HOT HADRONIC MATTER AND NUCLEAR COLLISIONS ☆

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Based on the statistical bootstrap model of strong interactions, we develop a description of hadronic matter with particular emphasis on hot nuclear matter as created in relativistic heavy ion collisions. We apply our theory to calculate temperatures and average transverse momenta of nucleons and pions from the decay of hadronic fireballs.

We propose a description of hot hadronic matter based on the statistical bootstrap model [1]. Our aim is to describe the gross properties of particle spectra emitted by heated and/or compressed nuclear matter. At present, the data relevant to our investigations begin to be available from experiments involving relativistic heavy ion collisions, triggered (by high multiplicities) for central collisions only [2]. In this paper we will focus on temperatures and average transverse momenta of nucleons and pions produced in such collisions.

While it is quite difficult to master the complexity of the strong interactions in detail, it has been shown [3] that hadronic resonance production dominates the interaction in all hadronic reactions. The central rôle is then assumed by the hadrnoic mass spectrum $\tau(m^2, b)$, which here, in addition to the usual dependence on the mass m of the resonance, is also a function of the conserved baryonic number b. $\tau(m^2, b)\, \mathrm{d}(m^2)$ is the number of bound states (resonances) in the mass interval $\mathrm{d}(m^2)$ at a given baryon number b.

Unlike the case of high energetic proton–proton scattering, the correct treatment à la Van der Waals of the volume occupied by each participating hadron is essential. We have found [1] that the "natural volume V_c" [‡1] of a hadronic fireball cluster grows proportional to its mass. As a consequence we see at the hadronic phase boundary a constant maximal energy density which is a free parameter of the model. The newly reformulated [1] bootstrap model provides us with the required hadronic mass spectrum $\tau(p^2, b)$ and with the particular energy dependence of the fireball volume necessary in the thermodynamic approach.

The description of the thermodynamic properties of hot hadronic matter begins with the grand partition function $Z(\beta, V, \lambda)$, as obtained from the level density $\sigma(p, V, b)$:

$$Z(\beta, V, \lambda) = \sum_{b=-\infty}^{\infty} \lambda^b \int \mathrm{e}^{-\beta\cdot p} \sigma(p, V, b)\, \mathrm{d}^4 p\,. \quad (1)$$

We use the covariant generalisation [4] of thermodynamics with the inverse temperature four-vector β_μ and the four-volume V_μ. In the rest frame of the system we have $\beta\cdot p = \beta_0 E = E/T$ ($\hbar = c = k = 1$) and $V_\mu = (V, 0)$. λ is the fugacity, related in the rest frame to the relativistic chemical potential μ: $\lambda = \exp(\mu/T)$;

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‡1 The "natural volume" is assumed when external forces are absent.

it is introduced in order to conserve the baryon number in the statistical ensemble. All quantities of physical interest can be derived as usual by differentiating ln Z with respect to its variables.

Assuming that the mass spectrum $\tau(m^2, b)$ is already known, the grand microcanonical level density is given by the invariant phase space integral [‡2]:

$$\sigma(p, V_{\text{ex}}, b) = \delta^4(p)\delta_{\text{K}}(b) + \sum_{N=1}^{\infty} \frac{1}{N!} \int \delta^4\left(p - \sum_{i=1}^{N} p_i\right) \sum_{\{b_i\}} \delta_{\text{K}}\left(b - \sum_{i=1}^{N} b_i\right) \times \prod_{i=1}^{N} \frac{2\Delta_\mu p_i^\mu}{(2\pi)^3} \tau(p_i^2, b_i)\, \mathrm{d}^4 p_i \,. \quad (2a)$$

Above, the first term corresponds to the vacuum state. The Nth term is the sum over all possible partitions of the total baryon number and the total momentum p among N boltzmannions, each having an internal number of quantum states given by $\tau(p_i^2, b_i)$. These boltzmannions are, in general, excited hadronic clusters of baryon number b_i ($-\infty < b_i < \infty$). Every cluster can move freely in the remaining volume Δ left over from the external volume V_{ex} after subtracting the proper volumina V_{c} of all clusters:

$$\Delta^\mu := V_{\text{ex}}^\mu - \sum_{i=1}^{N} V_{\text{c},i}^\mu \,. \quad (2b)$$

In the generalisation (2a) of the popular phase space formula, three essential features of hadronic interactions are now explicitly included [1]:

(a) The dense set of hadronic resonances dominating particle scattering via $\tau(m_i^2, b_i)$.

(b) The proper natural volumes of hadronic clusters via Δ^μ.

(c) Conservation of baryon number and clustering of hadrons into lumps of matter carrying their natural volume.

Further features naturally contained in eqs. (1) and (2) include:

(d) coexistence of a pion gas with nucleons,

(e) baryon–antibaryon pair creation,

(f) chemical equilibrium between all constituents (nucleons, isobars, mesons...).

‡2 The extreme richness of the spectrum $\tau(m^2, b) \sim \exp(m/T_0)$ enables us to neglect Fermi and Bose statistics above $T \approx 50$ MeV and treat all particles as "boltzmannions".

We assume that members of the same isospin multiplet are present in equal proportions [e.g. $N_{\pi^+}/N_{\pi^-}/N_{\pi^0} = 1/1/1$) and average over spin and isospin.

Eq. (2) leaves us with the task of finding the mass spectrum τ. Experimental knowledge of τ is limited to low excitations and/or baryon number. We therefore introduce here a theoretical model: "the statistical bootstrap", in order to obtain a complete mass spectrum consistent with direct and indirect experimental evidence. The qualitative arguments leading to an integral equation for $\tau(m^2, b)$ are the following: when V_{ex} in eq. (2) is just the proper volume V_{c} of a hadronic cluster, then σ in eq. (2), up to a normalization factor, is essentially the mass spectrum τ: indeed, how could we distinguish between a composite system [as described by eq. (2)] compressed to the natural volume of a hadronic cluster and an "elementary" cluster having the same quantum numbers? Thus we demand

$$\sigma(p, V, b)|_{V=V_{\text{c}}} \hat{=} H\tau(p^2, b) \,, \quad (3)$$

where the "bootstrap constant" H is to be determined below. It is not sufficient simply to insert eq. (3) into eq. (2) to obtain the bootstrap equation for τ; more involved arguments are necessary [1] in order to obtain the following "bootstrap equation" for the mass spectrum τ:

$$H\tau(p^2, b) = Hz_b\delta_0(p^2 - M_b^2) + \sum_{N=2}^{\infty} \frac{1}{N!} \int \delta^4\left(p - \sum_{i=1}^{N} p_i\right) \sum_{\{b_i\}} \delta_{\text{K}}\left(b - \sum_{i=1}^{N} b_i\right) \times \prod_{i=1}^{N} H\tau(p_i^2, b_i)\, \mathrm{d}^4 p_i \,. \quad (4)$$

The first term is the lowest one-particle contribution to the mass spectrum: the "input term" $b = 0, \pm 1$ corresponds to pions and (anti)nucleons respectively. The index "0" restricts the δ function to the positive root only. z_b is the multiplicity $(2I + 1)(2J + 1)$ of the ground state clusters. In the course of deriving [1] the bootstrap equation (4) it turns out that the cluster volume V_{c} grows proportional to the invariant cluster mass:

$$V_{\text{c}}(p^2) = (p^2)^{1/2}/(4B) \,, \quad (5)$$

$(4B)$ is a universal energy density of hadronic clusters chosen here to correspond to the quark bag [5] energy density with $B = (145\ \mathrm{MeV})^4$.

The bootstrap constant H and the bag constant B are the only free parameters of the model. We introduce the double integral transform [1] [already used in eq. (1)] of the one-particle term in eq. (4):

$$\varphi(\beta, \lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int H z_b \delta_0(p^2 - M_b^2) \mathrm{e}^{-\beta\cdot p}\, \mathrm{d}^4p\,, \tag{6a}$$

and the mass spectrum:

$$\phi(\beta, \lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int H\tau(p^2, b) \mathrm{e}^{-\beta\cdot p}\, \mathrm{d}^4p\,. \tag{6b}$$

Considering pions and nucleons only as input, we find:

$$\varphi(\beta, \lambda) = 2\pi HT[3m_\pi K_1(m_\pi/T) + 4(\lambda + 1/\lambda) m_\mathrm{N} K_1(m_\mathrm{N}/T)]\,. \tag{7}$$

Applying this same transform to the entire eq. (4) results in:

$$\varphi(\beta, \lambda) = 2\phi(\beta, \lambda) - \exp[\phi(\beta, \lambda)] + 1\,. \tag{8}$$

The well known properties of the inverse function of eq. (8), $G(\varphi) = \phi(\beta, \lambda)$, shown in fig. 1a, and in particular a singularity at $\varphi \to \varphi_0 = \ln(4/e)$ lead to the critical temperature [6] T_0 of hadronic interactions. We have chosen $H = 0.724\ \mathrm{GeV}^{-2}$, namely so that $T_0 = 190$ MeV. The reason for this choice will become obvious below. From here on everything is fixed; no adjustable parameters remain.

The point $\varphi = \varphi_0$ in fig. 1a defines through eq. (7) a singular curve $\varphi(\beta, \lambda) = \varphi_0$ in the $\mu - T$ plane as shown in fig. 1b. As we shall shortly see, this boundary to the hadronic world is characterized also by a constant energy density $4B$ and vanishing hadronic pressure. We note that $\mu = 0$ ($\lambda = 1$) implies zero baryon number, $\mu \to m_\mathrm{N}$ implies $T \to 0$ and leads to cold baryonic matter at about twice the nuclear matter density of $0.17\ m_\mathrm{N}\ \mathrm{fm}^{-3}$. However, as indicated by the shaded area in fig. 1b, our present approach is inapplicable for too low a temperature, since there Fermi and Bose statistics play an important rôle.

With this, the task of determining the mass spectrum is in principle completed, as we can now invert eqs. (8) and (6b). We then find that $\tau(m^2, b) \sim \mathrm{e}^{m/T_0}$, as is well known [6]. We will now show how $\ln Z$, eqs. (1) and (2), can be given directly in terms of the known function ϕ, without the need for an explicit calculation of τ. Indeed, we can use the formal similarity between eq. (4) and eq. (2a) in order to derive a relation between their integral transforms [1]; from here on, $\beta \equiv (\beta_\mu \beta^\mu)^{1/2}$:

$$\ln Z(\beta, V_\mathrm{ex}, \lambda) = -[2\Delta(V_\mathrm{ex})/H(2\pi)^3]\, \partial\phi(\beta, \lambda)/\partial\beta\,. \tag{9}$$

The remainder of our discussion is a simple application of the rules of statistical thermodynamics. By in-

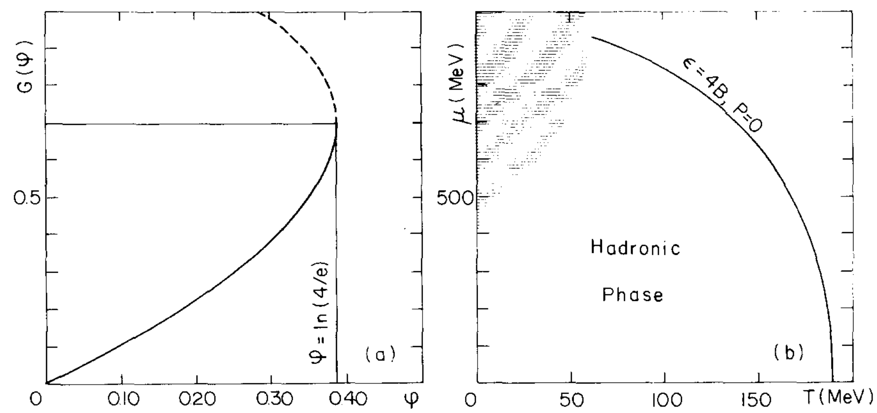

Fig. 1. (a) Bootstrap function $G(\varphi)$ – the dashed line represents the unphysical branch. The root singularity is at $\varphi_0 = \ln(4/e) = 0.3863$. (b) The critical curve corresponding to $\varphi(T, \mu) = \varphi_0$ in the $\mu - T$ plane. Beyond it the usual hadronic world ceases to exist.

vestigating the meaning of the thermodynamic averages it turns out that the apparent (β, λ) dependence of the available volume Δ in eq. (9) must be disregarded when differentiating $\ln Z$ with respect to β and λ. As eq. (2a) shows explicitly, the density of states of extended particles in V_{ex} is the same as that of point particles in Δ. Therefore also

$$\ln Z(\beta, V_{ex}, \lambda) = \ln Z_{pt}(\beta, \Delta, \lambda) . \qquad (10)$$

We thus first calculate the point particle energy and baryon number densities and pressure:

$$\epsilon_{pt} = -\frac{1}{\Delta}\frac{\partial}{\partial\beta}\ln Z_{pt} = \frac{2}{H(2\pi)^3}\frac{\partial^2}{\partial\beta^2}\phi(\beta, \lambda) , \qquad (11a)$$

$$\nu_{pt} = \frac{1}{\Delta}\lambda\frac{\partial}{\partial\lambda}\ln Z_{pt} = -\frac{2}{H(2\pi)^3}\lambda\frac{\partial^2}{\partial\lambda\,\partial\beta}\phi(\beta, \lambda) , \qquad (11b)$$

$$P_{pt} = \frac{T}{\Delta}\ln Z_{pt} = -\frac{2T}{H(2\pi)^3}\frac{\partial}{\partial\beta}\phi(\beta, \lambda) . \qquad (11c)$$

From this, we easily find the energy density, as

$$\epsilon = \frac{\langle E\rangle}{V_{ex}} = -\frac{1}{V_{ex}}\frac{\partial}{\partial\beta}\ln Z(\beta, V_{ex}, \lambda) = \frac{\Delta}{V_{ex}}\epsilon_{pt} . \qquad (12)$$

Using eq. (5), the available volume Δ of eq. (2b) becomes in the restframe:

$$\Delta = V_{ex} - \langle E\rangle/4B . \qquad (13)$$

Inserting eq. (13) into eq. (12) and solving for $\langle E\rangle$ we find:

$$\epsilon(\beta, \lambda) = \epsilon_{pt}(\beta, \lambda)/[1 + \epsilon_{pt}(\beta, \lambda)/4B] , \qquad (14a)$$

$$V_{ex} = \Delta[1 + \epsilon_{pt}(\beta, \lambda)/4B] , \qquad (14b)$$

and similarly for the baryon density and pressure:

$$\nu = \frac{\langle b\rangle}{V_{ex}} = \frac{1}{V_{ex}}\lambda\frac{\partial}{\partial\lambda}\ln Z(\beta, V_{ex}, \lambda) = \frac{\nu_{pt}}{1 + \epsilon_{pt}/4B} , \qquad (14c)$$

$$P = \frac{T}{V_{ex}}\ln Z(\beta, V_{ex}, \lambda) = \frac{P_{pt}}{1 + \epsilon_{pt}/4B} . \qquad (14d)$$

With eq. (14) we have a complete set of equations of state for observable quantities as functions of the chemical potential μ, temperature T and external volume V_{ex}. While these equations are semi-analytic, one has to evaluate the different quantities numerically due to the implicit definition of $\phi(\beta, \lambda)$ that determines $\ln Z$. However, when β, λ approach the critical curve, fig. 1b, we easily find from the singularity of ϕ that ϵ_{pt} diverges and therefore

$$\epsilon \to 4B , \quad P \to 0 , \quad \Delta \to 0 . \qquad (15)$$

These limits indicate that at the critical line, matter has lumped into one large cluster with the energy density $4B$. No available volume is left, and as only one cluster is present, the pressure has vanished. However, the baryon density varies along the critical curve; it falls with increasing temperature. This is easily understood: as temperature is increased, more mesons are produced that take up some of the available space. Therefore hadronic matter can saturate at lower baryon density. We further note here that in order to properly understand the approach to the phase boundary, one has to incroporate and understand the properties of the hadronic world beyond the critical curve – we believe [7] that a transition to the quark–gluon plasma phase occurs. We postpone the discussion of these aspects until a later paper, and turn now to the study of temperatures and average transverse momenta of particles emitted from hot hadronic matter in relativistic heavy ion collisions.

We assume that in relativistic collisions triggered to small impact parameters [2] a single fireball of hadronic matter can be produced [‡3]. Except for rare events, not all nucleons from projectile and target nuclei will participate in the formation of the fireballs [8]. However, in particular, in nearly symmetric collisions (projectile and target nuclei are similar) we can argue that the number of participants in the centre of mass of the fireball originating in the projectile or target are the same [2].

Therefore it is irrelevant how many nucleons do form the fireball – the above symmetry arguments leads in a straightforward way to a formula for the centre of mass energy per participating nucleon:

$$u := E_{cm}/A = m_N[1 + (E_{k,lab}/A)/2m_N]^{1/2} , \qquad (16)$$

‡3 We are aware of the whole problematics connected with such an idealization; a proper treatment should include collective motions and distributions of collective velocities, local temperatures and so on [3,9], triggering for small impact parameters hopefully eliminates some of the complications.

where $E_{k,lab}/A$ is the projectile kinetic energy per nucleon in the laboratory frame. While the fireball changes its chemical composition ($\pi + p \leftrightarrow \Delta$ etc.) during its lifetime through a change in temperature, the conservation of energy and baryon number assures us that u in eq. (16) remains constant. The influence on u of pre-equilibrium emission of hadrons is negligible.

From the ratio of eqs. (14a) and (14c) we find that u in the statistical average is:

$$\langle u \rangle = \langle E \rangle / \langle b \rangle = \epsilon_{pt}(\beta, \lambda)/\nu_{pt}(\beta, \lambda) \,. \tag{17}$$

Thus from eqs. (16) and (17) we see that the projectile kinetic energy fixes through u = const. a path ("cooling curve") in the μ–T diagram. In fig. 2a, some of these paths are shown for the following typical values of $E_{k,lab}/A$ = (1) 0.5; (2) 0.703; (3) 1.07; (4) 1.80; (5) 3.96 GeV. In fig. 2b, the baryon density of the fireballs along these lines is shown in units of $\nu_0 = 0.17\ \mathrm{fm}^{-3}$, the normal nuclear density. As the temperature decreases, the baryon density of the fireballs falls rapidly, while the entropy increases. This occurs essentially through a change in the chemical composition of hadronic matter and not through conversion of an important part of the internal energy into kinetic energy of the radial motion. The horizontal line in fig. 2b corresponds to normal nuclear baryon density – had we chosen a larger value of B it would correspond to a so much higher baryon density.

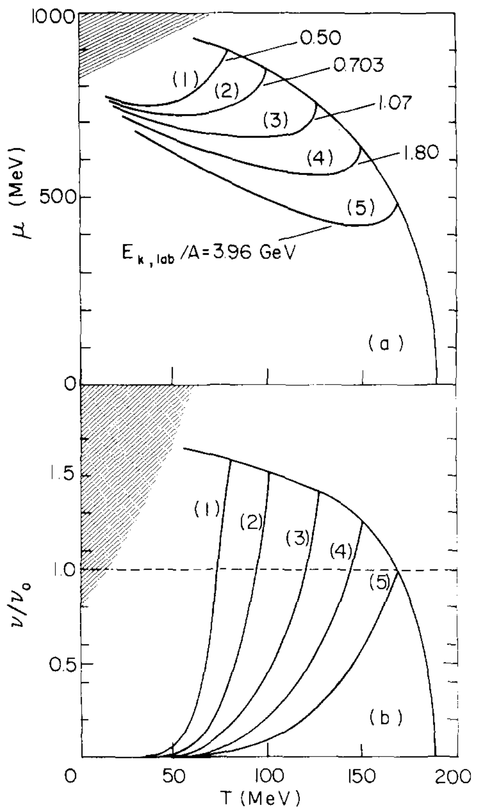

Fig. 2. (a) Paths of constant energy per baryon, per given projectile energies, and (b) the corresponding baryon densities in units of nuclear baryon density. The kinetic energies of projectiles per nucleon are (1) 0.5; (2) 0.703; (3) 1.07; (4) 1.8; (5) 3.96 GeV.

Along the cooling curves in fig. 2, particles are continuously emitted with characteristic momenta corresponding to the momentary temperature and relative intensity belonging to the chemical composition. The experimentally observable temperature is then obtained by averaging along the cooling curves, while assuming that there the temperature T decreases approximately linearly with the time, as long as particle emission is significant. As the emitted particles can be reabsorbed by the hadronic clusters present before reaching free space, we must include a factor Δ/V_{ex}, eq. (14b), which is the relative probability to escape. Thus we find:

$$\langle T \rangle_c := \frac{\int_c (\Delta/V_{ex}) T f(m, T, \mu)\, \mathrm{d}T}{\int_c (\Delta/V_{ex}) f(m, T, \mu)\, \mathrm{d}T} \,, \tag{18}$$

where f is the integrated momentum spectrum [10]:

$$f(m, T, \mu) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \exp\left[-(p^2 + m^2)^{1/2}/T + \mu/T\right]$$
$$= \frac{m^2 T}{2\pi^2} K_2(m/T)\, \mathrm{e}^{\mu/T} \,. \tag{19}$$

The integration along the cooling curves is carried out by inserting the function $\mu(T)$ that follows at constant given energy per baryon u. The average temperature, as a function of the range of integration described by T, reaches different limiting values for different particles. The thus obtained limiting value is the observable "average temperature" of the debris of the interaction, while the initial temperature is difficult to observe. When integrating along the cooling line, we can easily

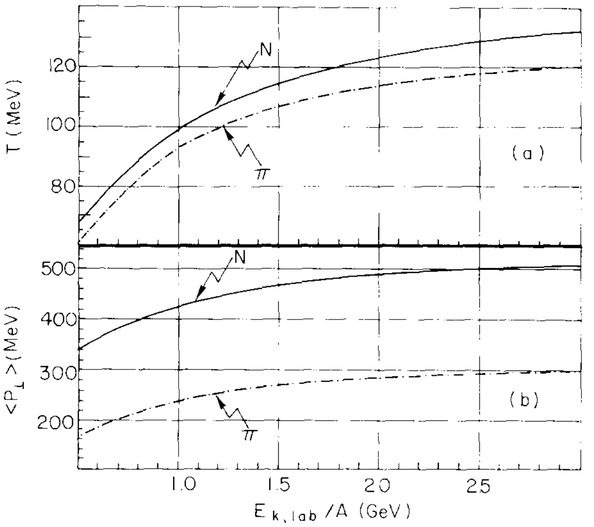

Fig. 3. (a) The average temperature and (b) the average transverse momentum for nucleons (full lines) and pions (dashed lines) as a function of the projectile laboratory energy per nucleon.

determine the average hadronic cluster mass. The integration for protons is interrupted (protons are frozen out) when the average cluster mass is about half the nucleon isobar mass, so that the nucleon-emitting clusters have essentially died out. This slightly model-dependent procedure will be described in more detail elsewhere. This is also the place to comment on our choice of the bootstrap constant H so that $T_0 \approx 190$ MeV: the observed [11] "experimental T_0" of the order of 160 MeV is an average in the above sense, our T_0 leads, for very large E_k, to an average value of the order of 160 MeV. In fig. 3a, we show the expected pion and nucleon average temperatures as a function of the heavy ion kinetic energy. The baryon average temperature is consistently higher than that of pions. Two effects contribute to this strange result:

(1) The particular shape of the cooling curves (fig. 2a): the chemical potential drops rapidly from the critical curve, thereby damping baryon emission. Hence, baryons are emitted dominantly earlier at the higher temperature, while pions, which do not feel the baryon chemical potential, continue being created also at lower temperatures.

(2) The freeze-out of baryons occurs when no significant number of clusters with a mass greater or equal to the nuclear isobar mass remain, while pions are still being emitted from lower mass clusters until *all* clusters have disappeared, and $\langle T \rangle$ has reached its lowest attainable value along the cooling line.

It may be more practical to discuss the average transverse momentum of the emitted particles. In principle, to obtain this result we have to perform a similar averaging as above; for the average transverse momentum at given T, μ we find [10]

$$\langle p_\perp(m, T, \mu)\rangle_\mathrm{p} = \frac{\int p_\perp \exp\{-[(p^2+m^2)^{1/2}-\mu]/T\}\,\mathrm{d}^3p}{\int \exp\{-[(p^2+m^2)^{1/2}-\mu]\,T\}\,\mathrm{d}^3p}$$

$$= \frac{(\pi m T/2)^{1/2} K_{5/2}(m/T)\mathrm{e}^{\mu/T}}{K_2(m/T)\mathrm{e}^{\mu/T}} . \tag{20}$$

The average over the cooling curve is then:

$$\langle\langle p_\perp(m, T, \mu)\rangle_\mathrm{p}\rangle_\mathrm{c} \tag{21}$$

$$= \frac{\int_\mathrm{c} (\Delta/V_\mathrm{ex}) T^{3/2} (\pi m/2)^{1/2} K_{5/2}(m/T)\mathrm{e}^{\mu/T}\,\mathrm{d}T}{\int_\mathrm{c} (\Delta/V_\mathrm{ex}) T K_2(m/T)\mathrm{e}^{\mu/T}\,\mathrm{d}T} .$$

We did verify numerically that the order of averages does not matter:

$$\langle p_\perp(m, \langle T_\mathrm{c}\rangle, \mu)\rangle_\mathrm{p} \approx \langle\langle p_\perp(m, T, \mu)\rangle_\mathrm{p}\rangle_\mathrm{c} , \tag{22}$$

which shows that the mean transverse momentum measurement is the simplest (and safest) method to determine the average temperature (indeed better than fitting ad hoc exponential type functions to $p_\perp$ distributions). In fig. 3b we show the dependence of the average transverse momenta of pions and nucleons on the kinetic energy of the heavy ion projectiles.

In this paper we have shown how a theory of hadronic matter that includes important features of hadronic interactions and limiting hadronic temperatures may be formulated and applied to the description of relativistic heavy ion collisions. While the measurement of the average temperature and transverse momenta already provide a certain test of our present understanding of hot hadronic matter, more work is needed to understand particle multiplicities and exclusive reactions. We intend to return to the discussion of these further features of our theory in the near future

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