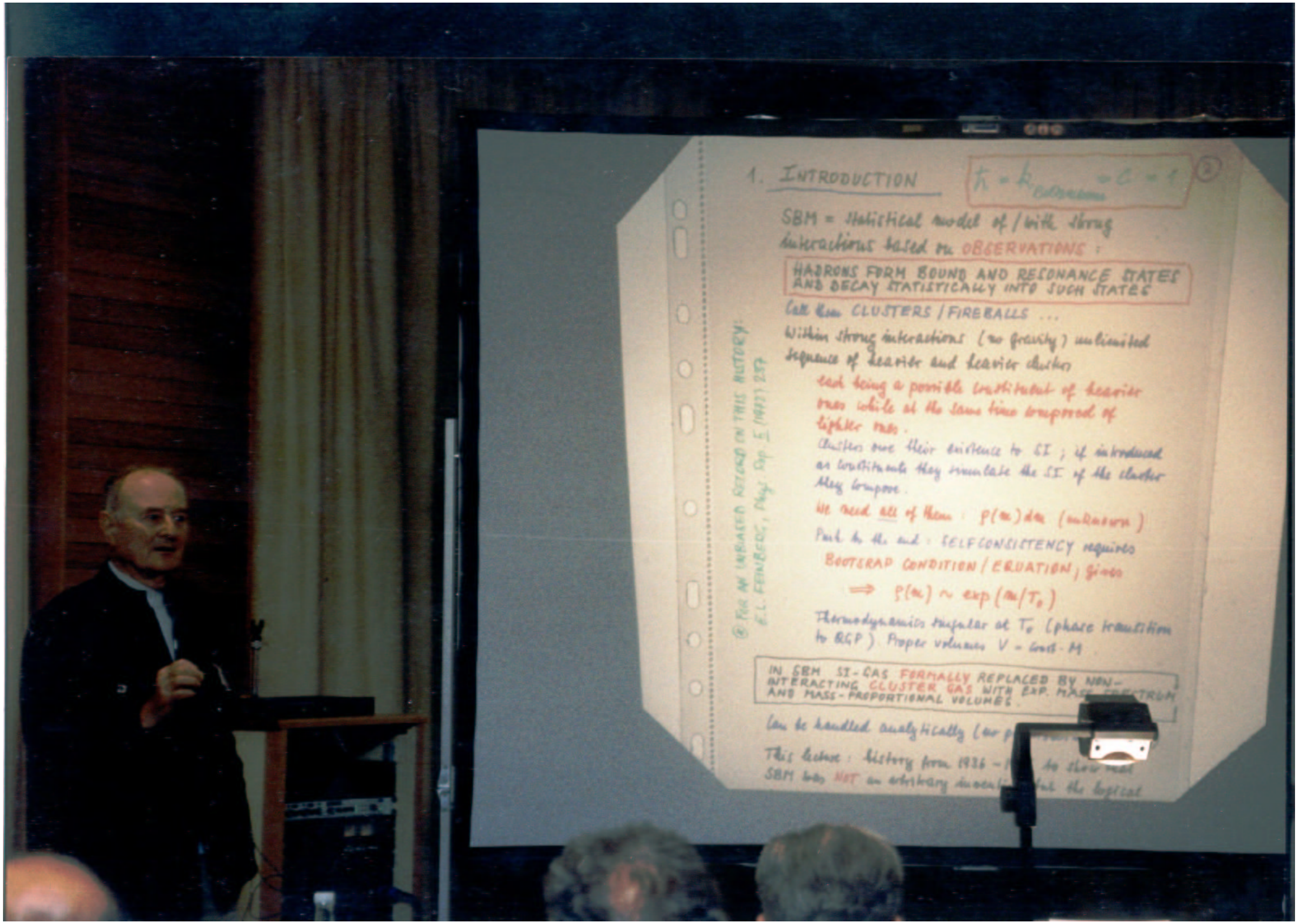
Rolf Hagedorn lecturing 1994 at Divonne: First Transparency

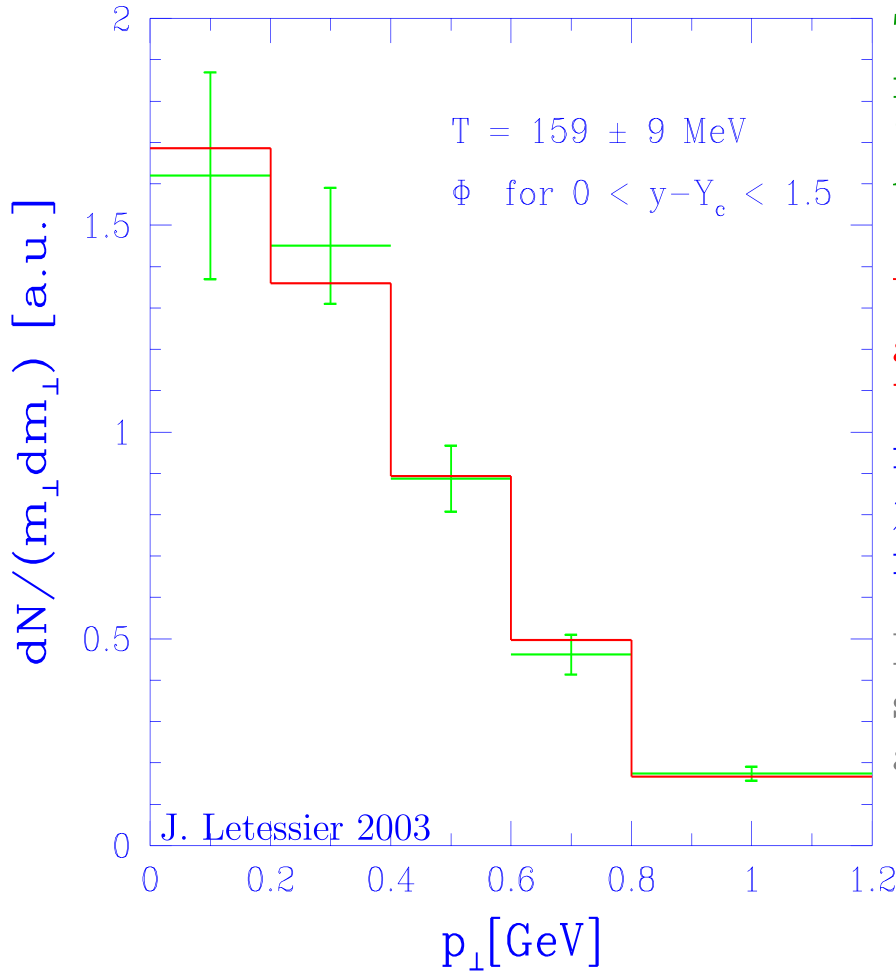


Nuclear Matter at Hagedorn Temperature

Rolf Hagedorn at CERN 1977

Thermal Hadron Spectra and Limiting Temperature

Multihadron production in *pp* reactions produced spectra with exponential fall-off as function of transverse momenta. The exponential (inverse) slope was the same for all particles, $T_{\rm H} \simeq 160$ MeV, and it did not change as the collision energy varied. Rolf Hagedorn (RH) recognized behavior akin to particle evaporation from boiling elementary matter. In the boiling process a maximum temperature cannot be exceeded even if we provide more and more heating.



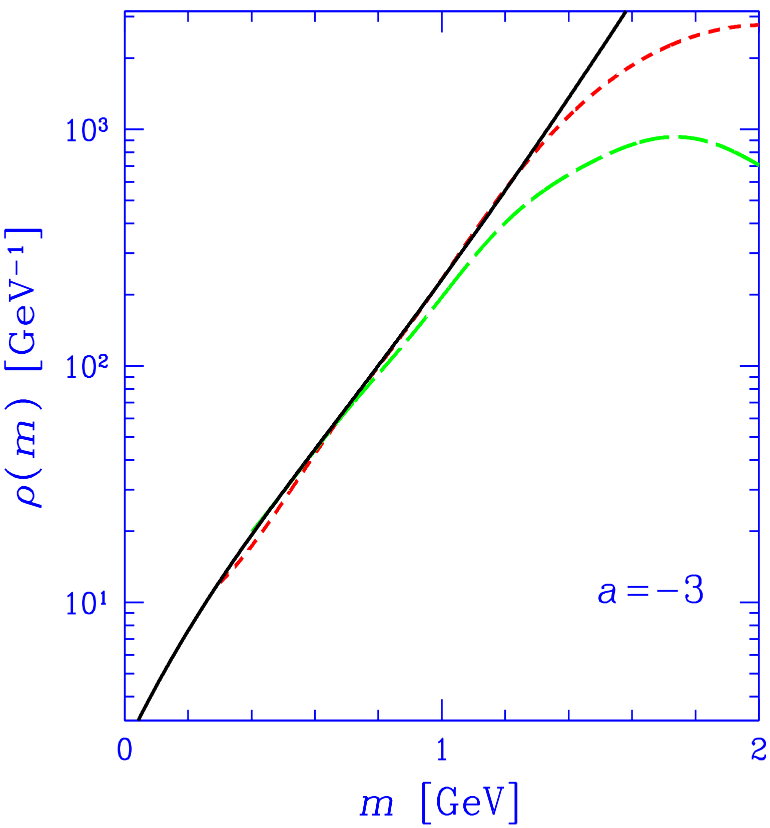
The spectrum here shown is for the recently (2000) measured NA49 $\Phi(s\bar{s})$ in 158 GeV ($\sqrt{s_{NN}} = 17.2$ GeV) pp reactions. 95% of $\Phi(s\bar{s})$ is directly produced (as opposed to being a decay product). We fit a Boltzmann thermal shape allowing for the cuts in rapidity and binning in $p_\perp$. There are 5 data points, fit parameters are: normalization and T. $\chi^2|dof = 1.2|3$ as required for high confidence fit with a few degrees of freedom.

In AA collisions we have greater slopes since the compressed bulk matter flows apart, use Doppler formula

$$T_{\rm slope} \simeq T\sqrt{\frac{1+v}{1-v}}$$

Exponential Hadron Mass Spectrum

RH discovered that the exponential growth of the hadronic mass spectrum could lead to an understanding of the limiting hadron temperature $T_{\rm H} \simeq 160$ MeV,



The solid line is the fit:

$$\rho(m) \approx c(m_a^2 + m^2)^{a/2} \exp(m/T_{\rm H})$$

with $a = -3$, $m_a = 0.66$ GeV, $T_{\rm H} = 0.158$ GeV.
Long-dashed line: 1411 states of 1967.
Short-dashed line: 4627 states of 1996.

Experimental lines include Gaussian smoothing:

$$\rho(m) = \sum_{m^*=m_\pi, m_\rho, \ldots} \frac{g_{m^*}}{\sqrt{2\pi}\sigma_{m^*}} \exp\left(-\frac{(m-m^*)^2}{2\sigma_{m^*}^2}\right).$$

$\sigma = \Gamma/2$, $\Gamma = \mathcal{O}(200)$ MeV is the assumed width of the resonance, excluding the 'stable' pion, a special case.

Note the missing resonances at $m > 1.4 GeV$.
The newly discovered 'pentaquark' resonances needed to fill this gap.

Limiting Hagedorn Temperature

A gas of hadrons with exponentially rising mass spectrum:

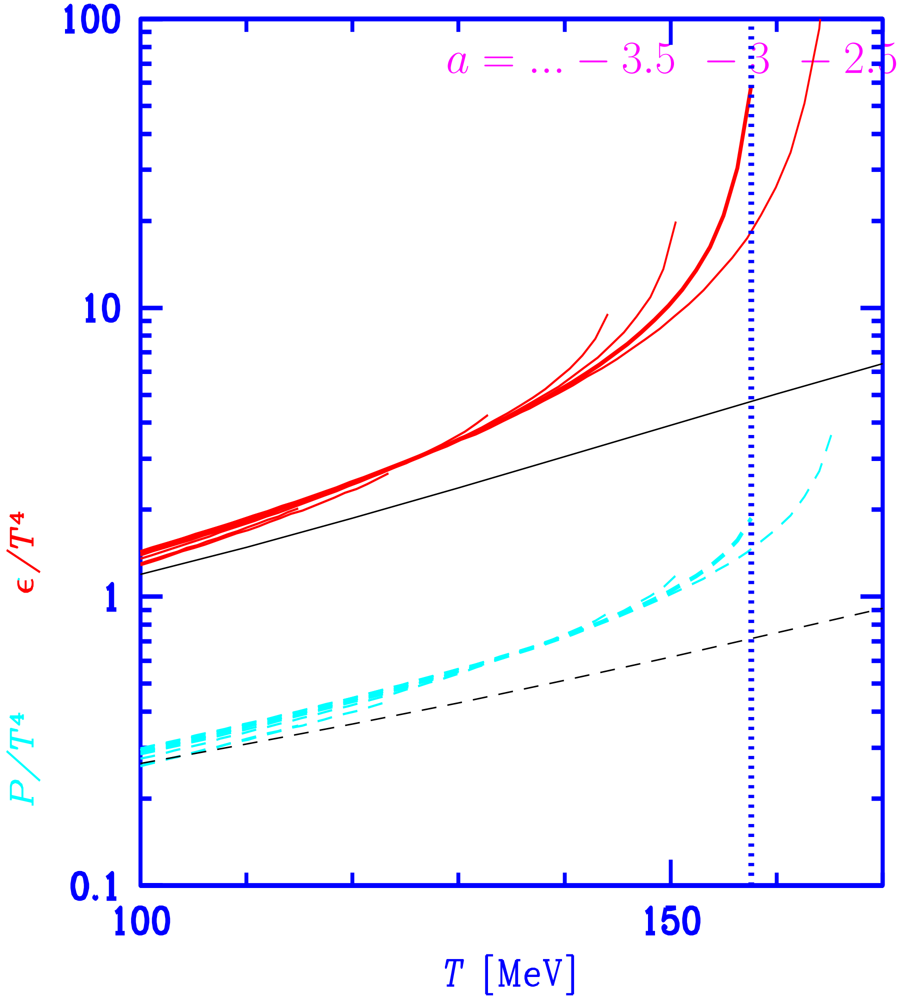
$$\ln \mathcal{Z}_{\rm HG}^{\rm cl} = cV\left(\frac{T}{2\pi}\right)^{3/2}\int_M^\infty m^a e^{m/T_{\rm H}} m^{3/2} e^{-m/T}\, dm + D(T,M),$$

Cutoff $M > m_a > T_{\rm H}$ is arbitrary, its role is to separate off $D(T,M) < \infty$. Because of the exponential factor, the first integral can be divergent for $T > T_{\rm H}$, and the partition function is singular for $T \to T_{\rm H}$ for a range of a:

$$P(T) \to \begin{cases} \left(\frac{1}{T} - \frac{1}{T_{\rm H}}\right)^{-(a+5/2)}, & \textbf{for} \quad a > -\frac{5}{2}, \\ \ln\left(\frac{1}{T} - \frac{1}{T_{\rm H}}\right), & \textbf{for} \quad a = -\frac{5}{2}, \\ \textbf{constant}, & \textbf{for} \quad a < -\frac{5}{2}; \end{cases} \qquad \epsilon \to \begin{cases} \left(\frac{1}{T} - \frac{1}{T_{\rm H}}\right)^{-(a+7/2)}, & \textbf{for} \quad a > -\frac{7}{2}, \\ \ln\left(\frac{1}{T} - \frac{1}{T_{\rm H}}\right), & \textbf{for} \quad a = -\frac{7}{2}, \\ \textbf{constant}, & \textbf{for} \quad a < -\frac{7}{2}. \end{cases}$$

The energy density ϵ goes to infinity for $a \geq -\frac{7}{2}$, when $T \to T_{\rm H}$.

Mass spectrum slope $T_{\rm H}$ appears as the limiting Hagedorn temperature beyond which we cannot heat a system which can have an infinite energy density. The partition function can be singular even when $V < \infty$.



energy density

$$\epsilon = -\frac{\partial}{\partial \beta} \ln \mathcal{Z}(\beta, V, \lambda)$$

exponentially growing mass spectrum

pressure

$$P = \frac{\partial}{\partial V} \ln \mathcal{Z}(\beta, V, \lambda)$$

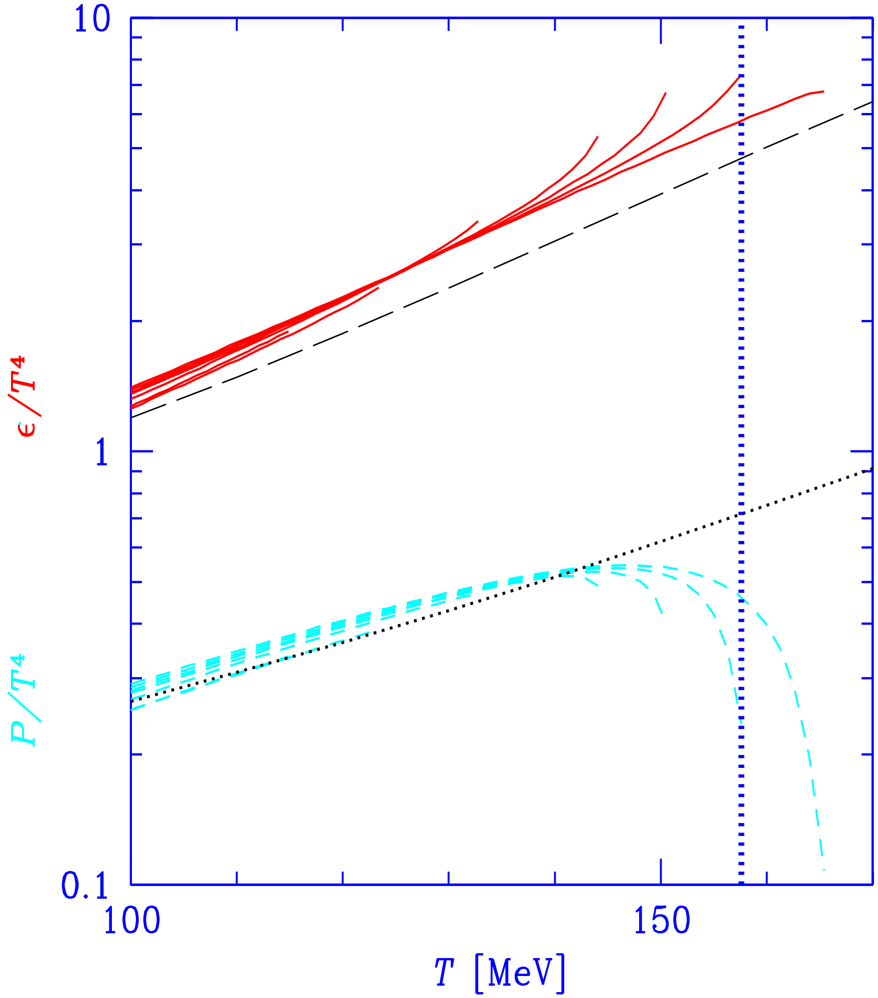
exponentially growing mass spectrum

Straight lines: P, ϵ based on actually known mass spectrum.

This is for point hadrons, year was 1965. Understanding of quark structure of hadrons implies their finite. This is important here given the exponential growth of mass spectrum which overfills a fixed volume with (different) particles.

Finite Volume Hadron Gas Model

Point hadron gas in free available volume Δ to have the properties of finite size hadron gas in total mean volume $\langle V\rangle$ (RH 1978+)



$$\ln \mathcal{Z}_{\rm pt}(T,\Delta,\lambda) \equiv \ln \mathcal{Z}(T,\langle V\rangle,\lambda)$$

Proper particle volume in the rest frame is assumed to be proportional to mass. For a gas of moving hadrons, in gas rest-frame: $\langle V\rangle = \Delta + \langle E\rangle/4\mathcal{B}$.

$$\langle E\rangle = \langle V\rangle \epsilon(\beta,\lambda) = -\frac{\partial}{\partial\beta}\ln \mathcal{Z}(\beta,\langle V\rangle,\lambda) =$$

$$= -\frac{\partial}{\partial\beta}\ln \mathcal{Z}_{\rm pt}(\beta,\Delta,\lambda) = \Delta\epsilon_{\rm pt}(\beta,\lambda)$$

$$\langle V\rangle = \Delta\big(1+\epsilon_{\rm pt}(\beta,\lambda)/4\mathcal{B}\big),$$

$$\frac{\langle E\rangle}{\langle V\rangle} \equiv \epsilon(\beta,\lambda) = \frac{\epsilon_{\rm pt}(\beta,\lambda)}{1+\epsilon_{\rm pt}(\beta,\lambda)/(4\mathcal{B})},$$

$$P = \frac{P_{\rm pt}(\beta,\lambda)}{1+\epsilon_{\rm pt}(\beta,\lambda)/4\mathcal{B}}.$$

The gas of finite size hadrons with exponential mass spectrum has nearly the same properties as a gas of point hadrons with today experimentally observed mass spectrum. That is why 'statistical hadronization works'.

Statistical Bootstrap Model (SBM)

Rolf Hagedorn constructed a theoretical model *Statistical Bootstrap* **in which the exponentially rising hadron mass spectrum naturally occurred.**

STATISTICAL THERMODYNAMICS OF STRONG INTERACTIONS AT HIGH-ENERGIES.
Nuovo Cim. Supp. 3 (2): 147 (1965)
citation classic

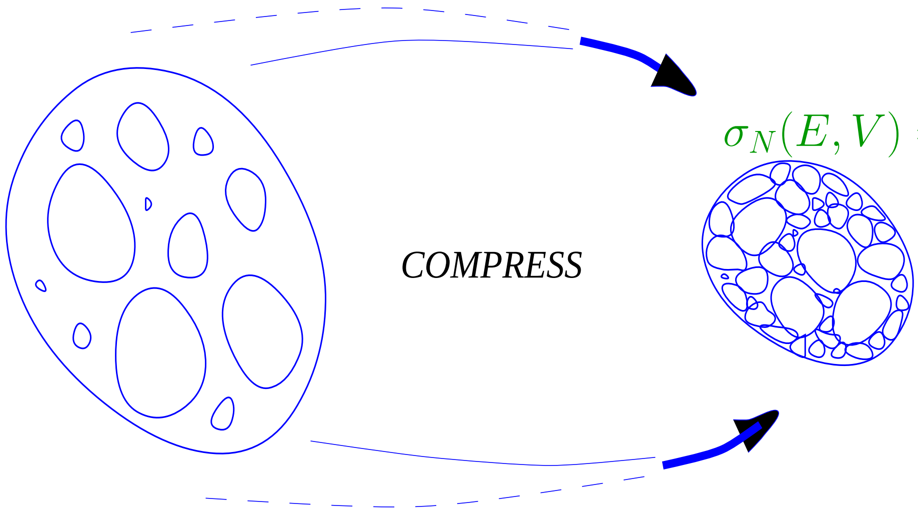
ON HADRONIC MASS SPECTRUM
Nuovo Cim. A 52 (4): 1336 (1967)

HADRONIC MATTER NEAR BOILING POINT
Nuovo Cim. A 56 (4): 1027 (1968)

75th Birthday, Divonne 1994

Statistical Bootstrap Equation

Resonant scattering comprises the essence of strong interaction and implies that there is clustering of hadrons into new hadrons. What happens if we compress? We look at N-particle level density,



$$\sigma_N(E,V) = \frac{1}{N!}\prod_{i=1}^{N}\int \frac{gV}{(2\pi)^3}\delta\Big(\sum_{i=1}^{N}\varepsilon_i - E\Big)\delta^3\Big(\sum_{i=1}^{N}\vec{p}_i\Big)d^3p_i,$$

Mix gases with different masses m e.g. $g \rightarrow \int \rho(m)dm$, with $\varepsilon_i = \sqrt{m^2+p_i^2}$. We sum over N allowing all microcanonical configurations which preserve the imposed constraints. We apply the Bootstrap hypothesis:

Macroscopic volume *V*

Natural cluster volume V_c (*m*,*b*)

$$\boxed{\sigma(E,V_c) \longrightarrow \rho(m)}$$

In covariant notation we find the Bootstrap (SBM) equation for ρ:

$$\mathcal{H}\rho(p^2) = \mathcal{H}\delta_0(m^2 - m_{\rm in}^2) + \sum_{N=2}^{\infty}\frac{1}{N!}\int \delta^4\Big(p - \sum_{i=1}^{N} p_i\Big)\prod_{i=2}^{N}\mathcal{H}\rho(p_i^2)d^4p_i,$$

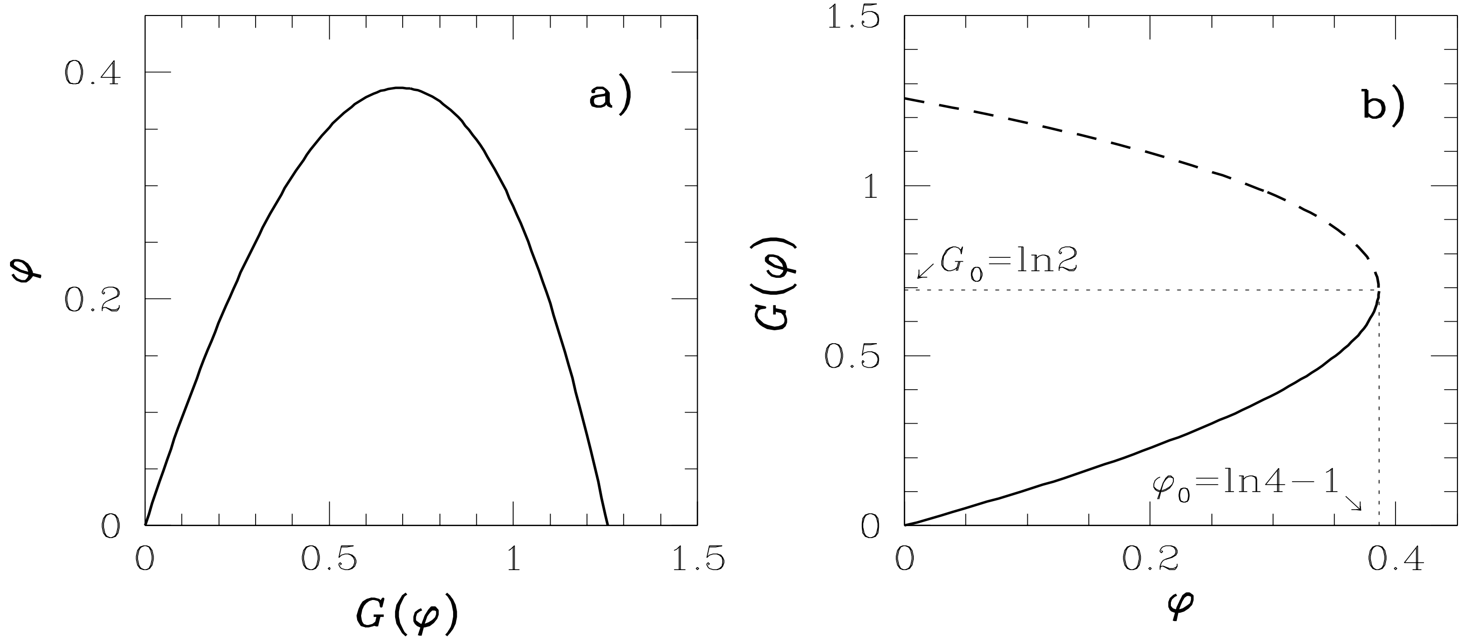
This is a nonlinear inhomogeneous integral equation. $\mathcal{H}$ to be fixed by experiment, related to proper volume V_c. For $m_{\rm in} \rightarrow m_\pi$, the lightest hadron, the pion generates higher hadron clusters. Further below we include baryons.

Singularity of the Bootstrap Equation

$$G \equiv \int e^{-\beta\cdot p}\,\mathcal{H}\rho(p^2)\,d^4p, \qquad \varphi(\beta) = \int e^{-\beta\cdot p}\,\mathcal{H}\delta_0(p^2 - m_{\rm in}^2)d^4p = \mathcal{H}2\pi m_{\rm in}^2\frac{K_1(\beta m_{\rm in})}{\beta m_{\rm in}},$$

Laplace Transform of the SBM equation yields $\boxed{G(\varphi) = \varphi + e^{G(\varphi)} - G(\varphi)}$

or $\varphi = 2G(\varphi) - e^{G(\varphi)} + 1$**.**



For $\varphi \to \varphi_0$**,** $G(\varphi) \approx \ln 2 \pm$ **constant** $\times \sqrt{\varphi_0 - \varphi} + \cdots$. **and since** φ **regular at** φ_0 **we have** $G(\beta \simeq \beta_0) = \ln 2 - \text{constant} \times \sqrt{\beta - \beta_0}$**. Such a singularity can only exist if**

$$\boxed{\rho \propto m^{-3} e^{m\beta_0} \qquad \beta_0 = 1/T_{\rm H}.}$$

Statistical Bootstrap with Mesons and Baryons

Each hadron 'i' has now baryon number b_i and we have the constraint that the baryon number in all partitions has to add up to b. The SBM equation becomes:

$$\mathcal{H}\tau(p^2,b)=\mathcal{H}g_{\rm b}\delta_0(p^2-m_{\rm b}^2)+\sum_{N=2}^{\infty}\frac{1}{N!}\int\delta^4\!\left(p-\sum_{i=1}^{N}p_i\right)\sum_{\{b_i\}}\delta_{\rm K}\!\left(b-\sum_{i=1}^{N}b_i\right)\prod_{i=1}^{N}\mathcal{H}\tau(p_i^2,b_i)\,d^4p_i,$$

The generalized Laplace transform method works, with :

$$\varphi(\beta,\lambda)\equiv\int e^{-\beta\cdot p}\sum_{b=-\infty}^{\infty}\lambda^b\mathcal{H}g_{\rm b}\delta_0(p^2-m_{\rm b}^2)\,d^4p=2\pi\mathcal{H}\sum_{b=-\infty}^{\infty}\lambda^b g_{\rm b}m_{\rm b}^2\frac{K_1(m_{\rm b}\beta)}{m_{\rm b}\beta},$$

$$\Phi(\beta,\lambda)\equiv\int e^{-\beta\cdot p}\sum_{b=-\infty}^{\infty}\lambda^b\mathcal{H}\tau(p^2,b)\,d^4p\quad\text{implies}\quad\Phi(\beta,\lambda)=\varphi(\beta,\lambda)+e^{\Phi(\beta,\lambda)}-\Phi(\beta,\lambda)-1.$$

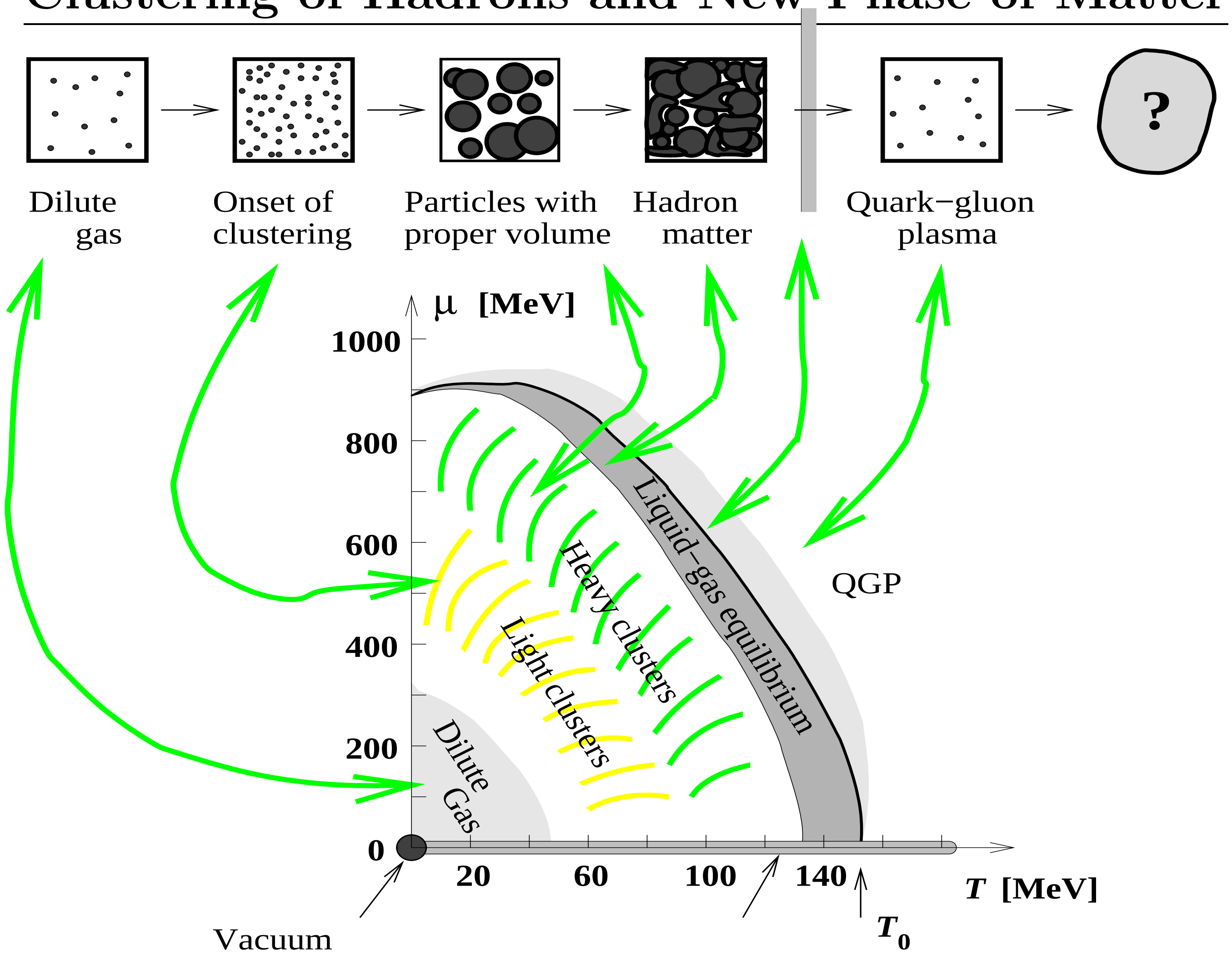
The singularity point becomes a line $\beta_{\rm cr}(\lambda_{\rm cr})$ derived from:

$$\ln(4/e)=\varphi(\beta_{\rm cr}=1/T_{\rm cr},\lambda_{\rm cr}=e^{\mu_{\rm b}^{\rm cr}/T_{\rm cr}})=2\pi\mathcal{H}T_{\rm cr}\left[3m_\pi K_1\!\left(\frac{m_\pi}{T_{\rm cr}}\right)+4\left(\lambda_{\rm cr}+\frac{1}{\lambda_{\rm cr}}\right)m_{\rm N}K_1\!\left(\frac{m_{\rm N}}{T_{\rm cr}}\right)\right].$$

This condition defines a critical curve $\mu_{\rm b}^{\rm cr}=f(T_{\rm cr})$ in the $T,\mu_{\rm b}$ plane. The value of $\mathcal{H}$ is fixed by the critical temperature $T_{\rm H}=160$ MeV associated with $\mu_{\rm b}^{\rm cr}=0$. Below the Hagedorn temperature we have matter made of baryons and mesons.

Approaching this critical curve for finite volume hadrons pressure vanishes and all hadrons combined to one only hadronic cluster.

Clustering of Hadrons and New Phase of Matter



At the critical curve for finite volume hadron pressure vanishes and all hadrons combine to one only hadronic cluster. This drop of matter is made of hadron constituents. The Hagedorn Temperature has become the condition in which the internal degrees of freedom of hadrons (quarks and gluons) become dynamical.

Hagedorn Temperature is:

1. **The intrinsic temperature at which hadronic particles are formed, in pp interactions seen as the inverse slope of hadron spectra.**
2. **This boiling point of hadrons which is the (inverse) slope of exponentially rising hadron mass spectrum.**
3. **The boundary value of temperature at which finite size hadrons coalesces into one cluster consisting of a new phase comprising hadron constituents.**

Statistical Bootstrap Model is:

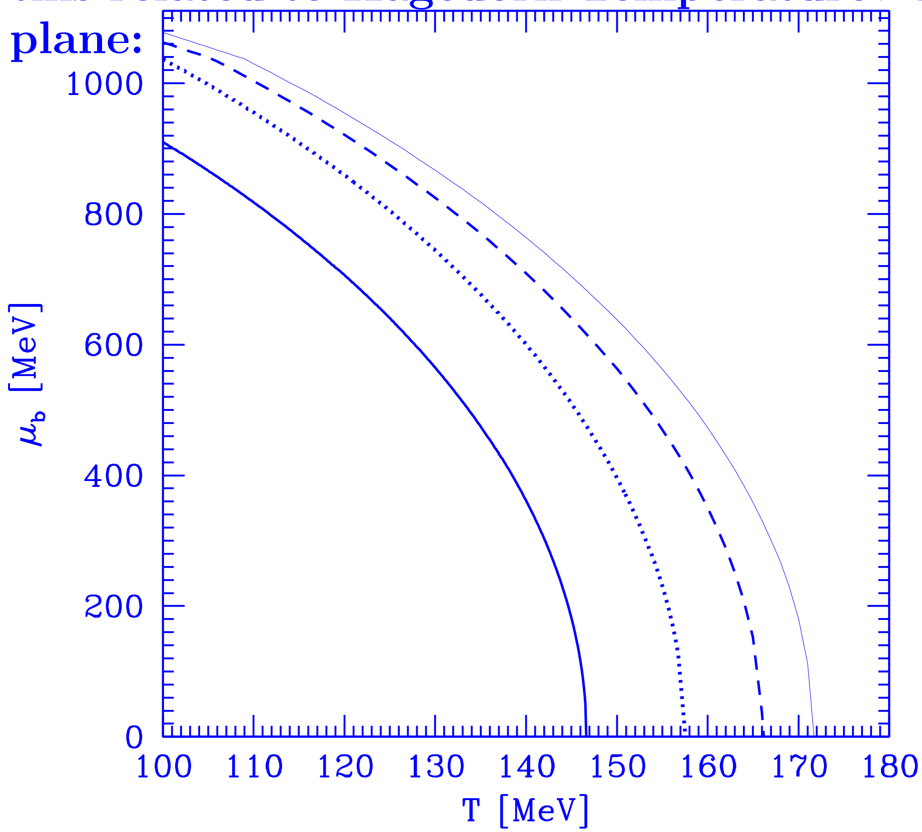
1. **A connection between hadronic particle momentum distribution and properties of hadronic interactions dominated by resonant scattering, and exponentially rising mass spectrum.**
2. **A theoretical framework for study of the properties of the equations of state of dense and hot baryonic matter (nuclear matter at finite temperature).**
3. **It is not a fundamental dynamical theory, in fact SBM is to be motivated today in terms of properties of the fundamental dynamical approach (QCD) (next talk).**

Measurement of Hagedorn Temperature

Particle spectra can be greatly distorted by post-formation reactions (resonance decays, rescattering) and even more by fast dynamical expansion present in collisions of large atomic nuclei. Hadron yields are less model dependent.

Use particle yields from boiling hadronic matter

We seek to determine the condition of chemical freeze-out for $\mu_b^{\rm cr} \to 0$. How is this related to Hagedorn Temperature? Look at the phase boundary in the μ_b, T plane:



For a system at rest:
point hadrons (solid thin line)
finite volume hadrons (dashed line)

With collective matter flow
finite size hadrons
$v_c = 0.32$, $\kappa = 0.6$ (dotted line)
$v = 0.54$, $\kappa = 0.6$ (thick solid line)
[based on PRL85, 4695 (2000)].

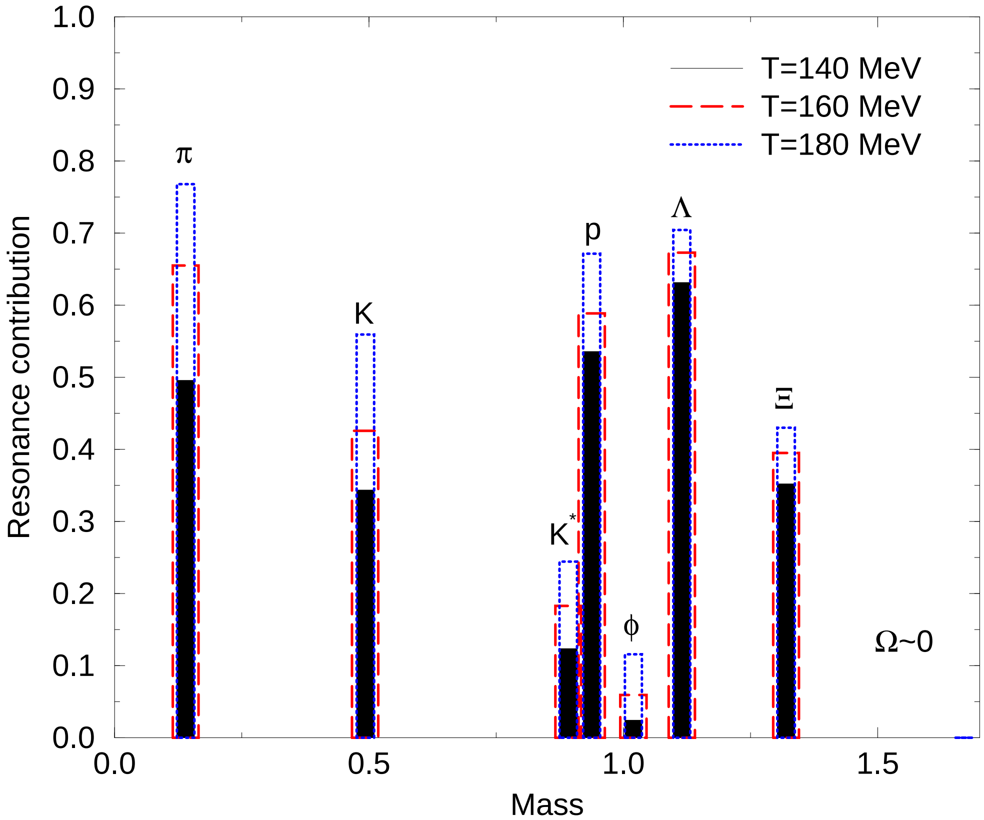
A global fit to all particle yields offers insight about both dynamics of hadronization and the value of Hagedorn Temperature.

STATISTICAL HADRONIZATION

Hypothesis (Fermi, Hagedorn): particle production can be described by evaluating the accessible phase space.

Verification of statistical hadronization:

Particle yields with same valance quark content are in relative chemical equilibrium, e.g. the relative yield of $\Delta(1230)/N$ as of K^*/K, $\Sigma^*(1385)/\Lambda$, etc, is controlled by chemical freeze-out i.e. Hagedorn Temperature $T_{\rm H}$:



$$\frac{N^*}{N}=\frac{g^*(m^*T_{\rm H})^{3/2}e^{-m^*/T_{\rm H}}}{g(mT_{\rm H})^{3/2}e^{-m/T_{\rm H}}}$$

Resonances decay rapidly into 'stable' hadrons and dominate the yield of most stable hadronic particles.

Resonance yields test statistical hadronization principles.

Resonances reconstructed by invariant mass; important to consider potential for loss of observability.

HADRONIZATION GLOBAL FIT:→

SPS 160/80/40 GeV Pb beam on Pb target

The chemical freeze-out statistical parameters (4π yields of NA49)

nonequilibrium (left) **semi equilibrium $\gamma_q = 1$ (right)**

$E_{\rm Lab}/A$ [GeV]	158	80	40	158	80	40
$\sqrt{s_{NN}}$ [GeV]	17.2	12.3	8.75	17.2	12.3	8.75
$T_{\rm H}$ [MeV]	135 ± 3	135 ± 3	133 ± 2	157 ± 4	156 ± 4	154 ± 3
λ_q	1.69(5)	1.98(6)	2.56(6)	1.74(5)	2.03(7)	2.69(8)
λ_s	1.23*	1.27*	1.31*	1.20*	1.24*	1.24*
γ_q	1.68*	1.68*	1.69*	1*	1*	1*
γ_s/γ_q	0.91(6)	0.83(4)	0.85(6)	0.66(4)	0.60(4)	0.67(5)
χ^2/dof	11.4/6	4.3/2	2.3/4	23/7	8.9/3	4.0/5

The star (*) indicates for λ_s that it is a value resulting from strangeness conservation constraint. For γ_q that there is an upper limit to which the value converged, $\gamma_q^2 < e^{m_\pi/T}$ (on left), or that the value of $\gamma_q = 1$ is set (on right).

Defer discussion of chemical parameters λ_i, γ_i)

ion to strangeness. Thus, assuming equilibrium in the quark plasma, we find the density of the strange quarks to be (two spins and three colours):

$$\frac{s}{V} = \frac{\bar{s}}{V} = 6\int \frac{d^3p}{(2\pi)^3}\, e^{-\sqrt{p^2+m_s^2}/T} = 3\,\frac{Tm_s^2}{\pi^2}\, K_2\left(\frac{m_s}{T}\right) \qquad (26)$$

(neglecting, for the time being, the perturbative corrections and, of course, ignoring weak decays). As the mass of the strange quarks, m_s, in the perturbative vacuum is believed to be of the order of 280 - 300 MeV, the assumption of equilibrium for $m_s/T \sim 2$ may indeed be correct. In Eq. (26) we were able to use the Boltzmann distribution again, as the density of strangeness is relatively low. Similarly, there is a certain light antiquark density ($\bar{q}$ stands for either $\bar{u}$ or $\bar{d}$):

$$\frac{\bar{q}}{V} \simeq 6\int \frac{d^3p}{(2\pi)^3}\, e^{-|p|/T-\mu_q/T} = e^{-\mu_q/T}\cdot T^3\,\frac{6}{\pi^2} \qquad (27)$$

where the quark chemical potential is, as given by Eq. (3) $\mu_q = \mu/3$. This exponent suppresses the $q\bar{q}$ pair production as only for energies higher than μ_q is there a large number of empty states available for the q.

What we intend to show is that there are many more $\bar{s}$ quarks than antiquarks of each light flavour. Indeed:

$$\frac{\bar{s}}{\bar{q}} = \frac{1}{2}\left(\frac{m_s}{T}\right)^2 K_2\left(\frac{m_s}{T}\right) e^{\mu/3T} \qquad (28)$$

The function $x^2K_2(x)$ is, for example, tabulated in Ref. 15). For $x = m_s/T$ between 1.5 and 2, it varies between 1.3 and 1. Thus, we almost always have more $\bar{s}$ than $\bar{q}$ quarks and, in many cases of interest, $\bar{s}/\bar{q} \sim 5$. As $\mu \to 0$ there are about as many $\bar{u}$ and $\bar{q}$ quarks as there are $\bar{s}$ quarks.

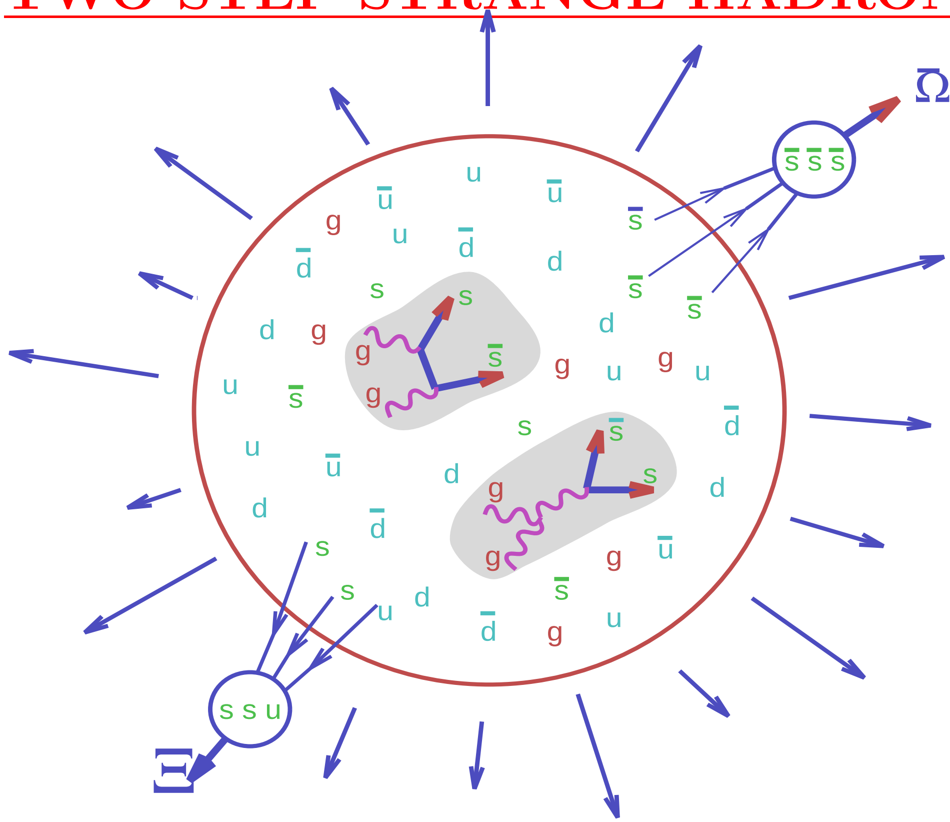
When the quark matter dissociates into hadrons, some of the numerous $\bar{s}$ may, instead of being bound in a $q\bar{s}$ Kaon, enter into a ($\bar{q}\bar{q}\bar{s}$) antibaryon and, in particular, a $\bar{\Lambda}$ or $\bar{\Sigma}^0$. The probability for this process seems to be comparable to the similar one for the production of antinucleons by the antiquarks present in the plasma.

Strangeness

First mention of strange particle production as probe of nuclear matter at Hagedorn temperature and as signature of QGP phase transition appears in the CERN Theory preprint CERN-TH-2969 of October 1980 (Hagedorn et al.). Both strangeness enhancement, and strange antibaryons are discussed as signatures of deconfined QGP phase.

Enhancement of chemical equilibrium $\bar{s}/\bar{q} \propto K^+/\pi^+$ yield is noted.

TWO STEP STRANGE HADRON FORMATION MECHANISM

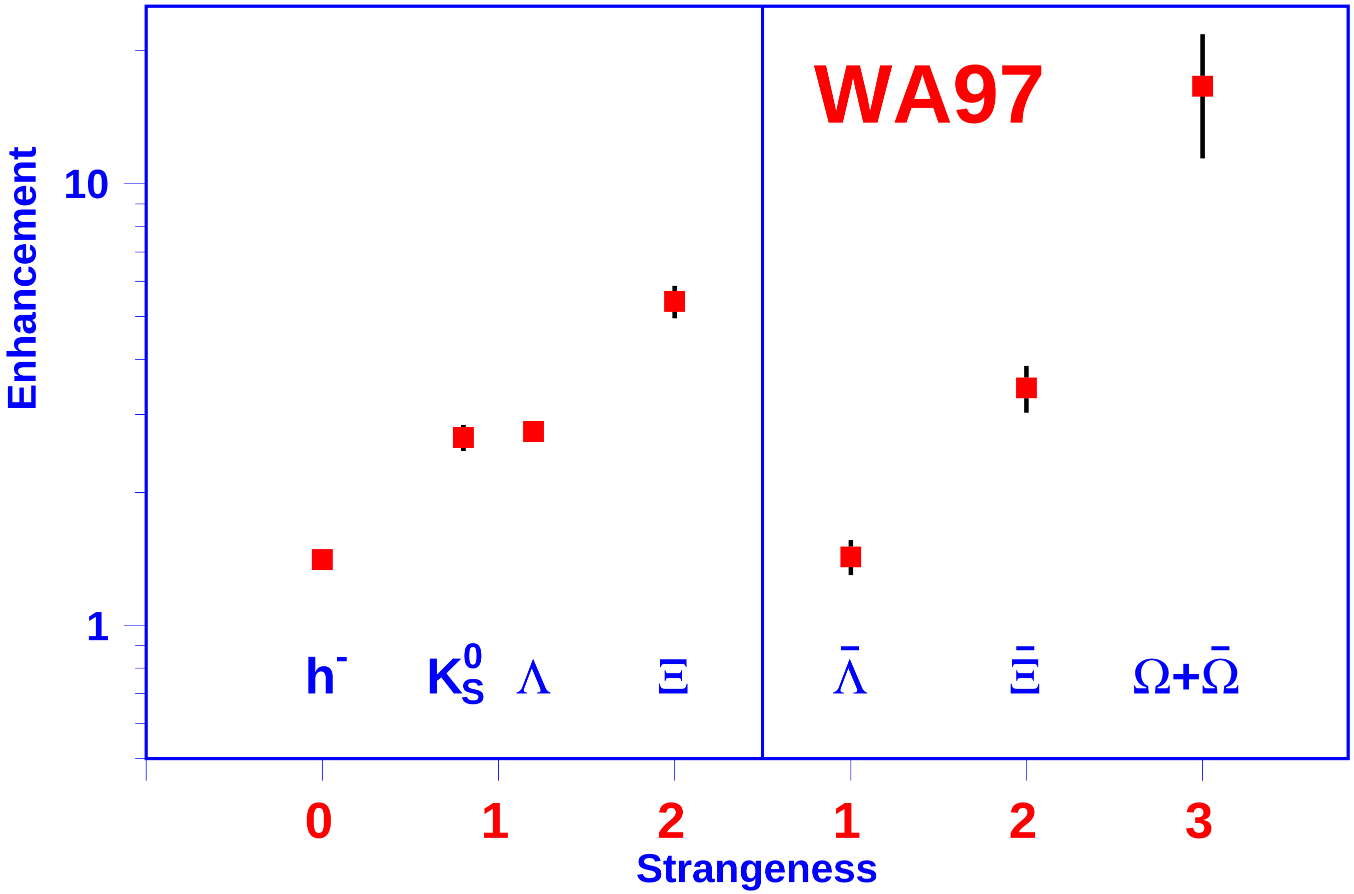


1. $gg \rightarrow s\bar{s}$ **in QG-plasma**
2. **hadronization of pre-formed** s, $\bar{s}$ **quarks**

Formation of complex rarely produced multi strange (anti)particles form QGP enabled by 'cross talk' between s, $\bar{s}$ **quarks made in different microscopic reactions; this is signature of deconfinement. Enhancement of strange antibaryons progressing with strangeness content.**

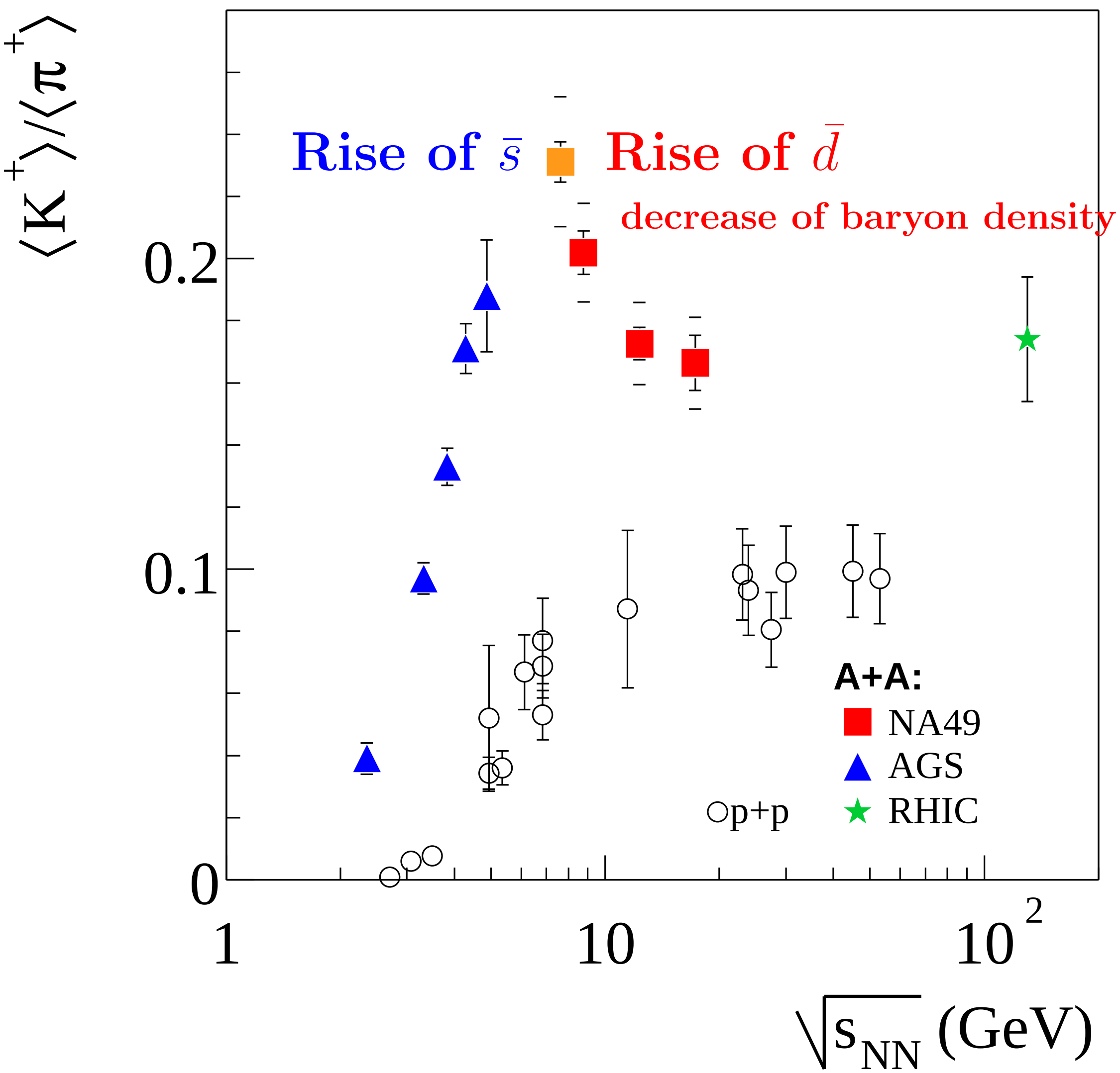
EXPERIMENTAL confirmation at CERN by WA97/NA57

MULTISTRANGE HYPERON ENHANCEMENT



Results of WA97/NA57 collaboration. Enhancement GROWS with a) strangeness b) antiquark content as predicted. Enhancement is defined with respect to the yield in p–Be collisions, scaled up with the number of 'wounded' nucleons.

$\bar{s}/\bar{d}$ ENHANCEMENT

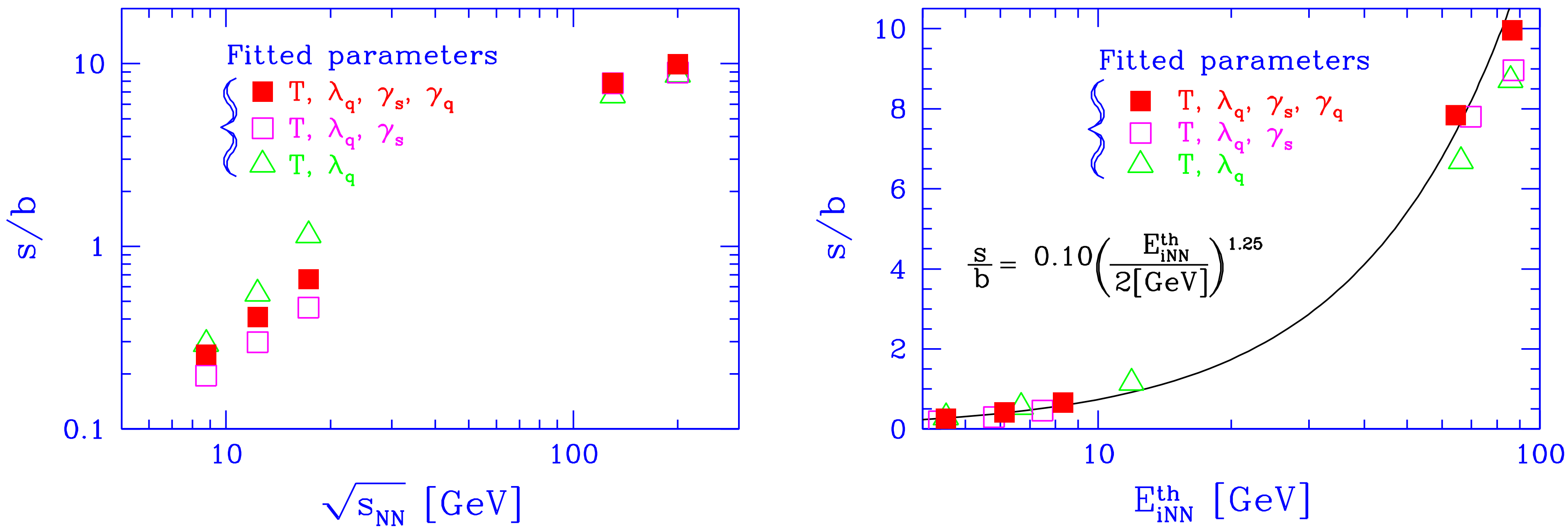


NA49 – Marek Gaździcki

Enhancement: specific strangeness / 'net' baryon

Compare SPS and RHIC results

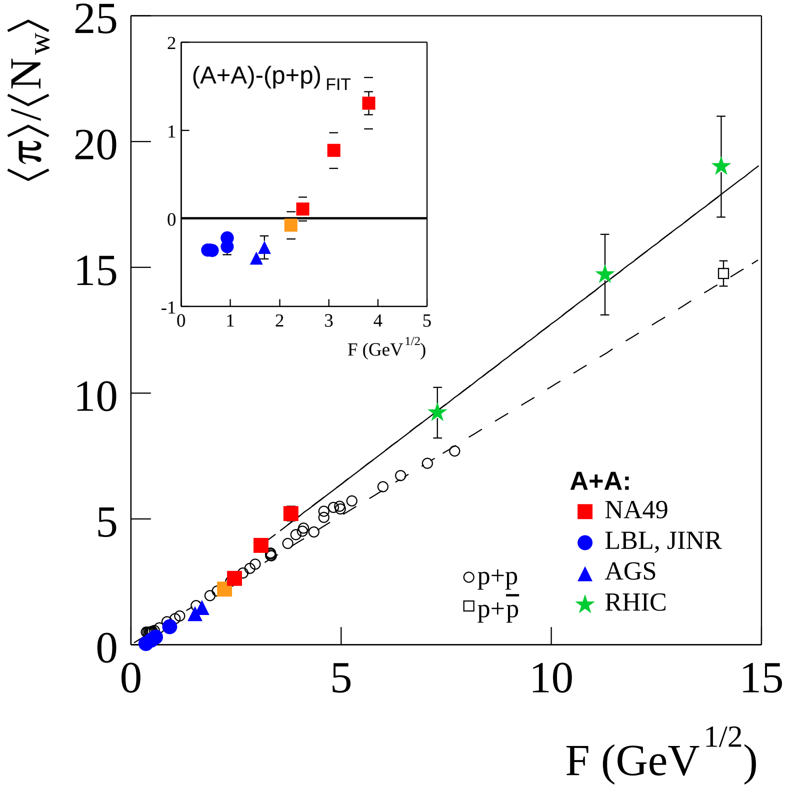
We use statistical hadronization to fix the hadronization conditions under three different chemical equilibrium scenarios. We evaluate the properties of the phase space at given parameter set, here in particular strangeness s, baryon number b and also local thermal energy content per nucleon pair, $E^{\rm th}_{\rm iNN}$.

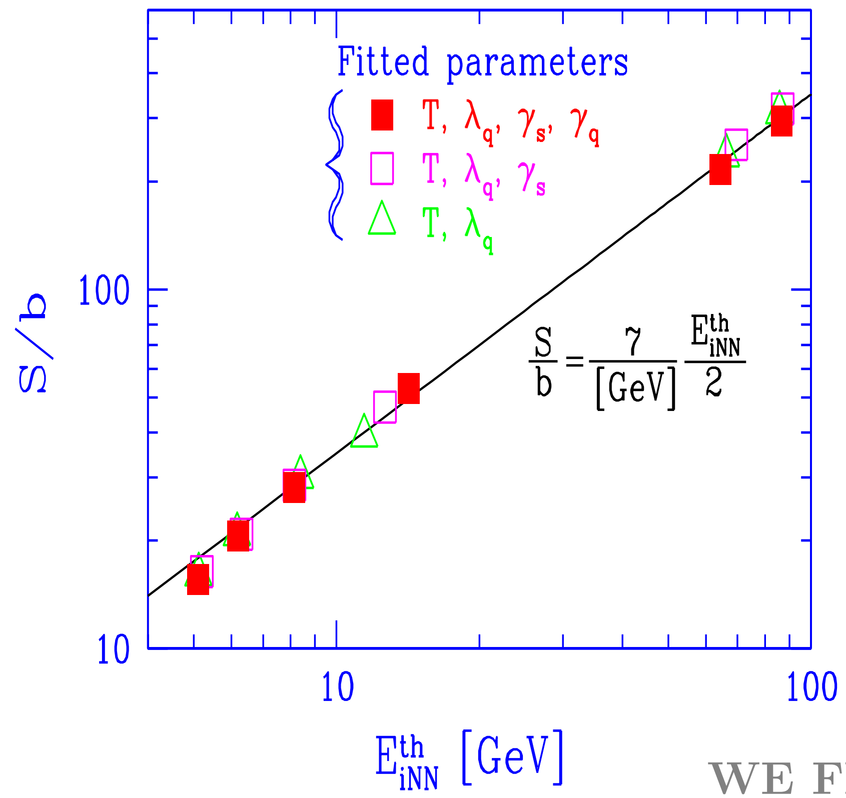


Strangeness per thermal baryon participating in the reaction grows rapidly (by an order of magnitude) and continuously from SPS to RHIC.

Specific strangeness yield appears to be significantly greater (by factor 5?) at RHIC compared to NN-reactions. We do not yet know precisely by how much, RHIC needs base-line experimental data.

Entropy Production at SPS and RHIC





WE FIND:

NA49–Marek Gaździcki: two different specific pion multiplicity slopes for: *pp*, AGS *AA*, and SPS,RHIC *AA* linear rise as function of ROOT of Fermi intrinsic fireball energy.

ENTROPY PRODUCTION per unit of available energy is a Const.

Both representations indicate begin of the onset of a new behavior near 40GeV

Entropy (like before strangeness) is in AA interactions smooth across SPS/RHIC energy range.NEW STATE OF MATTER AT CERN IS THE SAME AS AT RHIC

Influence

Before Hagedorn's period 1964-84, statistical method in study of hadronic interactions were not widely accepted, Fermi hadronization model was slowly falling into oblivion.

Hagedorn single-handedly and against significant and important opposition has opened up as a new field of physics the study of thermal properties of strongly interacting (nuclear) matter, in which we participate.

Hagedorn Temperature $T_{\rm H} = 160$ **MeV** **is today a'household brand'. Thermal equilibration in strongly interacting hadronic matter is an accepted research direction.**

Hagedorn has introduced methods in study of singular properties of equations of state which since have been adopted in 'more fundamental' fields which we will hear more about at the end of this meeting.

Personal Remarks

I first met Hagedorn (as he wanted to be called, not 'Rolf') in Winter 1975/76, when I attended one of his excellent Colloquium talks on the Statistical Bootstrap Model in Germany.

Even though I had a Ph.D. I missed all statistical and thermal physics classes, being a hard core 'nuclear' man. Thus after the lecture I asked him privately a few certainly quite ignorant questions, He took every matter very seriously, and followed up with very clear explanations. In our conversation he suggested I consider a short term position application. I arrived as fellow at CERN in September 1977.

Hagedorn was an extraordinary teacher in the many years of our very close collaboration.

Closing

In Summer and early Fall 2002 for the last time I had the privilage to work togather with Hagedorn. At that time, an annotated reprint volume *Quark-Gluon Plasma: Theoretical Foundations* was beeing readied. Rolf Hagedorn reviewed our selection of papers for the volume, agreed to the reprinting of his seminal work, and provided in writing very valuable remarks. This was very likely his last scientific activity before sickness had struck him down. As the vlume was about to go to Press in March 2003, we dedicated the "Foundations" to his memory.

Quark-Gluon Plasma: Theoretical Foundations

An Annotated Reprint Collection

Edited by:

J. Kapusta, University of Minnesota
B. Müller, Duke University
J. Rafelski, University of Arizona

ELSEVIER 2003 ISBN: 0-444-51110-5

Dedicated to Rolf Hagedorn