Time Evolution of the Hot Hagedorn Universe

Results obtained in collaboration with Jeremiah Birrell
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Johann Rafelski, presented at ICNFP2015, August 26, 2015
1965: Penzias and Wilson discover Alpher-Gamov CMB
1966-1968: Hot Big-Bang becoming conventional wisdom

IN RECENT YEARS the active frontiers of cosmology have widened and certain aspects of the subject are attracting more attention from physicists. Growing emphasis on physics has been stimulated by discovery of the universal black-body radiation and by growing realization that the composition of the universe was once extremely complex.

What was the universe like when it was very young? From a high-energy physicist’s dream world it has evolved through many eras to its present state of comparative darkness and emptiness.

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DOI: http://dx.doi.org/10.1063/1.3035005
SBM the only model providing initial singular condition
1967 many regard SBM as the Hot Big-Bang theory

Boiling Primordial Matter  Even though no one was present when the Universe was born, our current understanding of atomic, nuclear and elementary particle physics, constrained by the assumption that the Laws of Nature are unchanging, allows us to construct models with ever better and more accurate descriptions of the beginning.
Forward to the Universe 2015
The time evolution of the energy density composition

\[ t \text{ [s]} \]

Energy Density Fraction

- Dark Energy
- Dark Matter
- Hadrons
- \( e^\pm \)
- \( \gamma \)
- \( \nu \)
- \( \mu^\pm \)
- \( \pi \)
- \( K \)
- \( p+n \)
- \( \Delta, Y \)
- \( \eta, f_0 \)
- \( \rho+\omega \)
- \( c \)
- \( b \)

\[ T \text{ [eV]} \]

- \( T_{\text{recomb}} \)
- \( T_{\nu} \)
- \( T_{\text{BBN}} \)
- \( T_{\text{QCD}} \)

J. Birrell & J. Rafelski (2014/15)
Input into the image

- FRW Cosmology
- Disappearing Particles: Degrees of Freedom and Reheating – tracking $T_\gamma$
- Connecting the Eras
  - From the beginning to QGP – remarks
  - And matter free-streams, latest when:
    - QGP turns into disappearing hadrons, invisible radiation, ...
    - Onset of neutrino free-streaming
  - Big-Bang nucleosynthesis and disappearance of matter
  - Emergence of free streaming dark matter, baryons follow
  - Photon Free-streaming – Composition Cross-Point
  - Dark Energy Emerges – vacuum energy
- Open questions abound

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Overview: the Friedmann–Lemaitre–Robertson–Walker (FRW) cosmology assumes

- Homogeneous
- Isotropic

Einstein Universe

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2) \right]. \]

Flat \((k = 0)\) metric is favored [1] in the \(\Lambda\)CDM analysis. \(a(t)\) determines the distance between objects at rest in the Universe frame (comoving). Skipping \(g^{\mu\nu} \rightarrow R^{\mu\nu}\)

Einstein’s Equations, where \( R = g_{\mu\nu}R^{\mu\nu} \):

\[
G^{\mu\nu} = R^{\mu\nu} - \left( \frac{R}{2} + \Lambda \right) g^{\mu\nu} = 8\pi G_N T^{\mu\nu}, \quad T^{\mu\nu}_\nu = \text{diag}(\rho, -P, -P, -P),
\]

We absorb the vacuum energy (Einstein \( \Lambda \)-term) into the energy \( \rho \) and pressure \( P \)

\[
\rho \rightarrow \rho + \rho_\Lambda, \quad P \rightarrow P + P_\Lambda
\]

which contain other components in the Universe including CDM: cold dark matter. The \( \Lambda \)CDM speaks thus of Dark Energy, or equivalently, non-vanishing vacuum energy density

\[
\rho_\Lambda \equiv \Lambda/(8\pi G_N) = 25.6 \text{ meV}^4, \quad P_\Lambda = -\rho_\Lambda
\]

The pressure \( P_\Lambda \) has a) opposite sign from all matter contributions and b) \( \rho_\Lambda/P_\Lambda = -1 \).
Definitions: Hubble parameter $H$ and deceleration parameter $q$:

$$H(t) \equiv \frac{\dot{a}}{a}; \quad q \equiv -\frac{a \ddot{a}}{\dot{a}^2} = -\frac{1}{H^2} \frac{\ddot{a}}{a}, \Rightarrow \dot{H} = -H^2(1 + q).$$

Two dynamically independent Einstein equations arise

$$\frac{8\pi G_N}{3} \rho = \frac{\dot{a}^2 + k}{a^2} = H^2 \left(1 + \frac{k}{\dot{a}^2}\right), \quad \frac{4\pi G_N}{3} (\rho + 3P) = -\frac{\ddot{a}}{a} = qH^2.$$

solving both these equations for $8\pi G_N/3 \rightarrow$ we find for the deceleration parameter:

$$q = \frac{1}{2} \left(1 + 3\frac{P}{\rho}\right) \left(1 + \frac{k}{\dot{a}^2}\right).$$

In flat $k = 0$ Universe: $\rho$ fixes $H$; with $P$ also $q$ fixed, and thus also $\dot{H}$ fixed so also $\dot{\rho}$ fixed, and therefore also for $\rho = \rho(T(t)$ also $\dot{T}$ fixed.
The contents of the Universe:

- Matter coupled to photons: thermal matter = ideal Bose-Fermi gases
- Free-streaming matter: today
  - dark matter: since at or before QGP hadronization
  - neutrinos: since $T = \text{few MeV}$
  - photons: since $T = 0.25\text{eV}$
  - hypothetical darkness: quasi-massless particles, like neutrinos but due to earlier decoupling small impact on Universe dynamics
- dark energy = vacuum energy
Degrees of Freedom

The effective number of entropy degrees of freedom, $g^S_*$, defined by

$$S = \frac{2\pi^2}{45} g^S_* T_\gamma^3 a^3.$$ 

For ideal Fermi and Bose gases

$$g^S_* = \sum_{i=\text{bosons}} g_i \left( \frac{T_i}{T_\gamma} \right)^3 f_i^- + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left( \frac{T_i}{T_\gamma} \right)^3 f_i^+.$$ 

$g_i$ are the degeneracies, $f_i^{\pm}$ are functions varying valued between 0 and 1 that turn off the various species as the temperature drops below their mass.
Degrees of Freedom

Figure: Ideal gas approximation is not valid during QGP phase transition and equation of state from lattice QCD must be used [1]. At and above 300 MeV non-rigorous matching [2] with perturbation calculations may impact result.

Conservation of Entropy and Reheating

Once (example) Darkness decouples from SM particles at a photon temperature of $T_{d,s}$, a difference in its temperature from that of photons will build up during subsequent reheating periods. Conservation of entropy leads to a temperature ratio at $T_\gamma < T_{d,s}$ of

$$R_s \equiv T_s / T_\gamma = \left( \frac{g_*^S(T_\gamma)}{g_*^S(T_{d,s})} \right)^{1/3}.$$  

This can be used to determine the present day reheating ratio as a function of decoupling temperature throughout the Universe history.
Reheating and Particle Disappearance History

Figure: The reheating ratio reflects the disappearance of degrees of freedom from the Universe. At and above 300 MeV non-rigorous matching with perturbative calculations may impact result. These results are for adiabatic evolution of the Universe.
Free-streaming matter: solution of kinetic equations with decoupling boundary conditions at $T_k$ (kinetic freeze-out)

$$
\rho = \frac{g_\nu}{2\pi^2} \int_0^\infty \frac{(m_\nu^2 + p^2)^{1/2}}{\gamma_{\nu}^{-1} e^{\sqrt{p^2/T_\nu^2 + m_\nu^2/T_k^2}} + 1} \, p^2 \, dp,

P = \frac{g_\nu}{6\pi^2} \int_0^\infty \frac{(m_\nu^2 + p^2)^{-1/2}}{\gamma_{\nu}^{-1} e^{\sqrt{p^2/T_\nu^2 + m_\nu^2/T_k^2}} + 1} \, p^4 \, dp,

n = \frac{g_\nu}{2\pi^2} \int_0^\infty \frac{p^2 \, dp}{\gamma_{\nu}^{-1} e^{\sqrt{p^2/T_\nu^2 + m_\nu^2/T_k^2}} + 1}.
$$

These differ from the corresponding expressions for an equilibrium distribution by the replacement $m \rightarrow mT_\nu(t)/T_k$ only in the exponential. Only for massless photons free-streaming = thermal distributions.

Identifying Eras by Deceleration Parameter $q$

$q \equiv -\frac{\ddot{a}a}{\dot{a}^2}$.

Using Einstein's equations exact expression in terms of matter content

$$q = \frac{1}{2} \left( 1 + 3 \frac{P}{\rho} \right) \left( 1 + \frac{k}{\dot{a}^2} \right) \quad k = 0 \text{ favored}$$

- Radiation dominated universe: $P = \rho/3 \implies q = 1$.

- Matter dominated universe: $P \ll \rho \implies q = 1/2$.

- Dark energy ($\Lambda$) dominated universe: $P = -\rho \implies q = -1$. 
Hadron and QGP Era

- QGP era down to phase transition at $T \approx 150 \text{MeV}$. Energy density dominated by photons, neutrinos, $e^{\pm}$, $\mu^{\pm}$ along with u,d,s.
- 2 + 1-flavor lattice QCD equation of state must be used [1].
- u,d,s lattice energy density is matched by ideal gas of hadrons to sub percent-level at $T = 115 \text{MeV}$.
- Hadrons included: pions, kaons, eta, rho, omega, nucleons, delta, Y
- without all resonances hadron pressure is discontinuous at 10% level. Causes hard to notice discontinuity in $q$ (slopes match).

Figure: Evolution of temperature $T$ and deceleration parameter $q$ from QGP era until near BBN.

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Relic Neutrino Background:

At a temperature of 5 MeV the Universe consisted up to $10^{-9}$ of plasma made of $e^{\pm}$-pairs, photons, and neutrinos. At around 1 MeV neutrinos stop interacting or freeze-out and free stream through the universe. Today they comprise the relic neutrino background (CNB).

Direct measurement:
Relic neutrinos have not been directly measured.

Indirect measurement:
Impact on speed of Universe expansion can be seen in the CMB. This constrains: i) neutrino mass, ii) reheating of neutrinos by $e^{\pm}$-pair annihilation, and iii) the number of additional invisible relativistic degrees of freedom.
Reheating Ratio and $\delta N_\nu$

If a particle species with $g_s$ degrees of freedom (normalized to bosons) decouples from SM particles at a photon temperature of $T_{d,s}$ then the impact on the value of $N_\nu$ at a photon temperature of $T_\gamma$ is

$$\delta N_\nu = \frac{4g_s}{7R_\nu^4} \left( \frac{g_s^S(T_\gamma)}{g_s^S(T_{d,s})} \right)^{4/3}. \quad (1)$$

In particular, after $e^\pm$ annihilation the SM particles remaining are photons and SM neutrinos, the latter with temperature $R_\nu T_\gamma$, and so $g_s^S(T_\gamma) = 2 + 7/8 \times 6 \times 4/11$ and

$$\delta N_\nu \approx g_s \left( \frac{7.06}{g_s^S(T_{d,s})} \right)^{4/3}. \quad (2)$$
If Understanding of Neutrino Freeze-out Accurate

The computed best value is $N_\nu = 3.046$ (some flow of $e^\pm$-pair entropy into $\nu$) [1]. Only drastic changes in neutrino properties and/or physical laws can change this value noticeably [2].

- $\delta N_\nu$ probes ‘Darkness’ particle content in the Universe: new relativistic particles in the early Universe modify $N_\nu$, see e.g. [3].
- $\delta N_\nu$ limits variation of fundamental constants in early Universe [4]

How understanding the Universe enters laboratory experiments: Example

**Quark–gluon plasma as the possible source of cosmological dark radiation**

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doi:10.1016/j.physletb.2014.12.033

Open Access funded by SCOAP³ - Sponsoring Consortium for Open Access Publishing in Particle Physics

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Abstract

The effective number of neutrinos, $\nu_{\text{eff}}$, obtained from CMB fluctuations accounts for all effectively massless degrees of freedom present in the Universe, including but not limited to the three known neutrinos. Using a lattice-QCD derived QGP equation of state, we constrain the observed range of $\nu_{\text{eff}}$ in terms of the freeze-out of unknown degrees of freedom near to quark–gluon hadronization. We explore limits on the coupling of these particles, applying methods of kinetic theory, and discuss the implications of...
We showed quantitatively how freeze-out of a reasonable number of Bose or fermi DoF at $T_c$ during the QGP phase transition in the Universe leads to $\delta N_\nu$ in the range compatible with Planck.

The existence of such Dark QCD related particles should lead to observable effects in heavy ion collisions: search for missing energy in connection to dynamics of hadronization near to phase boundary as function of $\sqrt{s}$ with energy imbalance increasing with $A$. 
Radiation Dominated Era

- Neutrinos freeze-out at $T \approx 1\text{MeV}$.
- A small deviation from radiation dominated during $e^\pm$ annihilation.
- Energy density dominated by neutrinos, photons down through BBN ($T = 40 - 70\text{keV}$) until $T = O(1\text{eV})$
Dark energy and Matter Dominated Eras

- Present day on left of plot: 69% dark energy, 26% dark matter, 5% baryons, < 1% photons and neutrinos.
- Solid neutrino line shows massless neutrinos. Dashed line shows 1 massless and 2 $\times$ 0.1 eV neutrinos (Neutrino mass choice is just for illustration. Other values are possible)
- First vertical line on the left shows recombination at $T \approx 0.25$eV.
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Hot Hagedorn Universe
Figure: Evolution of temperature $T$ and deceleration parameter $q$ from soon after BBN to the present day
In Conclusion

- We connected the hot melted quark Universe, to the boiling hadron Universe, on to lepton Universe, and the ensuing matter emergence, and dark energy emergence.

- Limits on effects due to modifications of natural constants and any new radiance from the deconfined Universe were set.

- CMB fluctuations (PLANCK, WMAP data) have been for the first time connected to the QGP work in the laboratory.


