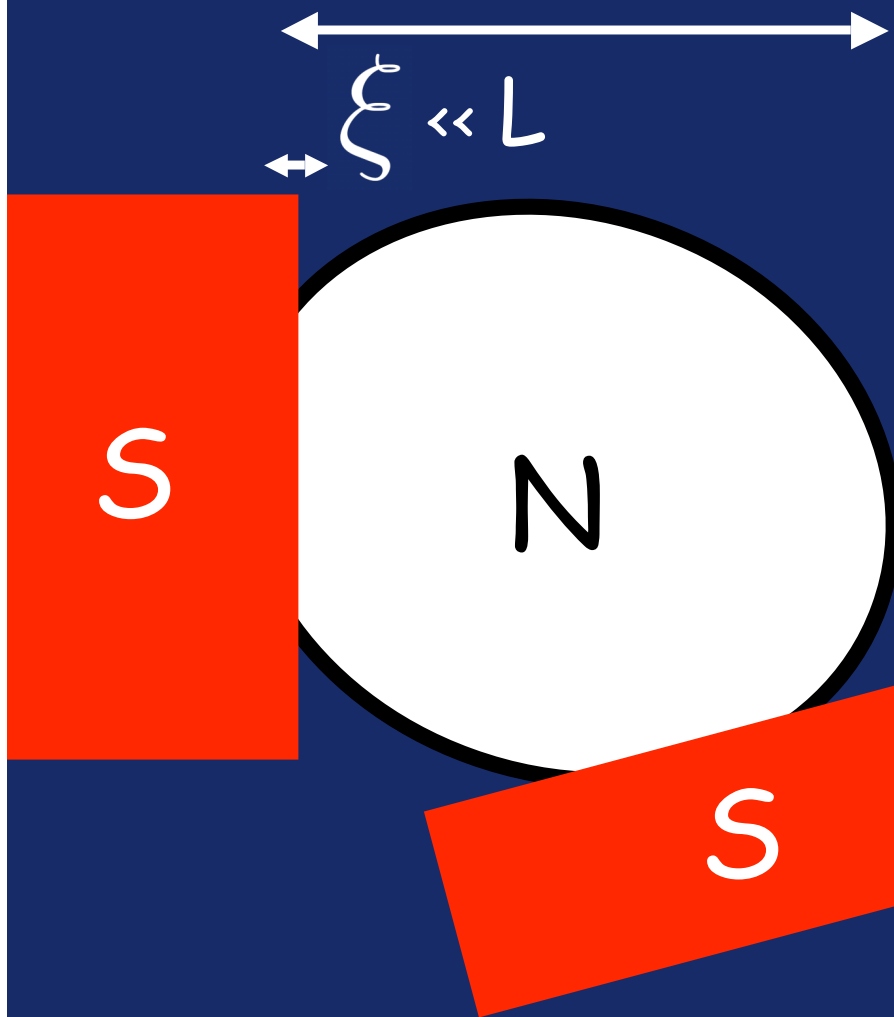


S invades N

But how ??



Mesoscopic Superconductivity



Effect of S in N depends on:

- (i) Electronic dynamics in N
- (ii) Symmetry of S state
(s- or d-wave; S phases...)
- (iii) τ_E/τ_D



Andreev reflection

$$\diamond (e, E_F + \varepsilon) \rightleftharpoons (h, E_F - \varepsilon)$$

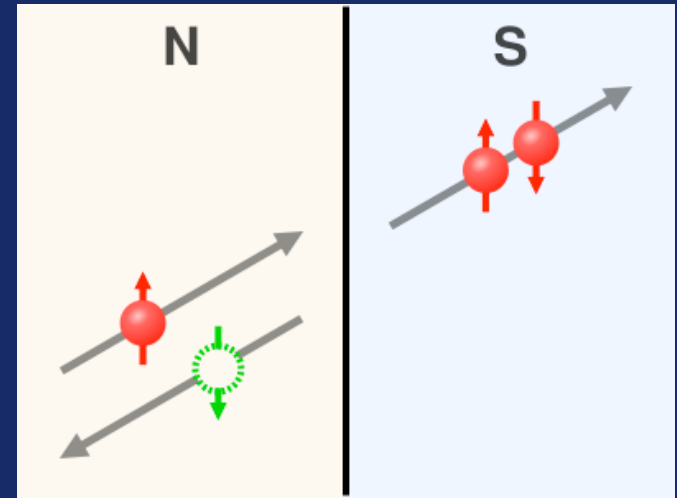
❖ Reflection phase :

$$\delta S_A = -\arccos[\varepsilon/\Delta] \mp \phi \simeq -\pi/2 \mp \phi$$

❖ Angle mismatch : Snell's law

$$k_+ \sin \theta_+ = k_- \sin \theta_-$$

$$\theta_- - \theta_+ \approx \frac{\varepsilon}{E_F} \tan \theta_+$$



(fig taken from Wikipedia)

S phase
 + : h → e
 - : e → h

Outline

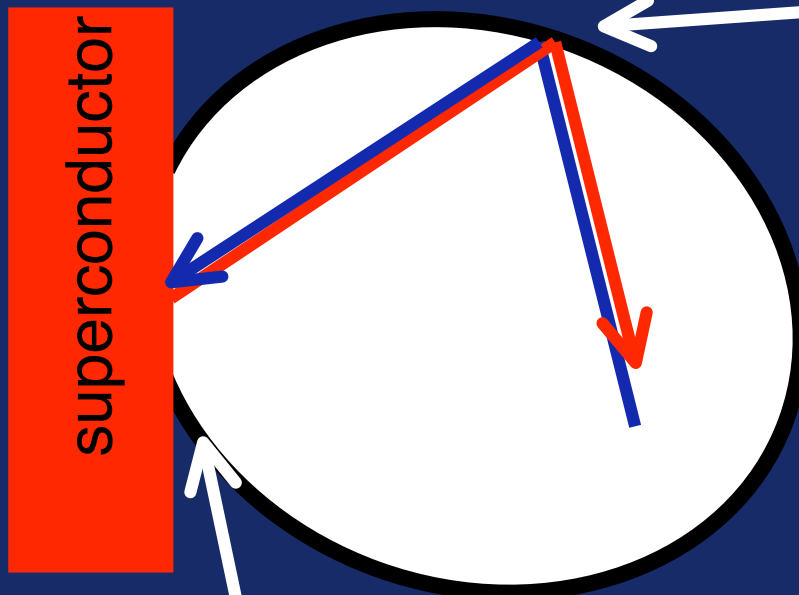
- Mesoscopic superconductivity - Andreev reflection
- Density of states in ballistic Andreev billiards
- Transport through ballistic Andreev interferometers
- Symmetries of charge transport in presence of superconductivity

PJ, H. Schomerus, and C. Beenakker, PRL '03

M. Goorden, PJ, and C. Beenakker, PRB '03; PRB '05



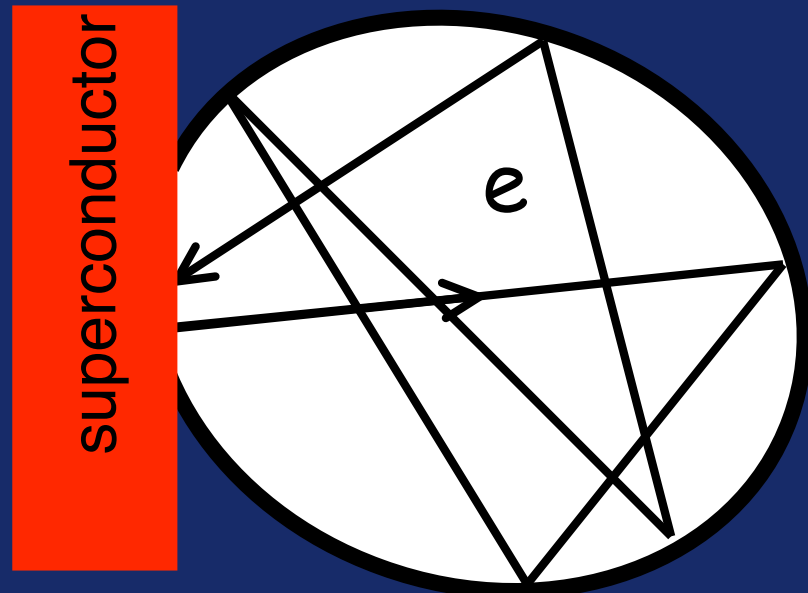
Andreev billiards: classical dynamics



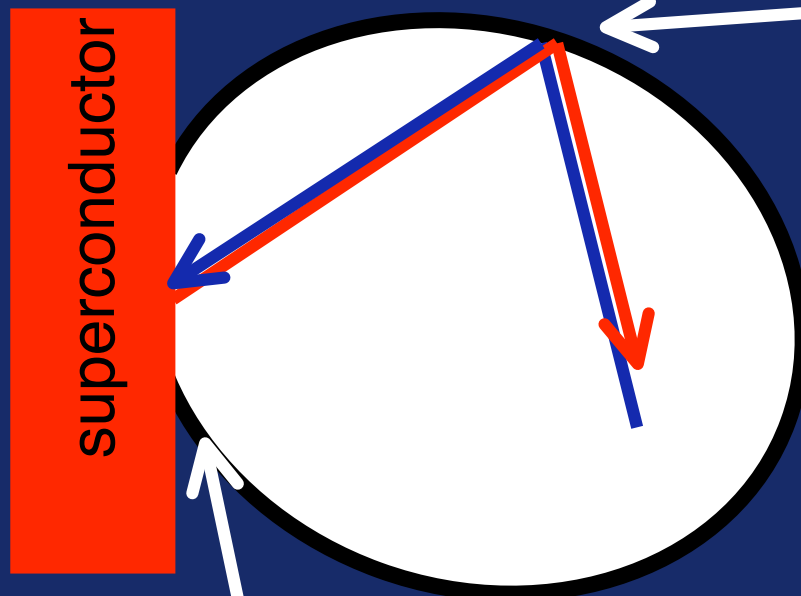
At NI interface:
Normal reflection

*Note #1: Billiard is chaotic
⇒ all trajectories
become periodic!*

At NS interface:
Andreev reflection



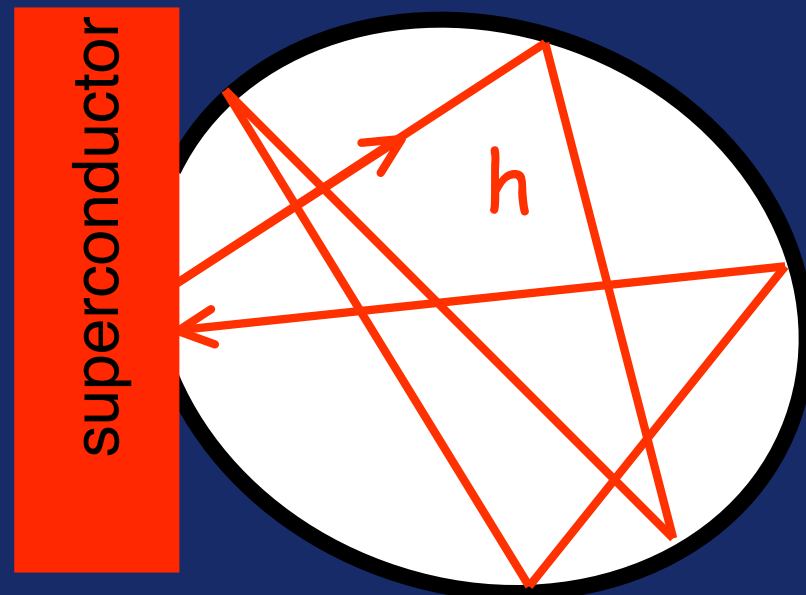
Andreev billiards: classical dynamics



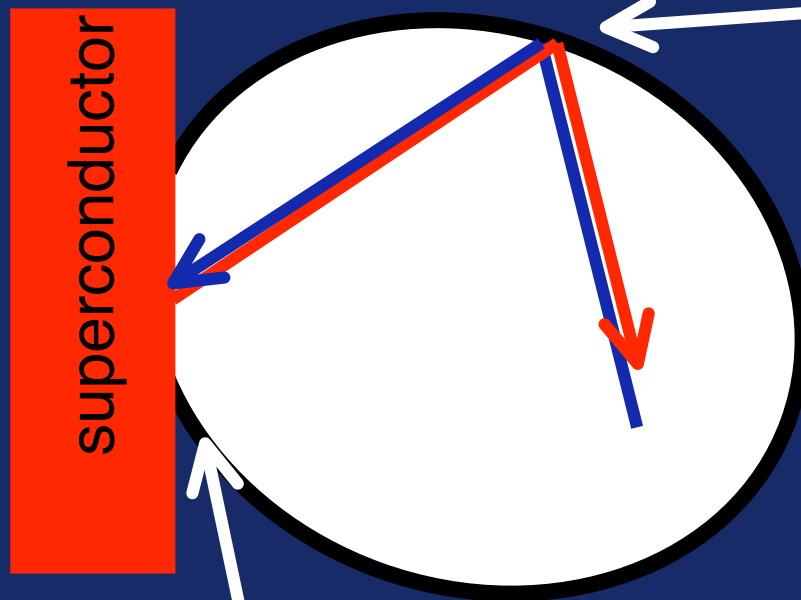
At NI interface:
Normal reflection

*Note #1: Billiard is chaotic
⇒ all trajectories
become periodic!*

At NS interface:
Andreev reflection



Andreev billiards: classical dynamics



superconductor

At NI interface:
Normal reflection

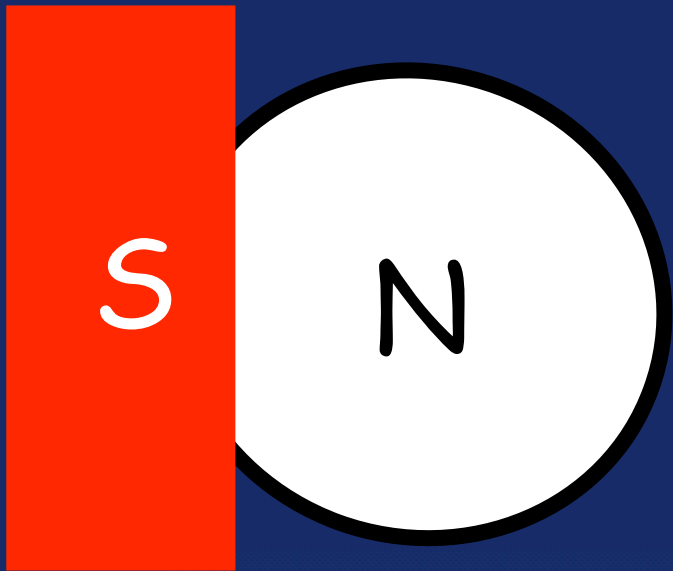
Note #2: Action on P.O.

$$\delta S_\gamma = 2\varepsilon T_\gamma - \pi\hbar$$

At NS interface:
Andreev reflection

Andreev reflection phase

Andreev billiards: semiclassical quantization



All orbits are periodic
→ Bohr-Sommerfeld

Andreev reflection phase

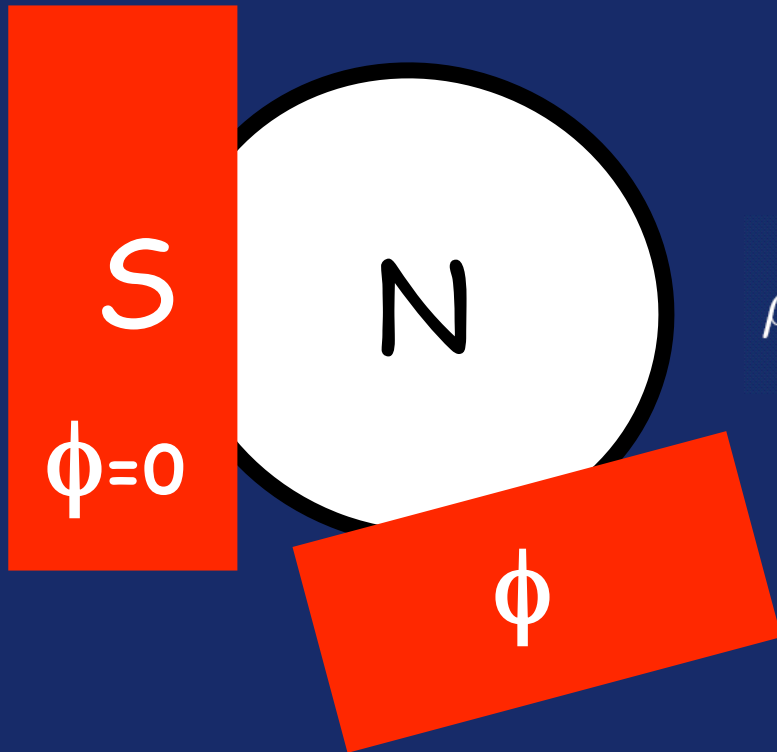
$$\rho_{\text{BS}}(E) = N_S \int_0^\infty dT P(T) \times 2 \sum_n \delta \left(E - \left[n + \frac{1}{2} \right] \frac{\pi \hbar}{T} \right)$$

Distribution of return times to S
chaos → exp. Suppression at $E=0$
regular → algebraic / others



See also: Melsen et al. '96; Ihra et al. '01; Zaitsev '06

Andreev billiards: semiclassical quantization



All orbits are periodic
 -> Bohr-Sommerfeld

$$\rho_{\text{BS}}(E) = N_S \int_0^\infty dT P(T)$$

$$\times \sum_n \left[\delta \left(E - \left[n + \frac{1}{2} \right] \frac{\pi \hbar}{T} \right) + \frac{1}{2} \delta \left(E - \left[n + \frac{1}{2} + \frac{\phi}{2\pi} \right] \frac{\pi \hbar}{T} \right) + \frac{1}{2} \delta \left(E - \left[n + \frac{1}{2} - \frac{\phi}{2\pi} \right] \frac{\pi \hbar}{T} \right) \right]$$

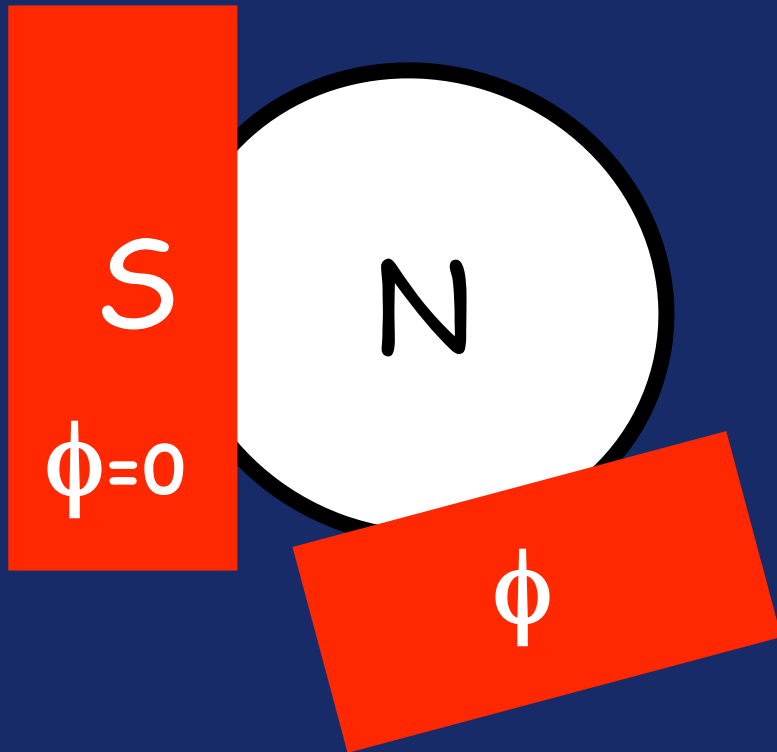
$|\phi| = \pi$: DoS has peak at $E=0$!!

All traj's touching both contribute to $n=0$ term

Andreev billiards: semiclassical quantization

Bohr-Sommerfeld for "chaotic" systems

$$\rho_{\text{BS}}(u) = \frac{2\pi}{\delta} \frac{\cosh\left(\frac{\phi}{2u}\right)}{\sinh\left(\frac{\pi}{u}\right)u^2} \left(\pi \cosh\left(\frac{\phi}{2u}\right) \coth\left(\frac{\pi}{u}\right) - \phi \sinh\left(\frac{\phi}{2u}\right) \right)$$



$$\lim_{u \rightarrow 0} \lim_{\phi \rightarrow \pi} \rho_{\text{BS}}(u) = \frac{2\pi}{\delta} \delta(u)$$

$$u = E/E_T$$

Andreev billiards: random matrix theory

$N = M \times M$ RMT Hamiltonians

$S \rightarrow$ particle-converting projectors

$$\mathcal{H} = \begin{pmatrix} H & -\pi W W^T \\ -\pi W W^T & -H^* \end{pmatrix}$$

$$W_{mn} = \delta_{mn} \left(\frac{M\delta}{\pi^2} \right)^{1/2}$$

$$m = 1, 2, \dots, M, \quad n = 1, 2, \dots, 2N_S$$

CONSTANT DOS EXCEPT:

\Rightarrow hard gap at $0.6 E_T$ for $\phi=0$

\Rightarrow linear "gap" of size δ for $\phi = \pi$

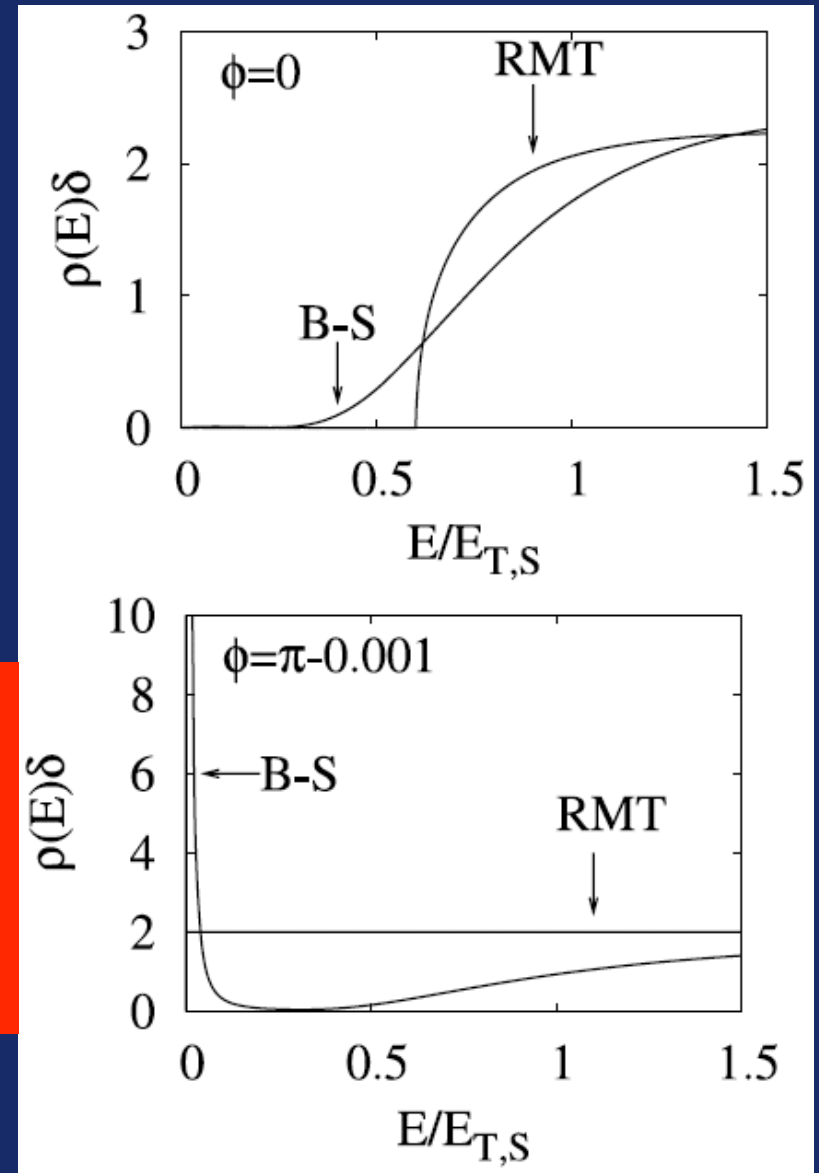
(class C1 with DoS: $\rho(E) = \frac{\pi}{\delta} \int_0^{2\pi E/\delta} dt J_0(t) J_1(t)/t$)

Melsen et al. '96, '97; Altland+Zirnbauer '97

Andreev billiards: RMT vs. B-Sommerfeld

At $\phi=0$: the "gap problem"
?: which theory is right ?
?: which theory is wrong ?

At $\phi=\pi$: macroscopic peak
(semiclassics) vs. minigap (RMT)
?: which theory is right ?
?: which theory is wrong ?



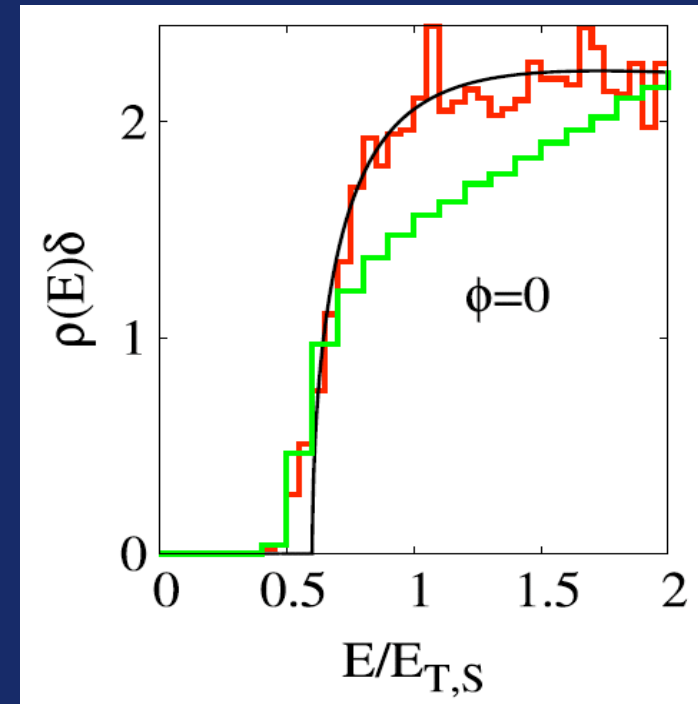
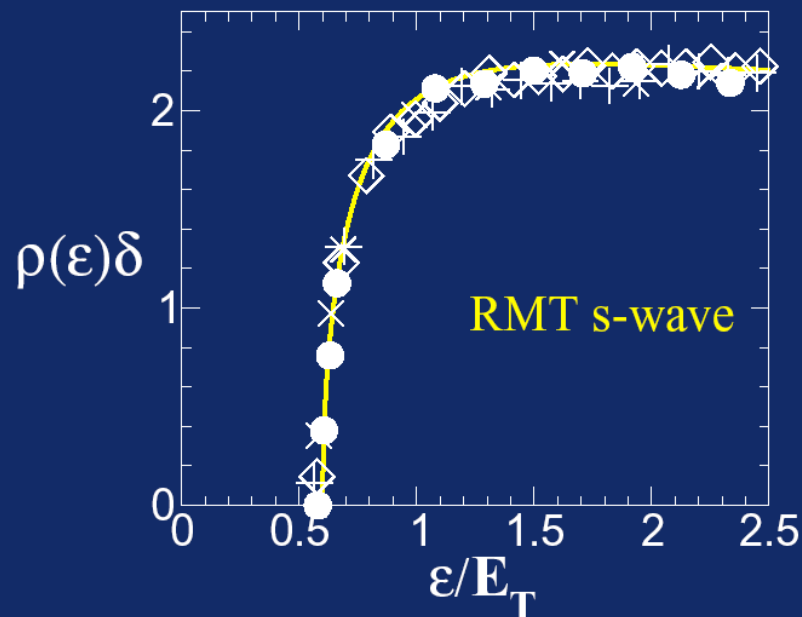
Andreev billiards - Solution to the "gap problem"

$$\tau_E \ll \tau_D$$

Universal, RMT regime

$$\tau_E \gtrsim \tau_D$$

Deep semiclassical regime



Note: numerics on "Andreev kicked rotator", PJ Schomerus and Beenakker '03
See also: Lodder and Nazarov '98; Adagideli and Beenakker '02; Vavilov and Larkin '03

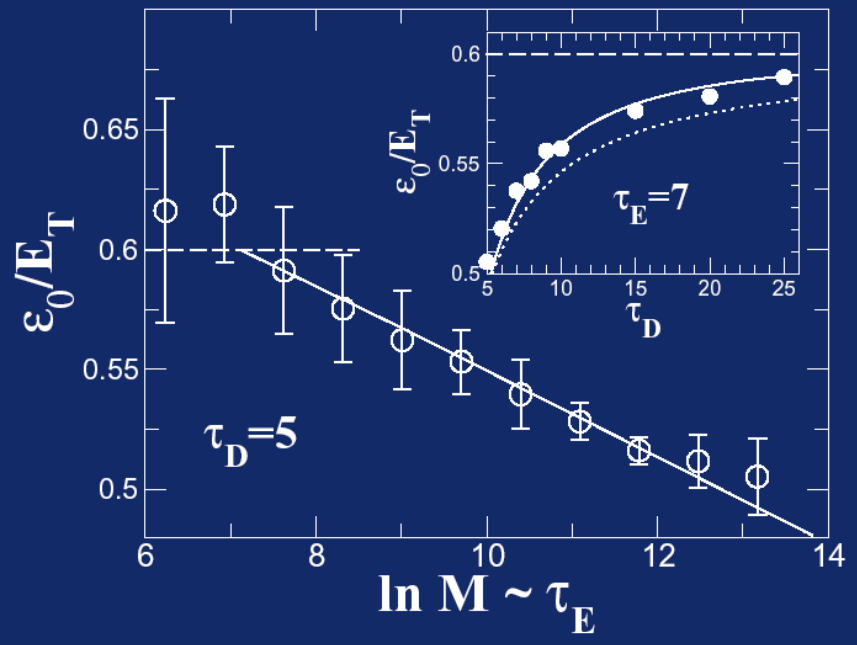
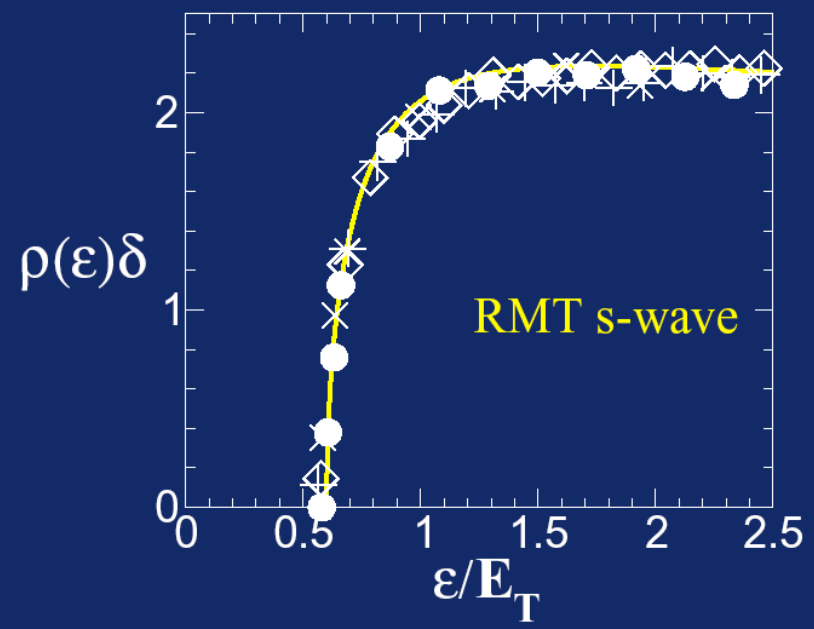
Andreev billiards - Solution to the "gap problem"

$$\tau_E \ll \tau_D$$

Universal, RMT regime:
Gap at Thouless energy

$$\tau_E \gtrsim \tau_D$$

Deep semiclassical regime:
Gap at Ehrenfest energy

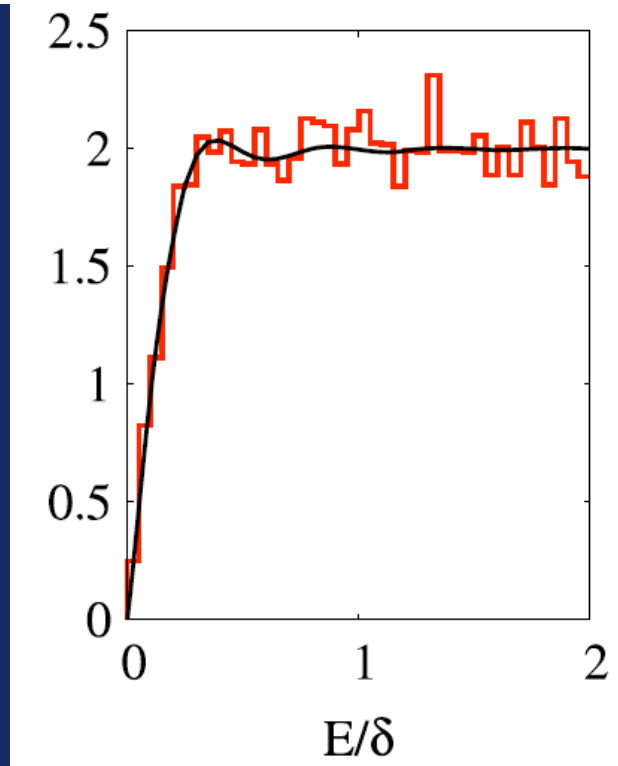


Note: numerics on "Andreev kicked rotator", PJ Schomerus and Beenakker '03
See also: Lodder and Nazarov '98; Adagideli and Beenakker '02; Vavilov and Larkin '03

Andreev billiards: DoS at $\phi=\pi$

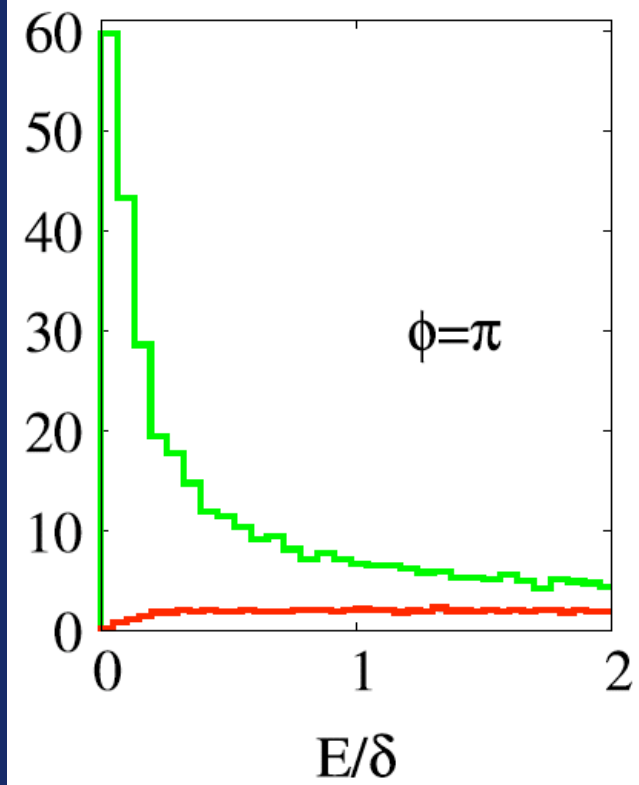
$$\tau_E \ll \tau_D$$

Universal, RMT regime:
Minigap at level spacing



$$\tau_E \gtrsim \tau_D$$

Deep semiclassical regime:
Large peak around $E=0$!



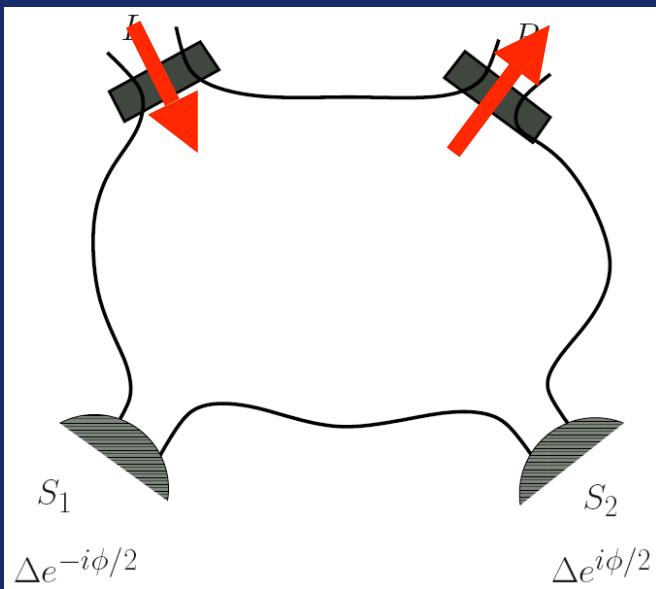
Outline

- Mesoscopic superconductivity - Andreev reflection
- Density of states in ballistic Andreev billiards
- Transport through ballistic Andreev interferometers
- Symmetries of charge transport in presence of superconductivity

**M. Goorden, PJ, and J. Weiss,
PRL '08, Nanotechnology '08**



Transport through Andreev interferometers



Lambert '93 formula

$$G/G_0 = T_{RL}^{ee} + T_{RL}^{he} + 2 \frac{T_{LL}^{he} T_{RR}^{he} - T_{LR}^{he} T_{RL}^{he}}{T_{LL}^{he} + T_{RR}^{he} + T_{LR}^{he} + T_{RL}^{he}}$$

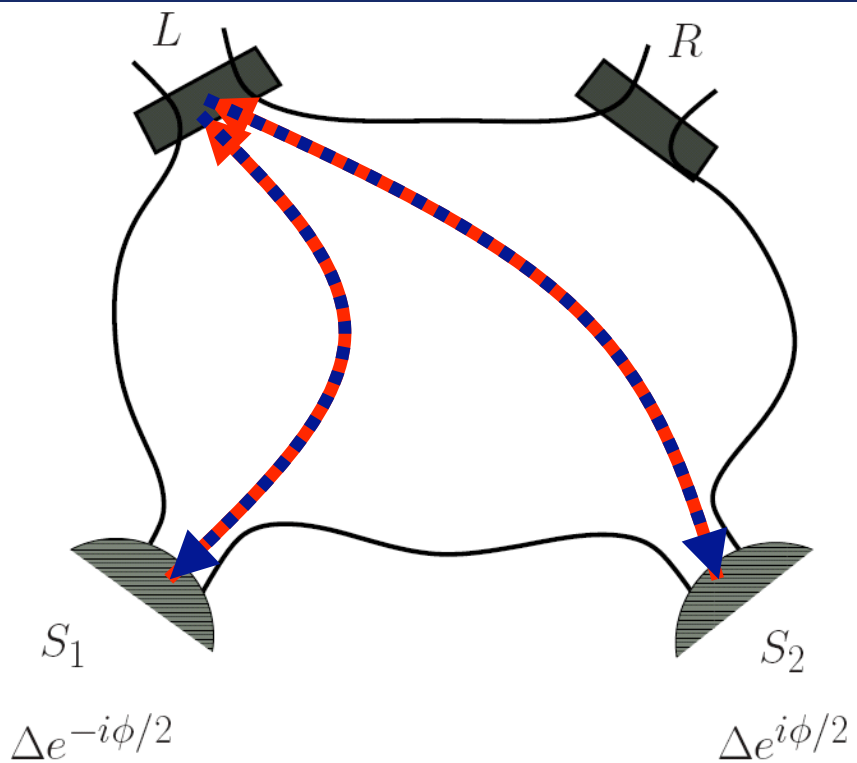
Average conductance for $N_L = N_R$

$$\langle G \rangle / G_0 = \langle T_{RL}^{ee} \rangle + \langle T_{LL}^{he} \rangle$$

- ❖ New, Andreev reflection term
- ❖ Gives classically large interference contributions



Transport through Andreev interferometers



At $\varepsilon=0$, any pair of Andreev reflected trajectories contributes to T_{LL}^{he} in the sense of a SPA !

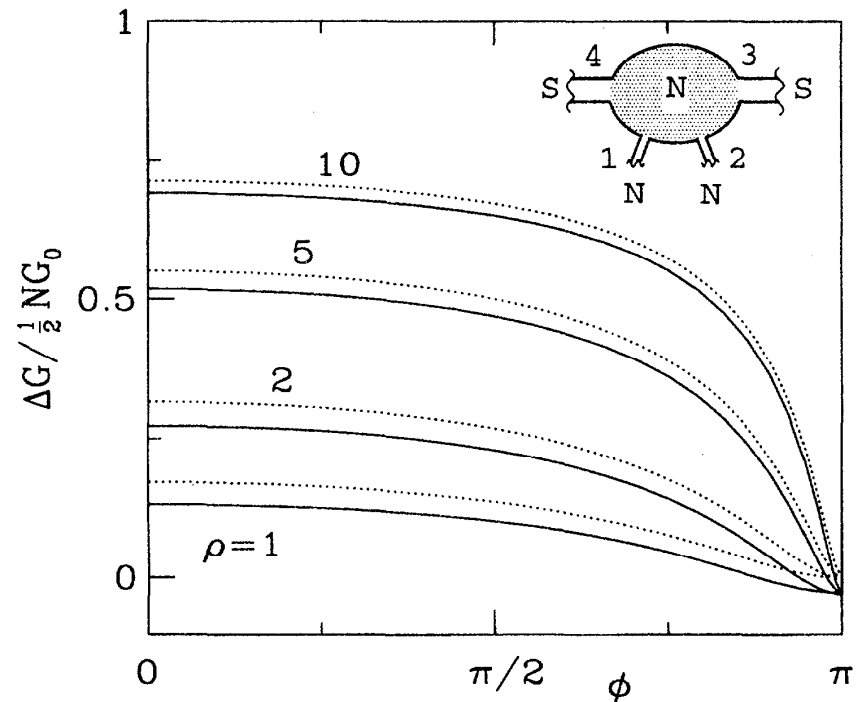
These pairs give classically large positive coherent backscattering at $\phi=0$, vanishing for $\phi=\pi$

$$\langle G \rangle / G_0 = \langle T_{RL}^{ee} \rangle + \langle T_{LL}^{he} \rangle$$

Transport through Andreev interferometers

No tunnel barrier :
Coherent backscattering is
- $O(N)$
-positive, increases G

This is (obviously) not
related to the DoS in
the Andreev billiard



**!! INTRODUCE TUNNEL BARRIERS
TUNNELING CONDUCTANCE \sim DOS !!**

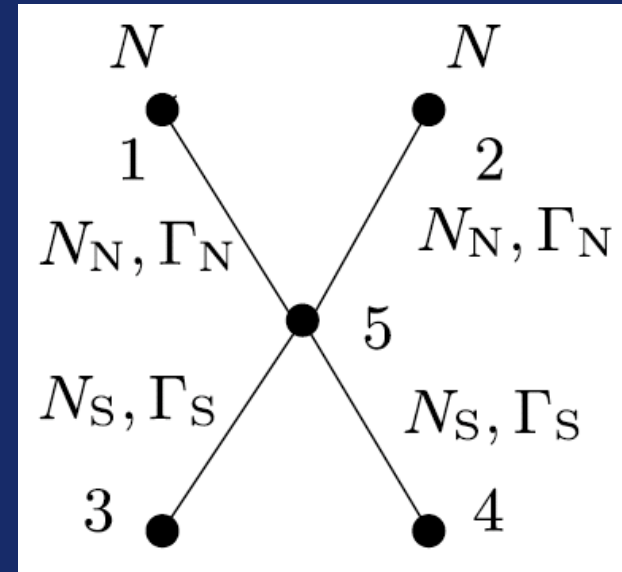
Beenakker, Melsen and Brouwer '95



Tunneling transport through Andreev interferometers

$$\langle G \rangle / G_0 = \langle T_{RL}^{ee} \rangle + \langle T_{LL}^{he} \rangle$$

Plan a) : extend circuit theory to tunneling



$$\langle G \rangle / G_0$$

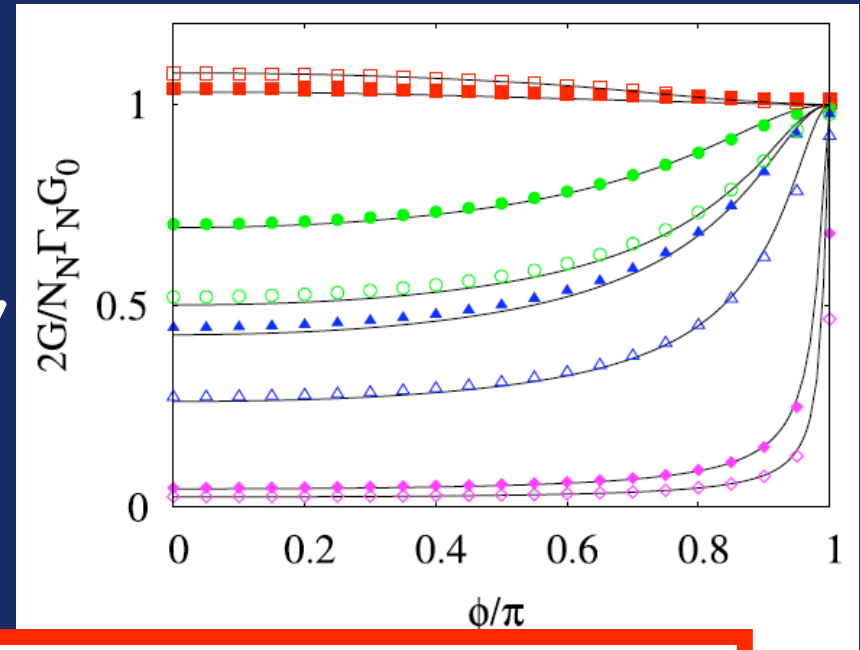
$$= \begin{cases} \Gamma_N^2 N_N \left(1 + \frac{N_N}{N_{S,\text{eff}}} \frac{1 + \cos \frac{\phi}{2}}{\cos \frac{\phi}{2}} \right) / 4, & \text{if } \delta\phi \gg \Gamma_N, \\ \Gamma_N N_N / 2 + \mathcal{O}(\delta\phi^2), & \text{if } \delta\phi \ll \Gamma_N. \end{cases}$$

Goorden, PJ and Weiss '08; inspired by : Nazarov '94; Argaman '97.

Tunneling transport through Andreev interferometers

$$\langle G \rangle / G_0 = \langle T_{RL}^{ee} \rangle + \langle T_{LL}^{he} \rangle$$

Plan a) : extend circuit theory to tunneling



$$\langle G \rangle / G_0$$

$$= \begin{cases} \Gamma_N^2 N_N \left(1 + \frac{N_N}{N_{S,\text{eff}}} \frac{1 + \cos \frac{\phi}{2}}{\cos \frac{\phi}{2}} \right) / 4, & \text{if } \delta\phi \gg \Gamma_N, \\ \Gamma_N N_N / 2 + \mathcal{O}(\delta\phi^2), & \text{if } \delta\phi \ll \Gamma_N. \end{cases}$$

Goorden, PJ and Weiss '08; inspired by : Nazarov '94; Argaman '97.

Tunneling transport through Andreev interferometers

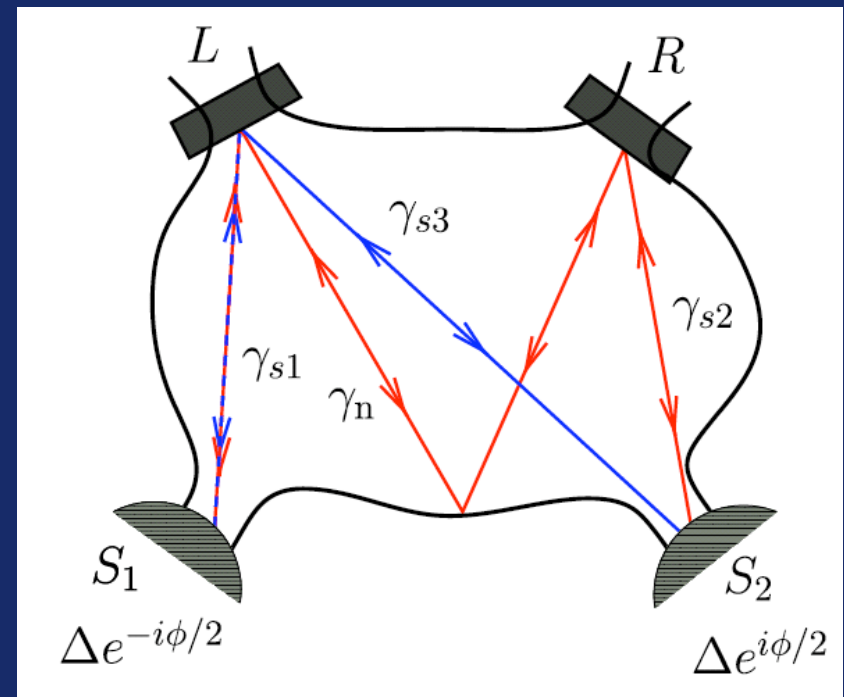
$$\langle G \rangle / G_0 = \langle T_{RL}^{ee} \rangle + \langle T_{LL}^{he} \rangle$$

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

— contribution to T_{RL}^{ee}
— contribution to T_{LL}^{he}

Why "macroscopic" ?
A: $O(N)$ effect !



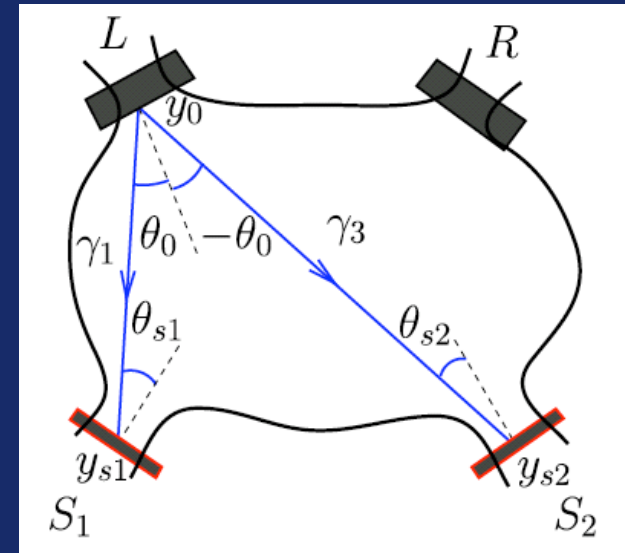
Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

Calculate transmission

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar} \int_i y_0 \int_j y'_0 \sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* \exp[i\delta S/\hbar]$$



on blue trajectories (i.e. for T_{LL}^{he})

$$\gamma_I^{(p)} = \underbrace{\gamma_{s1}^{(e)} + \gamma_{s1}^{(h)}}_{\text{"primitive traj."}} + p \times \underbrace{[\gamma_{s3}^{(h)} + \gamma_{s3}^{(e)} + \gamma_{s1}^{(e)} + \gamma_{s1}^{(h)}]}_{\text{"Andreev loop travelled p times"}}$$

"primitive traj." **"Andreev loop travelled p times"**

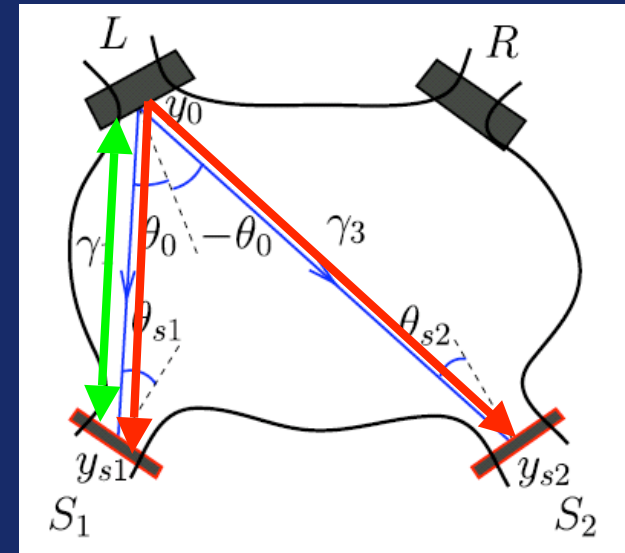
Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

Calculate transmission

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar} \int_i y_0 \int_j y'_0 \sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* \exp[i\delta S/\hbar]$$



on blue trajectories (i.e. for T_{LL}^{he})

$$\gamma_I^{(p)} = \underbrace{\gamma_{s1}^{(e)} + \gamma_{s1}^{(h)}}_{\text{primitive traj.}} + p \times \underbrace{[\gamma_{s3}^{(h)} + \gamma_{s3}^{(e)} + \gamma_{s1}^{(e)} + \gamma_{s1}^{(h)}]}_{\text{Andreev loop travelled p times}}$$

"primitive traj."

"Andreev loop travelled p times"

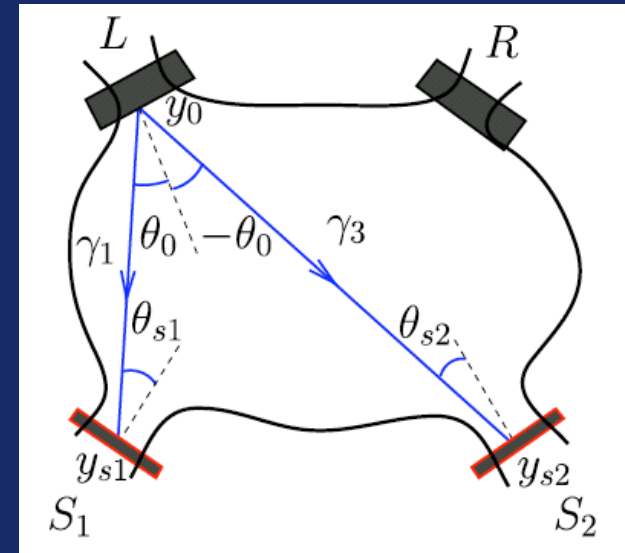
Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

Calculate transmission

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar} \int_i y_0 \int_j y'_0 \sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* \exp[i\delta S/\hbar]$$



on blue trajectories with action phase and stability

$$S_{\gamma, I} = p(-\pi + \phi + E t_{\ell, I}) + 2 E t_{\gamma_{s1}} - (\pi/2 - \phi/2)$$

$$A_{\gamma} = B_{\gamma} t_i t_j \prod_k [r_k]^{l_{\gamma}(k)}$$

Stability of trajectory

Sequence of transmissions and reflections at tunnel Barriers (Whitney '07)

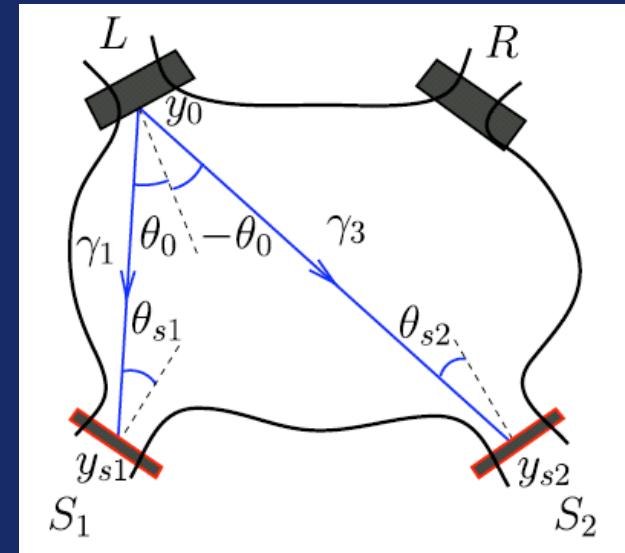
Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

Calculate transmission

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar} \int_i y_0 \int_j y'_0 \sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* \exp[i\delta S/\hbar]$$



One key observation :

Andreev reflections **refocus the dynamics**
for Andreev loops **shorter than Ehrenfest time**

➤ **Stability does not depend on p !**

➤ **Stability is determined only by**

$$\gamma_{s3}^{(e)} + \gamma_{s1}^{(e)}$$

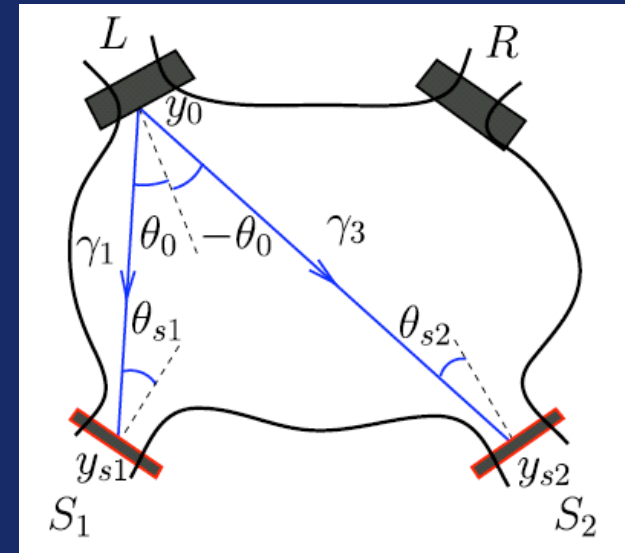
Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

Calculate transmission

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar} \int_i y_0 \int_j y'_0 \sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* \exp[i\delta S/\hbar]$$



-> Pair all traj. (w. different p's) on $\gamma_1 + \gamma_3$

-> Substitute

$$\sum_{\gamma_1, \gamma_2} A_{\gamma_1} A_{\gamma_2}^* [\dots]_{\gamma_1, \gamma_2} \longrightarrow \Gamma_N^2 \sum_{\gamma=\text{primitive}} B_\gamma^2 \sum_{p, p'=0}^{\infty} (1 - \Gamma_N)^{a(p+p')+b} \Gamma_S^{p+p'+c} [\dots]_{\gamma, p, p'}$$

Determine B_γ as for normal transport
 ~classical transmission probabilities

Tunneling transport through Andreev interferometers

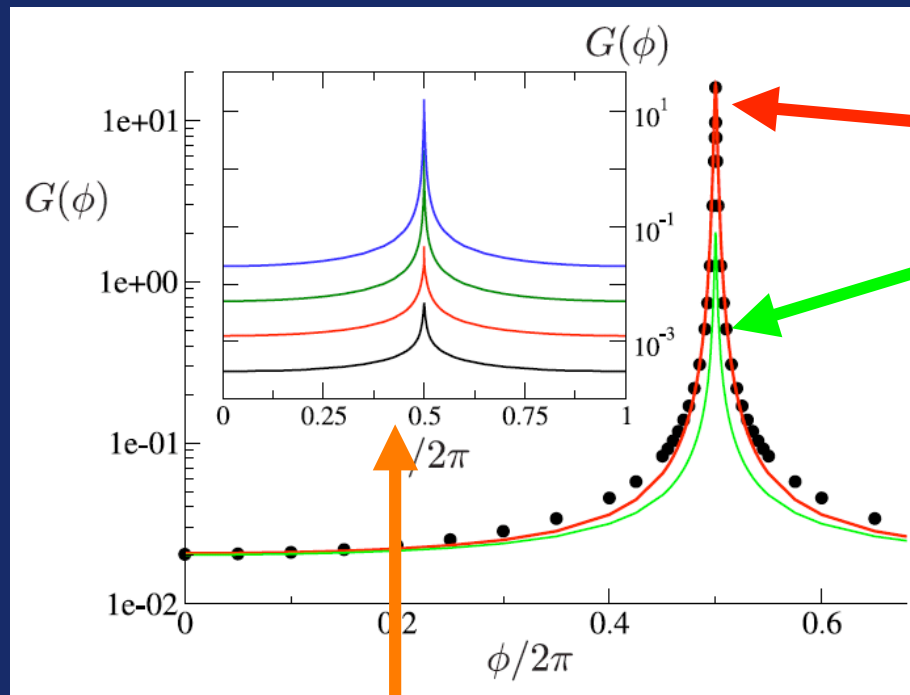
Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

$$\begin{aligned}
 \langle T_{LL}^{he} \rangle_r &= \frac{\pi \Gamma_N^2 N_N}{4} \left(\frac{N_S}{2\Gamma_N N_N + 2\Gamma_S N_S} \right)^2 \\
 &\times \left(1 - (1 + \tau_E/\tau_{D,S}) \exp[-\tau_E/\tau_{D,S}] \right) && \text{Measure of trajs.} \\
 &\times \frac{\Gamma_S}{1 - 2\Gamma_S (1 - \Gamma_N) \cos[\pi - \phi] + \Gamma_S^2 (1 - \Gamma_N)^2} && \text{Resonant tunneling} \\
 \\
 \langle T_{RL}^{ee} \rangle_r &= \frac{\pi^2 \Gamma_N^2 N_N^2}{8N_S} \left(\frac{N_S}{2\Gamma_N N_N + 2\Gamma_S N_S} \right)^3 \\
 &\times \left(1 - (1 + \tau_E/\tau_{D,S} + \tau_E^2/2\tau_{D,S}^2) \exp[-\tau_E/\tau_{D,S}] \right) && \text{Measure of trajs.} \\
 &\times \frac{1 + \Gamma_S^2 (1 - \Gamma_N)^2}{1 - 2\Gamma_S (1 - \Gamma_N)^2 \cos[\pi - \phi] + \Gamma_S^2 (1 - \Gamma_N)^4} && \text{Resonant tunneling}
 \end{aligned}$$

Tunneling transport through Andreev interferometers

Plan c) : numerics

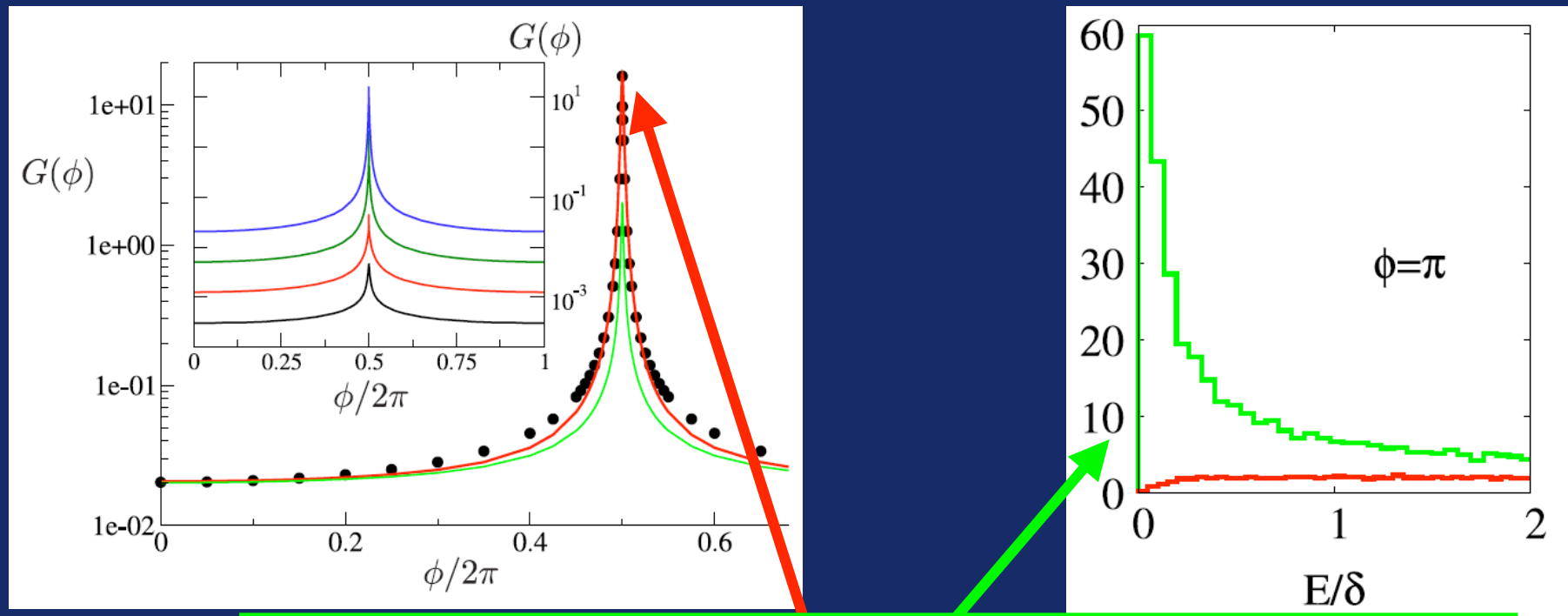


Order of magnitude enhancement from universal (green) to MRT (red)

Effect increases as $k_F L$ increases
Peak-to-valley ratio goes from Γ to Γ^2

Tunneling transport through Andreev interferometers

Plan c) : numerics



Tunneling through $\sim 10-15$ levels
i.e. half of those in the peak in the DoS
"TUNNELING THROUGH LEVELS AT $\varepsilon=0$ "

Outline

- Mesoscopic superconductivity - Andreev reflection
- Density of states in ballistic Andreev billiards
- Transport through ballistic Andreev interferometers
- Symmetries of charge transport in presence of superconductivity

J. Weiss and PJ, in progress



Symmetry of multi-terminal transport

NORMAL METAL:

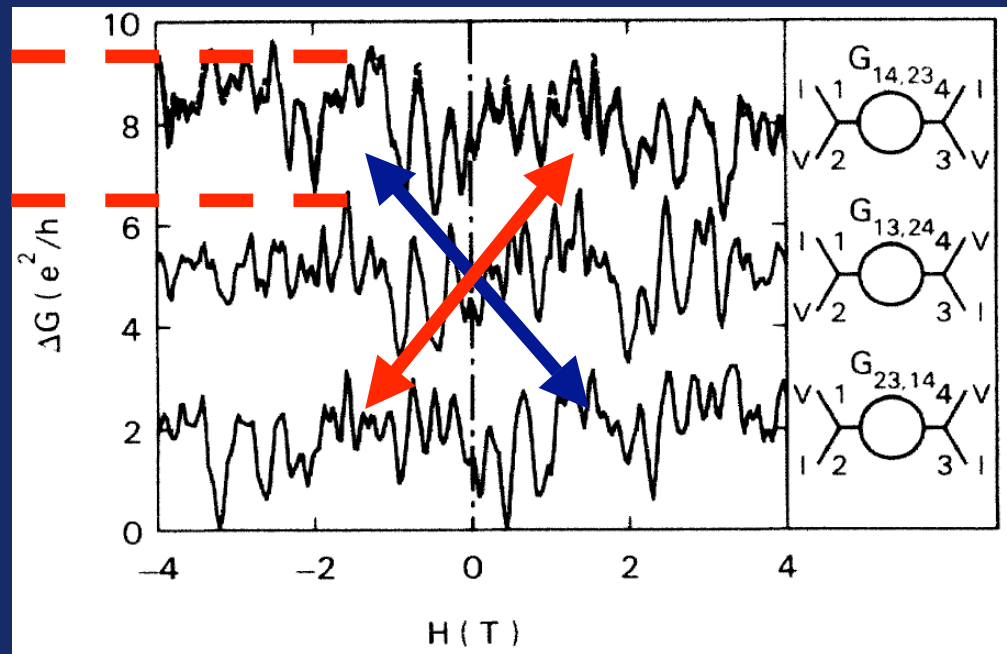
Two-terminal measurement

Four-terminal measurement

$$G(H) = G(-H)$$

$$G_{ij;kl}(H) = G_{kl;ij}(-H)$$

$O(e^2/h)$



Onsager, Casimir...
Buttiker '86
Benoit et al '86

Phase Dependent Thermopower in Andreev Interferometers

Jonghwa Eom, Chen-Jung Chien, and Venkat Chandrasekhar

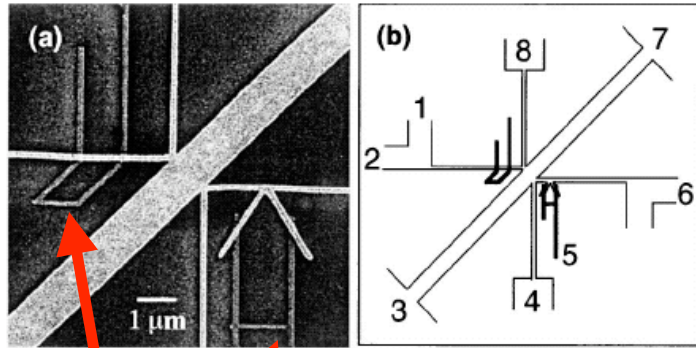
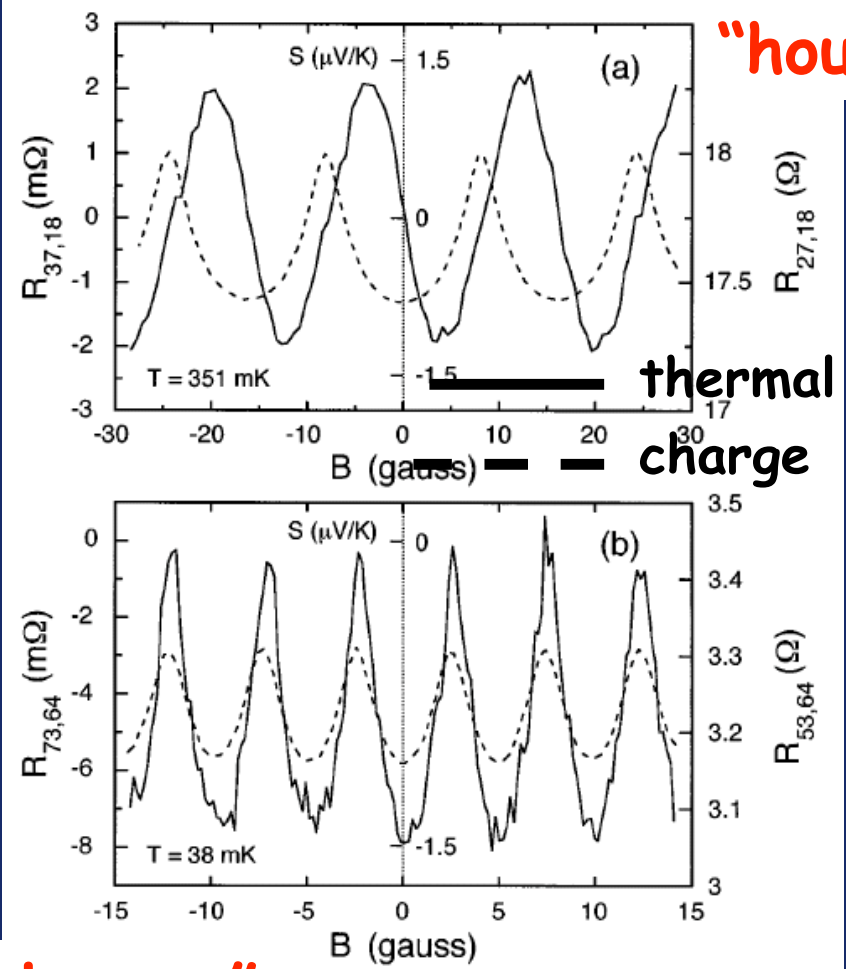


FIG. 1. (a) Scanning electron micrograph of a typical sample. (b) Schematic of the sample structure. Fat solid lines represent Al wires, while thin solid lines outline the Au film. The two hybrid loops are labeled as “parallelogram” (left) and “house” (right). The contacts are labeled with Arabic numerals.

S



“house”

thermal
charge

“parallelogram”

Symmetry of multi-terminal transport with superconductivity

VOLUME 76, NUMBER 24

PHYSICAL REVIEW LETTERS

10 JUNE 1996

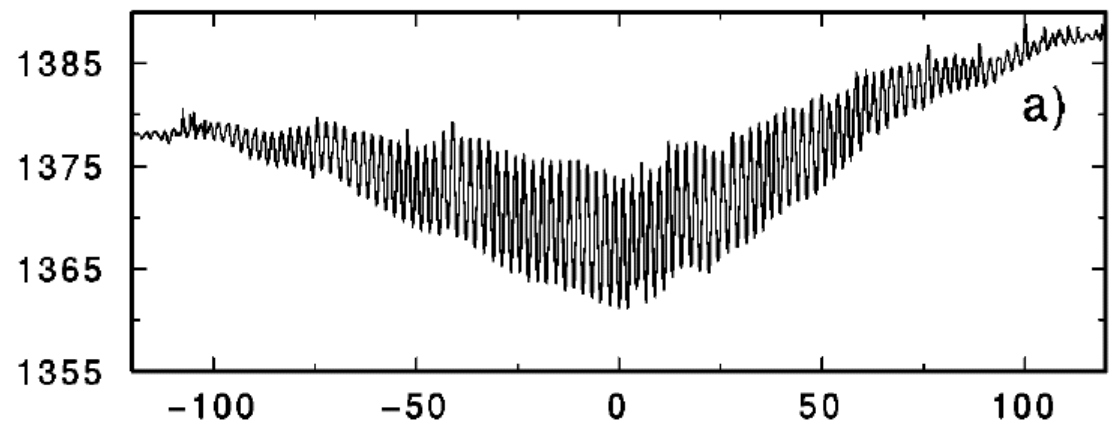
Sample-Specific Conductance Fluctuations Modulated by the Superconducting Phase

S. G. den Hartog, C. M. A. Kapteyn, B. J. van Wees, and T. M. Klapwijk

*Department of Applied Physics and Materials Science Centre, University of Groningen, Nijenborgh 4,
9747 AG Groningen, The Netherlands*

W. van der Graaf and G. Borghs

*Interuniversity Micro Electronics Centre, Kapeldreef 75, B-3030 Leuven, Belgium
(Received 20 October 1995)*

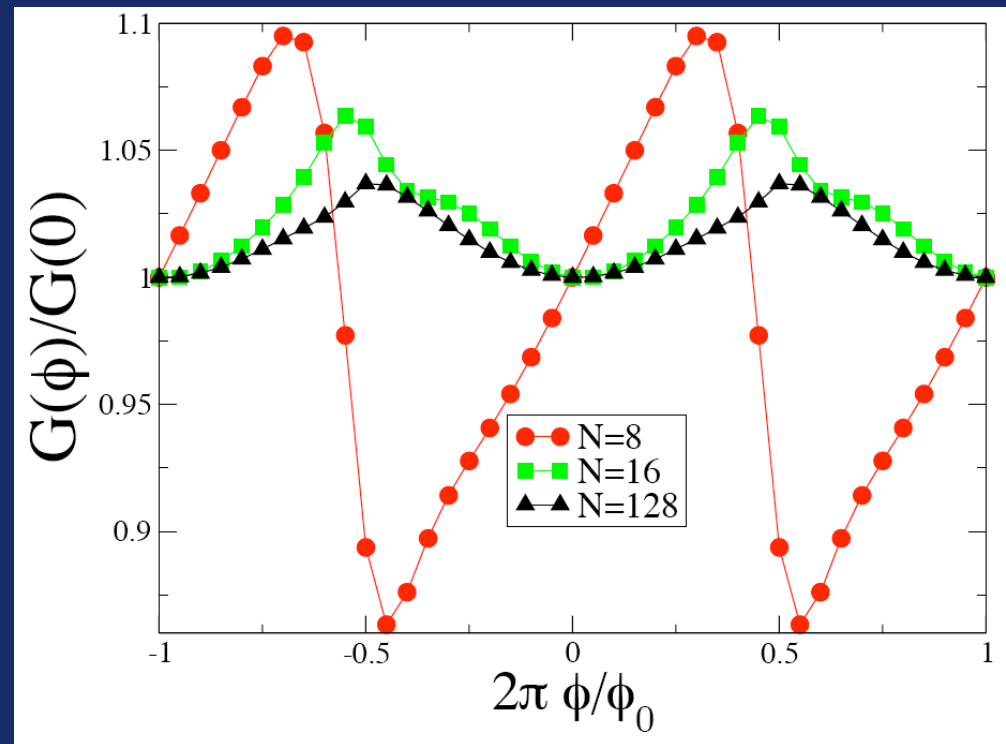


Symmetry of multi-terminal transport with superconductivity

Numerics :

- ❖ No particular symmetry
- ❖ AB-Amplitude is $O(N)$

➤ G looks more and more symmetric as N grows



Exps.: $\langle G \rangle = 1500 / 7700$
 $\delta G = 60 / 300$

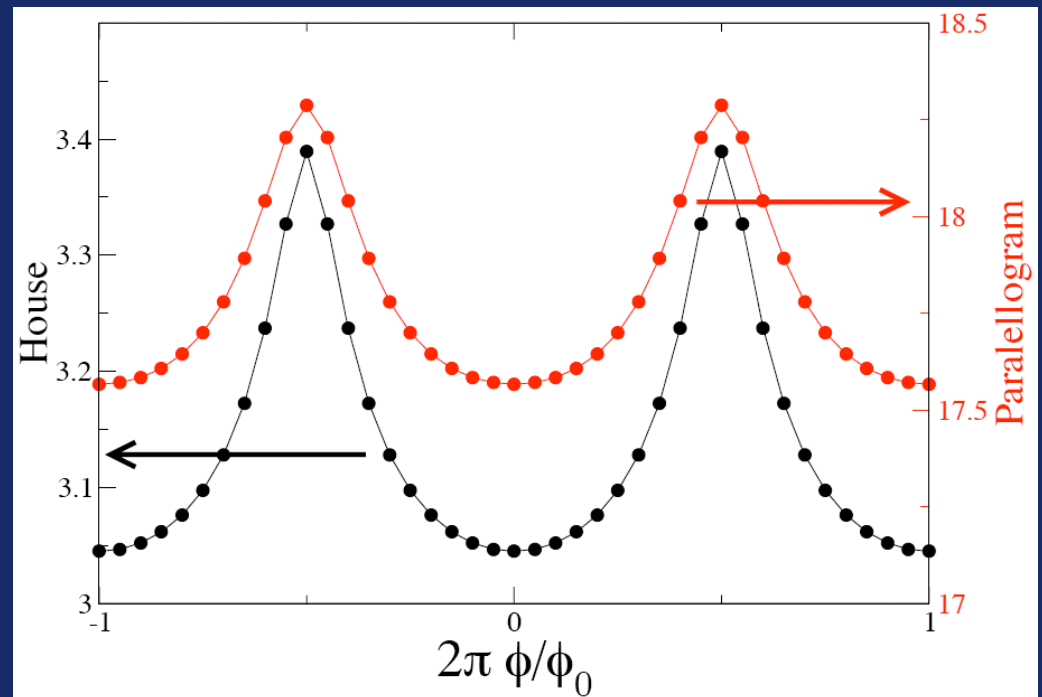
Unreachable numerically - use circuit theory!



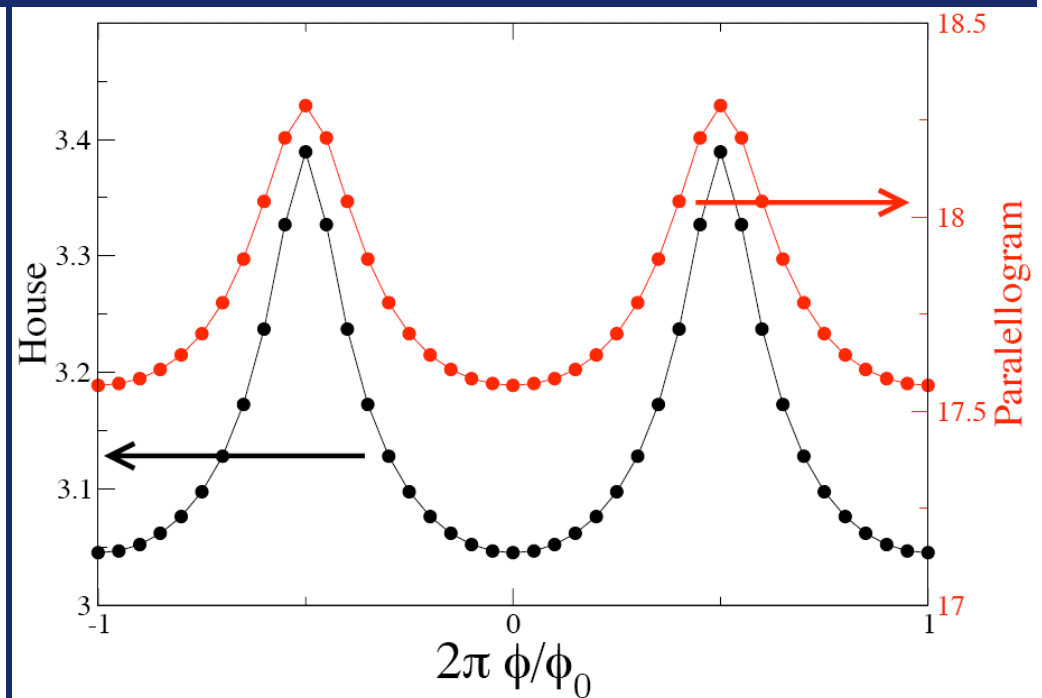
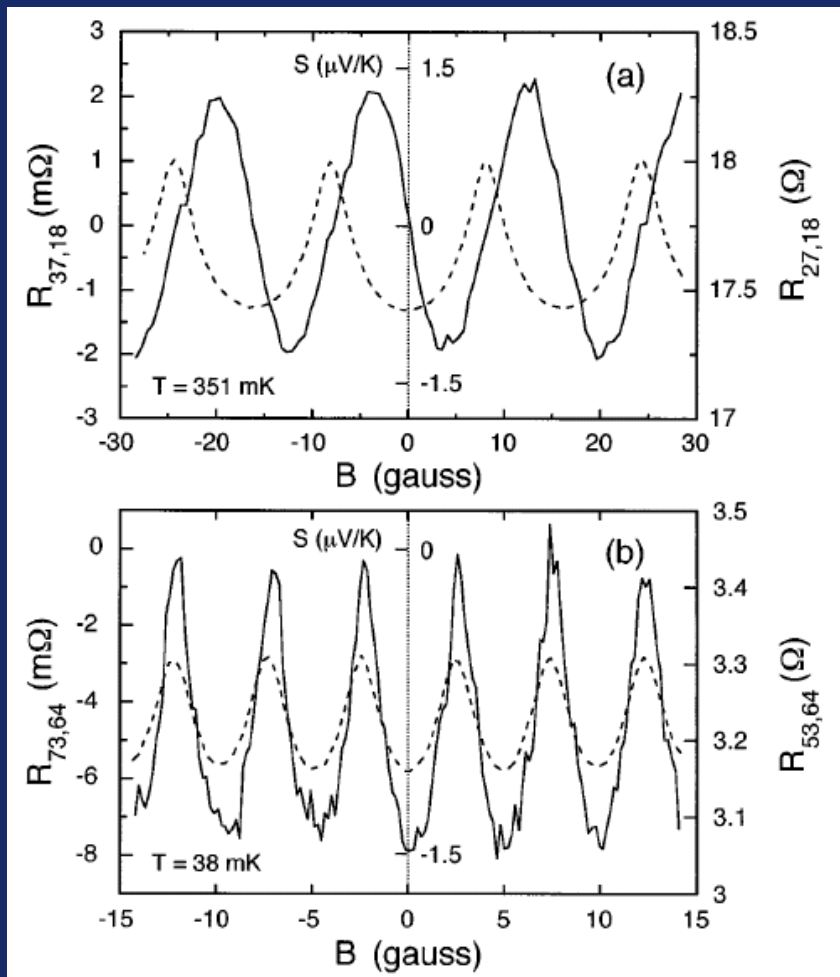
Symmetry of multi-terminal transport with superconductivity

Nazarov's circuit theory:

- ❖ Valid for $N \gg 1$
- ❖ Neglects "weak loc" effects
- symmetric 4-terminal "charge" conductance
- AB oscillations $O(N)$
- Minimum at $\phi=0$
- Ratio $\delta R / \langle R \rangle$ is in good agreement with exps

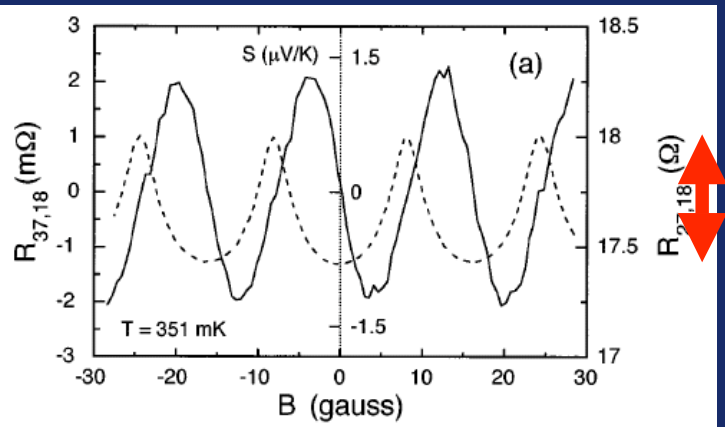


Symmetry of multi-terminal transport with superconductivity

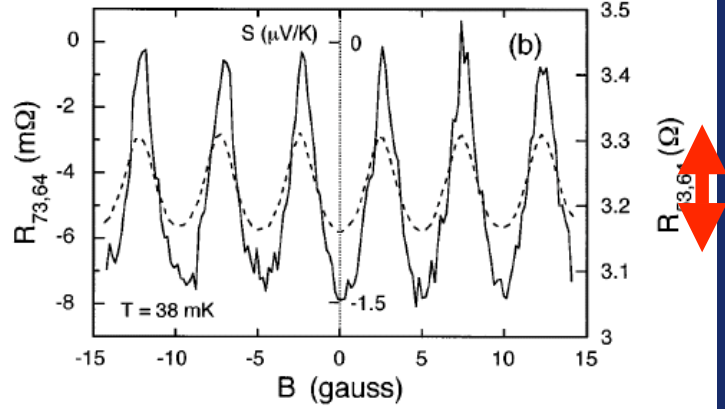


Nazarov '94; Argaman '97.



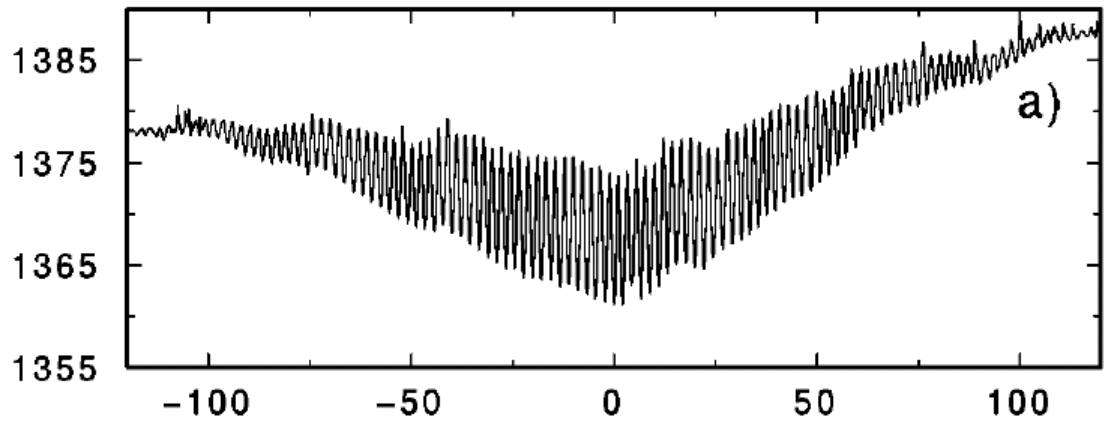


$\langle G \rangle = 1600$
 $dG = 70$



$\langle G \rangle = 7700$
 $dG = 300$

$\langle G \rangle = 18$
 $dG < 1$



Future perspectives

- Proximity effect with exotic superconductivity

