Strangeness in quark–gluon plasma

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It is argued that observation of the strange-particle abundance may lead to identification of the quark–gluon plasma and measurement of some of its properties. Approach to chemical equilibrium and competitive processes in the hadronic gas phase are discussed.
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'n Beredenering word gegee waarom die waarneming van die voorkoms van vreemde deeltjies tot die identifisering van die kwark–gluon-plasma en die meting van sommige van sy eienskappe mag lei. Die nadering van chemiese ewewig en kompeterende prosesse in die hadrongasfase word bespreek.
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1. Overview

I would like to argue in this paper that the nature and properties of quark–gluon plasma can be studied by observing the abundance of strange particles created in nuclear collisions [1]. Unlike hadron–hadron collisions, we anticipate that in an important fraction of nucleus–nucleus collisions each participating quark will scatter many times before joining in an asymptotic hadronic state. The associated simplification of the physics involved arises because the well-established methods of statistical physics can be used in such a case in order to connect the microscopic world with effects and properties visible to the experimentalist's eyes. Only the presumption of an approximate thermochemical equilibrium, to be studied below in more detail, frees us from the dependence on details of quark wave functions.

As a consequence of the statistical equilibrium, the available energy is equipartitioned among accessible degrees of freedom and, among others, $s\bar{s}$ pairs. This means that there exists a domain in space in which, in a proper Lorentz frame, the energy of the longitudinal motion has been largely transformed to transverse degrees of freedom. The basic question concerns the internal structure of this hadronic fireball: instead of consisting of individual hadrons it may be formed by quarks and gluons. In this new physical phase these colour-charged particles are deconfined and can move freely over the volume of the fireball. It appears that the phase transition from the hadronic gas phase to the quark–gluon plasma is mainly controlled by the energy density of the fireball. Several estimates [2] lead to 0.6–1 GeV/fm^3 for the critical-energy density, to be compared with *ca.* 0.25 GeV/fm^3 inside individual hadrons. Many theoretical questions about strong interactions will be settled once the parameters and nature of the phase transition have been determined.

Further development of this new field of research depends on the ability to observe plasma creation and its detailed physical properties. It is quite difficult to insert a thermometer and to measure baryon density at $T = 150$ MeV and threefold or even higher nuclear compressions. We must either use only electromagnetically interacting particles [3] (photons, lepton pairs) in order to get them out of the plasma or study the heavy quark flavour abundance, in particular strangeness, generated in the collision [1]. To obtain a better impression of what is meant, imagine that strange quarks are very abundant in the plasma (and indeed they are!). Then, for example, since the (sss)-state is bound and stable in the hot perturbative QCD-vacuum, it would be the most abundant baryon to emerge from the plasma. I doubt that such an omegazation of nuclear matter could leave any doubts about the occurrence of the phase transition. But even the enhancement of the more accessible abundance of $\bar{\Lambda}$ may already be sufficient for our purposes.

I will now explain in more detail why the strange-particle abundance is so useful [1] for observing properties of the quark–gluon plasma. First we note that, at a given temperature, the quark–gluon plasma will contain an equal number of strange (s) and antistrange ($\bar{s}$) quarks, naturally assuming that the hadronic collision time is much too short to allow for light-flavour weak-interaction conversion to strangeness. Thus, assuming equilibrium in the quark plasma (see section 2), we find the density of the strange quarks to be (two spins and three colours)

$$s/v \;=\; \bar{s}/v \;=\; 6\int\frac{\mathrm{d}^3p}{(2\pi)^3}\,\frac{1}{\exp(\sqrt{p^2+m_s^2}\,/T)+1}$$

$$\approx\; 3\,\frac{Tm_s^2}{\pi^2}\,K_2(m_s/T) \tag{1.1}$$

(neglecting, for the time being, the perturbative corrections). The mass of the strange quarks, m_s, in the perturbative vacuum is believed to be of the order of 180–300 MeV. Since the phase space density of strangeness is not too high, the Boltzmann limit is used in eq. (1.1). Similarly, there is a certain light antiquark density ($\bar{q}$ stands for either $\bar{u}$ or $\bar{d}$):

$$\bar{q}/v \;\triangleq\; 6\int\frac{\mathrm{d}^3p}{(2\pi)^3}\,\frac{1}{\exp(|p|/T+\mu_q/T)+1}$$

$$\approx\; \mathrm{e}^{-\mu_q/T}\,T^3\,\frac{6}{\pi^2}\,, \tag{1.2}$$

where the quark chemical potential is $\mu_q = \mu/3$ and μ is the baryonic chemical potential. This exponent suppresses the $q\bar{q}$ pair production. It reflects the chemical equilibrium between q–$\bar{q}$ and the presence of a light quark density associated with the net baryon number.

Alternative, but physically equivalent, ways to understand these factors are the following two statements:

(a) $\bar{q}$ is fermi-blocked, since in its production the partner q quark has to go on top of a Fermi sphere ($T\to 0$ limit);

(b) $\bar{q}$ quarks are easily destroyed by the abundant q quarks in the plasma.

What we now intend to show is that there are often more $\bar{s}$ quarks than antiquarks of each light flavour. Indeed,

$$\bar{s}/\bar{q} \;=\; \tfrac{1}{2}\left(\frac{m_s}{T}\right)^2 K_2\left(\frac{m_s}{T}\right)\mathrm{e}^{\mu/3T}. \tag{1.3}$$

This ratio is shown in Figure 1. Thus, we almost always have more $\bar{s}$ than $\bar{q}$ quarks and, in many cases of interest, $\bar{s}/\bar{q} \approx 5$. As $\mu\to 0$ there are about twice as many $\bar{u}$ or $\bar{d}$ quarks as there are $\bar{s}$ quarks at $T \gtrsim m_s$.

When the quark matter dissociates into hadrons, some of the numerous $\bar{s}$ quarks may, instead of being bound in a $q\bar{s}$ kaon, enter into a ($\bar{q}\bar{q}\bar{s}$) or ($\bar{q}\bar{s}\bar{s}$) antibaryon and, in particular, a $\bar{\Lambda}$, $\bar{\Sigma}$ or $\bar{\Xi}$. The probability for this process seems to be comparable to the similar one for the production of $\bar{\Lambda}$, $\bar{\Sigma}$ or $\bar{\Xi}$ by the quarks present in the plasma. What is particularly noteworthy about the $\bar{s}$ carrying antibaryons is that they can conventionally only be produced in direct pair production reactions. Up to high energies this process is suppressed by the energy-momentum conservation and phase space considerations. This leads me to argue that a study of the $\bar{\Lambda}$, $\bar{\Sigma}$ and $\bar{\Xi}$ in high energetic nuclear collisions could shed light on the early stages of the nuclear collisions in which quark matter may be formed.

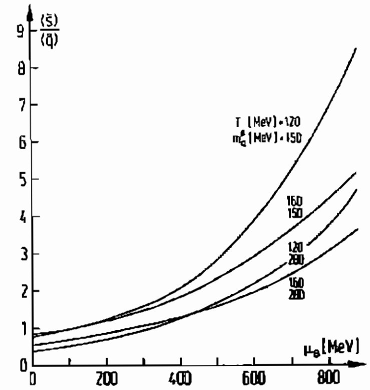

Figure 1 Abundance of strange (antistrange) quarks relative to light quark abundance as function of μ for several choices of T (= 120, 160 MeV) and strange quark mass (m_q^s = 150, 280 MeV).

As is apparent from the above remark that the crucial aspects of the proposal to use strangeness as a tag of quark–gluon plasma, involve

(a) assumption of thermal *and* chemical equilibrium (see next section);

(b) comparison between results anticipated in both hadronic phases at given T and μ, the chemical potential to be determined by other considerations (see section 3).

The theoretical techniques required for the description of the two quite different hadronic phases, the hadronic gas and the quark–gluon plasma, must allow for the formation of numerous hadronic resonances, which then dissolve at sufficiently high partial density in the state consisting of its constituents. At this point we must appreciate the importance of and help provided by finite temperature. To obtain high particle density we may, instead of compressing the matter (which as it turns out is quite difficult), heat it up; many pions are generated easily, leading to the occurrence of the transition at moderate (even vanishing) baryon density [1].

2. Strangeness production in the quark–gluon plasma

In this section we investigate the abundance of strangeness as function of the lifetime and excitation of the plasma state [4]. This investigation was motivated by the observation that light quarks could not by themselves lead to chemical equilibrium of strange quarks [5]. After identifying the strangeness-producing mechanisms we compute the relevant rates as function of the energy density ('temperature') of the plasma state and compare them with those for light u and d quarks.

In lowest order in perturbative QCD, $s\bar{s}$ quark pairs can be created by annihilation of light quark–antiquark pairs

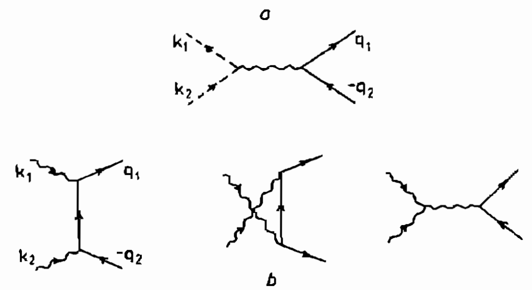

Figure 2 Lowest order QCD diagrams for $s\bar{s}$ production: (a) $q\bar{q} \to s\bar{s}$; (b) $gg \to s\bar{s}$.

(Figure 2a) and in collisions of two gluons (Figure 2b). The averaged total cross-sections for these processes were calculated by Combridge [6]. For fixed invariant mass-squared $s = (k_1 + k_2)^2$, where k_i are the four momenta of the incoming particles $[w(s) = (1 - 4M^2/s)^{1/2}]$,

$$\bar{\sigma}_{q\bar{q}\to s\bar{s}} = \frac{8\pi\alpha_s^2}{27s}\left(1 + \frac{2M^2}{s}\right)w(s) \tag{2.1a}$$

$$\sigma_{gg\to s\bar{s}} = \frac{2\pi\alpha_s^2}{3s}\left[\left(1 + \frac{4M^2}{s} + \frac{M^4}{s^2}\right)\tanh^{-1}w(s) - \left(\frac{7}{8} + \frac{31}{8}\frac{M^2}{s}\right)w(s)\right]. \tag{2.1b}$$

For the mass of the strange quark we assume the following:
(a) the value fitted within the MIT bag model: M = 280 MeV; and
(b) the value found in the study of quark currents: M = 150 MeV.

When discussing light quark production we use M = 15 MeV. The effective QCD coupling constant $\alpha_s = g^2/4\pi$ is an average over space- and timelike domains of momentum transfers in reactions shown in Figure 2. We use (a) $\alpha_s = 2.2$, the value consistent with M = 280 MeV in the MIT bag model, and (b) the value $\alpha_s = 0.6$, expected at the involved momentum transfer.

Given the averaged cross-sections it is easy to calculate the rate of events per unit time, summed over all final and initial states:

$$\frac{dN}{dt} = \int d^3x \int \frac{d^3k_1}{(2\pi)^3|k_1|}\sum_i \rho_i(k_1,x)\int\frac{d^3k_2}{(2\pi)^3|k_2|} \times \sum \rho_i(k_2,x)\int_{4M^2}^{\infty} ds\,\delta[s-(k_1+k_2)^2]k_1^{\mu}k_{2\mu}\bar{\sigma}(s). \tag{2.2}$$

The sum over initial states involves the discrete quantum numbers i (colour, spin, etc.) over which eq. (2.1) is averaged. The factor $k_1 \cdot k_2/|k_1||k_2|$ is the relative velocity for massless particles, and we have introduced a dummy integration over s in order to facilitate the calculations. We now replace the phase space densities $\rho_i(k,x)$ by momentum distributions $f_g(k)$, $f_q(k)$ and $f_{\bar{q}}(k)$ of gluons, quarks and antiquarks that can still have a parametric x-dependence, *i.e.* through a space dependence of temperature $T = T(x)$. The (invariant) rate per unit time and volume for the elementary processes shown in Figure 2 is then

$$A = \frac{dN}{dt\,d^3x} = \tfrac{1}{2}\int_{4M^2}^{\infty} s\,ds\,\delta[s-(k_1+k_2)^2]\int\frac{d^3k_1}{(2\pi)^3|k_1|} \times \int\frac{d^3k_2}{(2\pi)^3|k_2|}\,[(2\times8)^2 f_g(k_1)f_g(k_2)\bar{\sigma}_{gg\to s\bar{s}}(s) + 2(2\times3)^2 f_q(k_1)f_{\bar{q}}(k_2)\bar{\sigma}_{q\bar{q}\to s\bar{s}}(s)], \tag{2.3}$$

where the numerical factors count the spin, colour and isospin degrees of freedom.

We furthermore assume that in the rest frame of the plasma the distribution functions f depend only on the absolute value of the momentum, $|k| = k_0 \equiv k$. We then evaluate angular integrals in eq. (2.3):

$$A = \frac{8}{\pi^4}\int_{4M^2}^{\infty} s\,ds\,\bar{\sigma}_{gg\to s\bar{s}}\left[\int_0^{\infty}dk_1\int_0^{\infty}dk_2\Theta(4k_1k_2 - s) \times f_g(k_1)f_g(k_2)\right] + \frac{9}{4\pi^4}\int_{4M^2}^{\infty} s\,ds\,\bar{\sigma}_{q\bar{q}\to s\bar{s}}\left[\int_0^{\infty}dk_1\int_0^{\infty}dk_2\Theta(4k_1k_2 - s) \times f_q(k_1)f_{\bar{q}}(k_2)\right], \tag{2.4}$$

where the step function Θ requires that $k_1k_2 \geq \frac{s}{4} \geq M^2$. We now turn to the discussion of the momentum distribution and related questions. We note that the anticipated lifetime of the plasma created in nuclear collisions is of the order 6 fm/c = 2 × 10^{-23} s. After this time the high internal excitation will most likely have dissipated to below the energy density required for the global restoration of the perturbative QCD vacuum state [7, 8]. The transition between the hadronic and the quark–gluon phase is expected at an energy density of approximately 1 GeV/fm^3. Under these conditions, it is possible to estimate that each perturbative quantum (light quark, gluon) in the plasma state will rescatter several times during the lifetime of the plasma. Hence the momentum distribution functions $f(p)$ can be approximated by the statistical Bose (Fermi) distribution functions,

$$f_g(p) \approx (e^{\beta\cdot p} - 1)^{-1} \qquad \text{(gluons)} \tag{2.5}$$

$$f_{q/\bar{q}}(p) \approx (e^{\beta\cdot p}\lambda^{(\pm)} + 1)^{-1} \qquad \text{(quarks–antiquarks)}, \tag{2.6}$$

where $\beta\cdot p = \beta_0|p| - \boldsymbol{\beta}\cdot\boldsymbol{p}$ for massless particles, $(\beta\cdot\beta)^{-1/2} = T$ is the temperature-like parameter and $\lambda^{(\pm)}$ is the baryon number (antibaryon number) fugacity. In the rest frame of the plasma, $\beta\cdot p = |p|/T$. The distributions (eqs. 2.5 and 2.6) can only be taken seriously for $|p|$ not much larger than T; to populate the high-energy tail of the distributions too many collisions are required, for which there may not be enough time during the lifetime of the plasma. While in each individual nuclear collision the momentum distribution may vary, the ensemble of many collisions may lead to better statistical distributions.

Finally, let us discuss the values of the fugacities $\lambda^{(\pm)}$ in eq. (2.6). As quarks are brought into the reaction by the colliding nuclei, baryon number conservation makes it possible to relate the baryon density v to the fugacities by integrating eq. (2.6) over all momenta:

$$v(T,\lambda^+,\lambda^-) = \tfrac{1}{3}\times 12\int \frac{d^3p}{(2\pi)^3}\,[(e^{|p|/T}\lambda^+ + 1)^{-1} - (e^{|p|/T}\lambda^- + 1)^{-1}]. \qquad (2.7)$$

The factor 1/3 takes into account the fractional baryon number of quarks. As we will show, the gg → q$\bar{\text{q}}$ reaction time is much shorter than that for q$\bar{\text{q}}$ → s$\bar{\text{s}}$ production, since the light quark masses are only of the order of ≈15 MeV. Consequently, we may assume chemical equilibrium between q and $\bar{\text{q}}$ ($\mu = 3\mu_q$):

$$\lambda^+ = \frac{1}{\lambda^-} = e^{-\mu_q/T} \qquad (2.8a)$$

$$v(T,\mu_q) = \frac{2}{3\pi^2}\,[\mu_q^3 + \mu_q(\pi T)^2]. \qquad (2.8b)$$

As long as gluons dominate the plasma state, conditions at the phase transition, such as abundance of q and $\bar{\text{q}}$, will not matter for the s$\bar{\text{s}}$ abundances at times comparable to the lifetime of the plasma. Hence for the purpose of this study we will use the value μ_q = 300 MeV in order to estimate the quark densities at given temperature. We can now return to the evaluation of the rate integrals, eq. (2.4).

In the glue part of the rate A, eq. (2.4), the k_1, k_2 integral can be carried out exactly by expanding the Bose function, eq. (2.5), in a power series in $\exp(-k/T)$:

$$A_g = \frac{8}{\pi^4}\,T\int_{4M^2}^{\infty} ds\, s^{3/2}\bar{\sigma}_{gg\to s\bar{s}}(s)\sum_{n,n'=1}(nn')^{-1/2} \times K_1\left(\frac{(nn's)^{1/2}}{T}\right). \qquad (2.9)$$

In the quark contribution an expansion of the Fermi function is not possible and the integrals must be evaluated numerically. It is found that the gluon contribution, eq. (2.9), dominates the rate A. For $T/M \gtrsim$ we find

$$A \approx A_g = \frac{7}{3\pi^2}\,\alpha_s^2 M T^3 e^{-2M/T}\left(1 + \frac{51}{14}\,\frac{T}{M} + \ldots\right). \qquad (2.10)$$

The abundance of s$\bar{\text{s}}$ pairs cannot grow forever; at some point the s$\bar{\text{s}}$ annihilation reaction will deplete the strange quark population.* This loss term is proportional to the square of the density n_s of strange and antistrange quarks. With $n_s(\infty)$ being the saturation density at large times, the following differential equation determines n_s as function of time [10]:

$$\frac{dn_s}{dt} \approx A\{1 - [n_s(t)/n_s(\infty)]^2\}. \qquad (2.11)$$

The solution for $n_s(t=0) \approx 0$

$$n_s(t) = n_s(\infty)\tanh(t/\tau) \qquad (2.12)$$

[where $\tau = n_s(\infty)/A$] is a monotonically rising, saturating function, controlled by the characteristic time constant τ. In a thermally equilibrated plasma the asymptotic strangeness density, $n_s(\infty)$, is that of a relativistic Fermi gas ($\lambda = 1$),

$$n_s(\infty) = \frac{2\times 3}{2\pi^2}\,TM^2\sum_{n=1}^{\infty}\frac{(-)^{n-1}}{n}\,K_2(nM/T), \qquad (2.13)$$

provided the volume V is large. We find that the relaxation time

$$\tau \approx \tau_g = \frac{9}{7}\left(\frac{\pi}{2}\right)^{1/2}\alpha_s^{-2}M^{1/2}T^{-3/2}\,e^{M/T} \times\left(1 + \frac{99}{56}\,\frac{T}{M} + \ldots\right)^{-1} \qquad (2.14)$$

is falling rapidly with increasing temperature.

We now discuss the numerical results for the rates, time constants and the expected strangeness abundance. In Figure 3a we compare the rates for strangeness production by the processes depicted in Figure 2 for the two different choices of parameters discussed below eq. (2.1). The rate for q$\bar{\text{q}}$ → s$\bar{\text{s}}$ alone [5] (shown separately) contributes less than 10% to the total rate. In Figure 3b we show the corresponding characteristic relaxation times towards chemical equilibrium, τ, defined in eq. (2.12). While our results for strangeness production by light quarks agree in order of magnitude with those of Biró and Zimányi [5] (considering their choice of parameters), it is here obvious that gluonic strangeness production, which was not discussed by these authors, is the

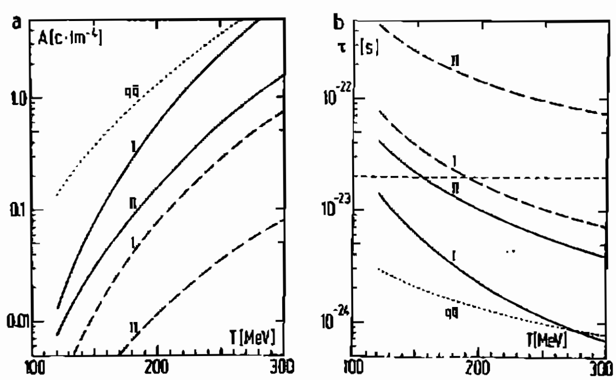

Figure 3 (a) Rates A; Time constants τ as function of temperature T. Full lines: q$\bar{\text{q}}$ → s$\bar{\text{s}}$ *and* gg → s$\bar{\text{s}}$; dashed lines: q$\bar{\text{q}}$ → s$\bar{\text{s}}$; dotted lines gg → q$\bar{\text{q}}$ (M = 15 MeV). Curves marked I are for α_s = 2.2 and M = 280 MeV, those marked II are for α_s = 0.6 and M = 150 MeV.

*The s$\bar{\text{s}}$ pair annihilations may not only proceed *via* the two-gluon channel, but instead through γG final states [9]: the noteworthy feature of such a reaction is the production of relatively high energetic γ's at the fixed energy of about 700–900 MeV (T = 160 MeV). These γ's will leave the plasma without further interactions. To some degree, this process is stimulated by coherent glue emission.

dominant process. If we compare the time constant τ with the estimated lifetime of the plasma state we find that the strangeness abundance will be chemically saturated for temperatures of 160 MeV and above, *i.e.* for an energy density above 1 GeV/fm^3. We note that τ is quite sensitive to the choice of the strange quark mass parameter and the coupling constant α_s which must, however, be chosen consistently. A measure of the uncertainty associated with the choice of parameters is illustrated by the difference between our results for the two parameter sets taken here.

Also included in Figures 3a and 3b are our results for gluon conversion into light quark–antiquark pairs. The shortness of τ for this process indicates that gluons and light quarks reach chemical equilibrium during the beginning stage of the plasma state, even if the quark/antiquark (i.e. baryon/meson) ratio was quite different in the prior hadronic compression phase.

The evolution of the density of strange quarks, eq. (2.12), relative to the baryon number content of the plasma state, is shown in Figure 4 for various temperatures. The saturation of the abundance is clearly visible for $T \geqslant 160$ MeV. To obtain the measurable abundance of strange quarks, the corresponding values reached after the typical lifetime of the plasma state, 2×10^{-23} s, can be read off in Figure 4 as a function of temperature. The strangeness abundance shows a pronounced threshold behaviour at $T \approx 120$–160 MeV.

I thus conclude that strangeness abundance saturates in sufficiently excited quark–gluon plasma ($T > 160$ MeV, $\varepsilon > 1$ GeV/fm^3).

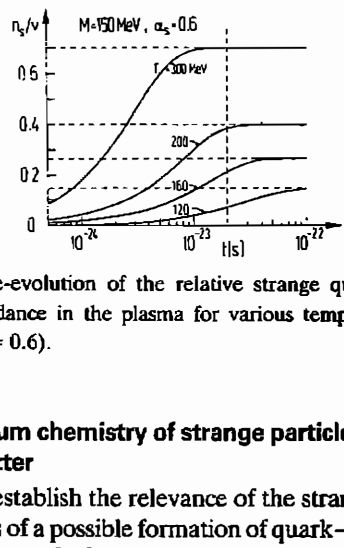

Figure 4 Time-evolution of the relative strange quark to baryon number abundance in the plasma for various temperatures (M = 150 MeV, $\alpha_s = 0.6$).

3. Equilibrium chemistry of strange particles in hot nuclear matter

In order to establish the relevance of the strangeness signal for diagnosis of a possible formation of quark–gluon plasma we must establish [11] relevant particle rates originating from highly excited matter but consisting of individual hadrons — the hadronic gas phase. The main hypothesis which makes it possible to simplify the situation is to postulate the resonance dominance of hadron–hadron interactions [12] — in this case the hadronic gas phase is practically a superposition of an infinity of different hadronic gases and all information about the interaction is hidden in the mass spectrum $\tau(m^2, b)$ which describes the number of hadrons of baryon number b in a mass interval dm^2 [13]. When considering strangeness carrying particles, all we need to consider is the baryon chemical potential established by the non-strange particles. The total partition function is approximately additive in the independent degrees of freedom:

$$\ln Z = \ln Z^{\text{non-strange}} + \ln Z^{\text{strange}}. \tag{3.1}$$

For our purposes it is sufficient to list strange particles separately, and we find

$$\ln Z^{\text{strange}}(T, V, \lambda_s, \lambda_q) = \frac{VT^3}{2\pi^2}\{2W(x_k)[\lambda_s\lambda_q^{-1} + \lambda_s^{-1}\lambda_q]$$

$$+ 2[W(x_\Lambda) + 3W(x_\Sigma)][\lambda_s\lambda_q^2 + \lambda_s^{-1}\lambda_q^{-2}]\} \tag{3.2a}$$

$$W(x_i) = \left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right). \tag{3.2b}$$

We neglected to write down quantum statistics corrections as well as the multi-strange particles, Ξ and Ω^-, as it turns out that our arguments remain valid in this simple approximation. Interactions are effectively included through explicit reference to the baryon spectrum in the spirit of the hadronic statistical bootstrap approach [12, 13] as just discussed. Non-strange hadrons influence the strange fraction by establishing the value of λ_q at given temperature and baryon density.

The fugacities λ_s and λ_q are introduced here to control the strangeness and the baryon number, respectively. While λ_s counts the strange quark content, the up and down quark content is counted by $\lambda_q = \lambda_B^{1/3}$. Usually one replaces the fugacity by a chemical potential,

$$\lambda_q = e^{\mu_q/T} = e^{\mu_B/3T},$$

$$\lambda_B = e^{\mu_B/T} = \lambda_q^3. \tag{3.3}$$

Using the partition function, eq. (3.2), we calculate for given μ_B, T and V, the mean strangeness by evaluating

$$\langle n_s - n_{\bar{s}} \rangle = \lambda_s \frac{\partial}{\partial \lambda_s} \ln Z^{\text{strange}}(T, V, \lambda_s, \lambda_q), \tag{3.4}$$

which is the difference between strange and anti-strange components. This expression must be equal to zero due to the fact that the strangeness is a conserved quantum number with respect to strong interactions. From this condition we get:

$$\lambda_s = \lambda_q \left[\frac{W(x_k) + \lambda_B^{-1}[W(x_\Lambda) + 3W(x_\Sigma)]}{W(x_k) + \lambda_B[W(x_\Lambda) + 3W(x_\Sigma)]}\right]^{1/2} = \gamma\lambda_q. \tag{3.5}$$

We notice a strong dependence of γ on the baryon number. For large μ_B the term with λ_B^{-1} will tend to zero and the term with λ_B will dominate the expression for λ_s and γ. As a consequence the particles with fugacity λ_s and strangeness $S = -1$ are suppressed by a factor γ which is always smaller than one. In the opposite case the production of particles which carry the strangeness $S = +1$ will be favoured by γ^{-1}. This is the consequence of the presence of nuclear matter; for $\mu = 0$ we find $\gamma = 1$.

In order to calculate the mean abundance of strange particles we must introduce for each species its own fugacity, which subsequently must be set equal to one since all different strange particles are in mutual chemical equilibrium by assumption. Using eq. (3.5) we find, from eq. (3.2), the grand canonical partition sum for zero (average) strangeness:

$$\begin{aligned}\ln Z_0^{\text{strange}} = \frac{VT^3}{2\pi^2}\{&2W(x_k)[\gamma\lambda_k + \gamma^{-1}\lambda_{\bar{k}}] \\ &+ 2W(x_\Lambda)[\gamma\lambda_{\rm B}\lambda_\Lambda + \gamma^{-1}\lambda_{\rm B}^{-1}\lambda_{\bar{\Lambda}}] \\ &+ 6W(x_\Sigma)[\gamma\lambda_{\rm B}\lambda_\Sigma + \gamma^{-1}\lambda_{\rm B}^{-1}\lambda_{\bar{\Sigma}}]\}. \end{aligned} \tag{3.6}$$

The strange-particle multiplicities follow from

$$\langle n_i \rangle = \lambda_i \frac{\partial}{\partial\lambda_i} \ln Z_0^{\text{strange}}\Big|_{\lambda_i=1} . \tag{3.7}$$

Explicitly we find

$$\langle n_{K^\mp} \rangle = \frac{VT^3}{2\pi^2}\gamma^{\mp 1} W(x_k) \tag{3.8}$$

$$\langle n_{\Lambda/\bar{\Lambda}} \rangle = \frac{VT^3}{2\pi^2}\gamma^{\pm 1} W(x_\Lambda) e^{\pm\mu_{\rm B}/T} \tag{3.9}$$

and hence the ratio $\langle n_{K^+} \rangle / \langle n_{K^-} \rangle = \gamma^{-2}$, shown in Figure 5 as a function of the baryo-chemical potential $\mu_{\rm B}$ for several temperatures.

We record that this particular particle ratio is a good measure of the baryon chemical potential when temperatures are approximately known. This is due to the competitive processes of associated and direct K^+-production which are quite sensitive to baryon density (chemical potential).

We turn our interest directly to the rarest of all singly strange particles, and show in Figure 6 the ratio $\langle n_{\bar{\Lambda}} \rangle / \langle n_\Lambda \rangle$. We notice an expected suppression of $\bar{\Lambda}$ due to the baryo-chemical potential as well as strangeness chemistry. This ratio exhibits both a strong temperature and $\mu_{\rm B}$ dependence. The remarkably small abundance of $\bar{\Lambda}$, e.g. 10^{-4} Λ, under conditions likely to be reached in an experiment [$T \approx 120$–180 MeV, $\mu_{\rm B} \approx (4$–$6)T$] is characteristic of the nuclear nature of the hot hadronic matter phase under consideration here. Our estimates for quark–gluon plasma based on flavour content are two to three orders of magnitude higher.

In summary to this section we note that K^+/K^- abundance is a sensitive measure of the hadrochemical potential, while the relative abundance of strangeness carrying antibaryons is greatly suppressed in the hadronic gas phase. Hence enhancements observed in nuclear collisions may be useful indications of the reactions leading to the formation of the quark–gluon plasma. The study of multistrange hadrons is in progress.

4. Discussion

Only some selected aspects of the strangeness production in hot hadronic matter have been studied in detail. The results are quite encouraging and suggest interesting future perspectives. It was shown in section 2 that strangeness abundance reaches chemical equilibrium in the plasma. The subsequent depletion of the strangeness during the plasma

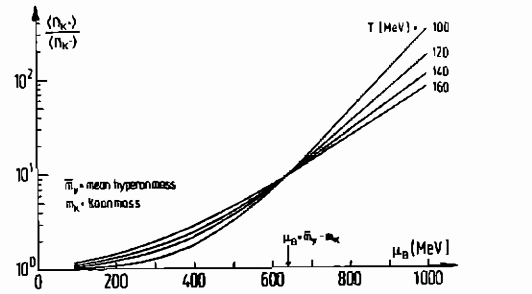

Figure 5 The ratio $\langle n_{K^+} \rangle / \langle n_{K^-} \rangle = \gamma^{-2}$ as function of the baryo-chemical potential for several temperatures.

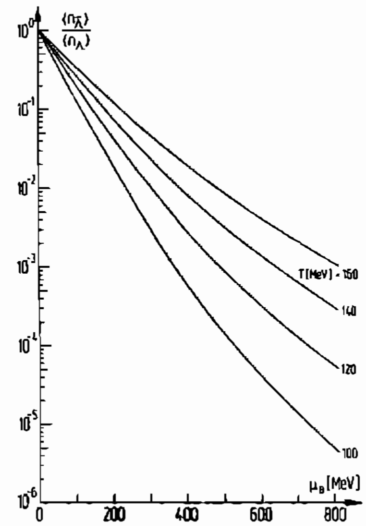

Figure 6 The ratio $\langle n_{\bar{\Lambda}} \rangle / \langle n_\Lambda \rangle$ as function of $\mu_{\rm B}$ for several T.

disintegration as well as its preferred hadronization channels have not yet been studied in detail. However, only if the plasma disintegration is an extremely slow process, lasting in the order of 10^{-22} s, a significant feedback on the high s abundance created at the maximum temperature reached in the collision can be anticipated. As shown in Figure 3, the invariant rates drop quite rapidly with decreasing temperature, leading to a rapid increase of the equilibrium time constant τ; hence the strangeness abundance decouples from the equilibrium and remains a witness of the hot collision period.

While we cannot yet discuss in detail the abundance of multistrange antihadrons, which are influenced also by the possible ss, $\bar{s}\bar{s}$, sss, $\bar{s}\bar{s}\bar{s}$ and $s\bar{s}$ bound states in the plasma, it is apparent from the calculations performed in section 3 that measurement of the production cross-section of the antistrange baryons could already be quite helpful in the observation of the phase transition. The high suppression of these degrees of freedom in the hadronic gas phase for obvious physical reasons is not maintained in the plasma phase, where $\bar{s}$ abundance is larger than $\bar{u}$, $\bar{d}$ abundance, as shown in section 1. Measurement of the relative K^+/K^- yield, while indicative for the value of chemical potential, may carry less specific information about the plasma.

The K/π ratio may indeed also contain relevant information. However, it will be much more difficult to decipher the message. The π abundance will originate from diverse sources needed to be understood for that purpose.

It is more appropriate to concentrate the attention on those reaction channels which will be particularly strongly populated when the quark plasma dissociates into hadrons. Here, in particular, it appears that otherwise quite rare multistrange hadrons will be enhanced, on the one hand by the relative high phase space density of strangeness in the plasma, on the other hand by the attractive ss QCD interaction in the $\bar{3}_c$ and $\bar{s}s$ in 1_c channels. Hence we should search for the rise of the abundance of particles like Ξ, $\bar{\Xi}$, Ω, $\bar{\Omega}$ and ϕ, and perhaps in highly strange pieces of baryonic matter, rather than in the *K*-channels. It seems that such experiments would uniquely determine the existence of the phase transition to the quark–gluon plasma.

It is important to appreciate that the experiments discussed above would certainly be complementary to the measurement with the help of electromagnetically interacting probes, e.g. dileptons or direct photons. Strangeness-based measurements have the advantage that they are based on the observation of a strongly interacting particle (s,$\bar{s}$ quark) originating from the hot plasma phase; these are much more abundant than the electromagnetic particles.

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